

# Market Transparency and Collusion under Imperfect Monitoring

Luke Garrod

School of Business and Economics, Loughborough University

Matthew Olczak

Aston Business School, Aston University

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### Contact Details:

Luke Garrod

[l.garrod@lboro.ac.uk](mailto:l.garrod@lboro.ac.uk)

# Market Transparency and Collusion under Imperfect Monitoring\*

Luke Garrod<sup>†</sup> and Matthew Olczak<sup>‡</sup>

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**JEL classification:** D43, D80, K21, L40

**Key words:** transparency, tacit collusion, imperfect information, price wars

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<sup>†</sup>Corresponding author, School of Business and Economics, Loughborough University, LE11 3TU, UK, email: l.garrod@lboro.ac.uk

<sup>‡</sup>Aston Business School, Aston University, Birmingham, B4 7ET, UK, email: m.olczak@aston.ac.uk

# 1 Introduction

In 1993, the Danish Competition Council began publishing transaction prices in the ready-mixed concrete industry to increase market transparency. It intended to enhance competition by making consumers more responsive to prices; instead, it facilitated collusion by limiting producers' ability to implement secret price cuts (Albæk *et al.*, 1997). Since then, theory has analysed how market transparency affects collusion to guide policy (e.g. OECD, 2001). The main focus has been on the effects via the consumer side of the market (e.g. Schultz, 2005). However, some work has also tried to understand the effects when transparency is imperfect on both the consumer and producer side (e.g. Møllgaard and Overgaard, 2003; Schultz, 2016).

The main approach of this literature to producer-side transparency has been to allow for imperfect monitoring of firms' actions off the equilibrium path (i.e. only deviations are detected imperfectly). Consequently, the effects of market transparency on collusion are analysed in a standard infinitely repeated game, where sufficiently patient firms can share the monopoly profit. However, this approach rules out equilibrium price wars that would occur if firms' actions on the equilibrium path were also imperfectly monitored. Thus, little is known of how collusive profits vary through the length and frequency of price wars when a change in transparency affects both sides of the market. This is an significant gap in the literature, because price wars are an important feature of collusion when firms can implement secret price cuts due to limited producer-side transparency (Tirole, 1988, p.262-264).

To fill this gap, we develop a general model of a simple imperfect monitoring setting, where price wars occur in equilibrium. This encompasses other simple models of imperfect monitoring (e.g. Tirole, 1988; Harrington and Skrzypacz, 2007, p.323-324) as well as the collusive models of market transparency mentioned above. One novel result is that, despite minimal structure on the competition game and despite the usual ambiguous effect on the critical discount factor, increased consumer-side transparency strictly lowers the best collusive profits. In contrast, and as standard, increased producer-side

transparency lowers the critical discount factor and raises the best collusive profits. Consequently, to analyse the counteracting consumer- and producer-side effects further, following Schultz (2005, 2016), we use Varian (1980) to place more structure on the competition game. Here, increasing transparency always strictly lowers the critical discount factor, but the effect on the best collusive profits depends upon the setting.

## 2 Imperfect Monitoring

Consider an infinitely repeated game with  $n \geq 2$  symmetric firms and discount factor  $\delta \in (0, 1)$ . In any period  $t$ , firms set prices simultaneously, where  $p_{it}$  is firm  $i$ 's price,  $\mathbf{p}_{-it}$  is the vector of firm  $i$ 's rivals' prices, and  $\mathbf{p}_t \equiv \{p_{it}, \mathbf{p}_{-it}\}$ . Suppose a symmetric static Nash equilibrium exists, where each firm's profits are  $\pi^N \geq 0$ . Firms want to collude on some  $p^c$  to receive expected per-period profits of  $\pi^c > \pi^N$ . However, each firm can deviate to some  $p^d < p^c$ , where the deviant's expected per-period profits are  $\pi^d > \pi^c$ . Firms never observe their rivals' prices. Instead, they monitor each other through a noisy public signal of  $\mathbf{p}_t$ , denoted  $y_t \in \{\underline{y}, \bar{y}\}$ , where:

$$\Pr(y_t = \underline{y} | p_{it}, \mathbf{p}_{-it}) = 1 - \Pr(y_t = \bar{y} | p_{it}, \mathbf{p}_{-it}) = \begin{cases} \alpha \in (0, 1] & \text{if } p_{it} = p \forall i \\ \beta \in (0, 1] & \text{otherwise} \end{cases}$$

Similar to Tirole (1988), we analyse a class of perfect public equilibrium (PPE) strategies commonly known as 'trigger strategies'. Formally, in any 'collusive' period  $t$ , each firm should set  $p^c$ . If  $y_t = \bar{y}$ , then period  $t + 1$  is a collusive period. If  $y_t = \underline{y}$ , then firms enter a 'punishment phase' in period  $t + 1$ . In a punishment phase, each firm should play the static Nash equilibrium for  $T$  periods, and then period  $t + T + 1$  is a collusive period. The sequence repeats.<sup>1</sup>

Let  $V^c$  and  $V^p$  denote each firm's expected (normalised) profit in a collusive period

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<sup>1</sup>While such strategies are not always optimal, they are optimal for each specific example in this paper where they generate the maximal PPE profits and the lowest critical discount factor. More details are provided in the appendix.

and at the start of a punishment phase, respectively, where:

$$\begin{aligned} V^c &= (1 - \delta) \pi^c + \delta [\alpha V^p + (1 - \alpha) V^c] \\ V^p &= (1 - \delta) \sum_{t=0}^{T-1} \delta^t \pi^N + \delta^T V^c \end{aligned}$$

Solving yields:

$$V^c(T) = \pi^N + \frac{(1 - \delta)}{1 - \delta + \alpha \delta (1 - \delta^T)} (\pi^c - \pi^N) \quad (1)$$

$$V^p(T) = \pi^N + \frac{(1 - \delta) \delta^T}{1 - \delta + \alpha \delta (1 - \delta^T)} (\pi^c - \pi^N),$$

where  $\pi^c > V^c(T) > V^p(T) \forall T > 0$  and  $V^p(T) > \pi^N \forall T < \infty$ .

The profile of trigger strategies is a PPE if, for each period  $t$  and any public history  $h^t = (y_0, y_1, \dots, y_{t-1})$ , the strategies yield a Nash equilibrium from  $t$  onwards. By definition each period of a punishment phase is a Nash equilibrium. So, consider a collusive period where each firm's incentive compatibility constraint is:

$$V^c(T) \geq (1 - \delta) \pi^d + \delta [\beta V^p(T) + (1 - \beta) V^c(T)] \quad (2)$$

Substituting in  $V^p(T)$  and  $V^c(T)$ , then rearranging yields:

$$\beta (\pi^c - \pi^N) - \alpha (\pi^d - \pi^N) - (\pi^d - \pi^c) \left( \frac{1 - \delta}{\delta} \right) \geq \delta^T [\beta (\pi^c - \pi^N) - \alpha (\pi^d - \pi^N)] \quad (3)$$

Consistent with Tirole (1988), (3) implies that a PPE in trigger strategies with  $V^c(T) > \pi^N$  must satisfy three conditions. First, the RHS of (3) must be non-negative:

$$\alpha \leq \beta \left( \frac{\pi^c - \pi^N}{\pi^d - \pi^N} \right) \equiv \alpha^* \in (0, \beta)$$

Second, the LHS of (3) must be non-negative, where for  $\alpha < \alpha^*$ :

$$\delta \geq \frac{\pi^d - \pi^c}{\pi^d - \pi^c + \beta (\pi^c - \pi^N) - \alpha (\pi^d - \pi^N)} \equiv \delta^* \in (0, 1),$$

Third,  $T \geq T^*$  where  $T = T^*$  ensures (3) holds with equality, so  $T^* \rightarrow \infty$  if  $\delta = \delta^*$  and

$T^* < \infty \forall \delta > \delta^*$ .

**Proposition 1.** For  $\pi^d > \pi^c > \pi^N$  and  $0 < \alpha < \alpha^* < \beta \leq 1$ , if  $\delta \geq \delta^*$ , then each firm's best PPE profits under trigger strategies are:

$$V^* = \pi^c - \frac{\alpha}{\beta - \alpha} (\pi^d - \pi^c) \in (\pi^N, \pi^c). \quad (4)$$

*Proof.* Given  $\frac{\partial V^c}{\partial T} < 0$ , it follows that  $V^* = V^c(T^*)$ . From (3):

$$1 - \delta^{T^*} = \frac{(1 - \delta) (\pi^d - \pi^c)}{\delta [\beta (\pi^c - \pi^N) - \alpha (\pi^d - \pi^N)]}.$$

Substituting into (1) yields (4), where  $V^* \in (\pi^N, \pi^c) \forall 0 < \alpha < \alpha^*$ . ■

In addition to Tirole (1988), our model encompasses Harrington and Skrzypacz (2007, p.323-324). More specifically, suppose  $b \geq 1$  discrete buyers each wish to purchase  $\frac{m}{b}$  units of a homogeneous product with a reservation price  $\bar{p}$ . In contrast to firms, buyers observe all prices. If  $p_i < p_j \forall j \neq i$ , then all buyers purchase from firm  $i$ . Otherwise, buyers randomly choose between the lowest-priced firms and their decisions are i.i.d. Thus, given marginal costs  $c = 0$ :

$$\pi^N = 0, \pi^c = \bar{p} \frac{m}{n}, \pi^d = \bar{p} m$$

Firms' realised sales are publicly observable. Consequently, if all buyers purchase from one firm, then rivals are uncertain whether this results from chance or a deviation. Thus,  $y_\tau = \underline{y}$  if all buyers purchase from one firm, and  $y_\tau = \bar{y}$  otherwise, so  $\alpha = (\frac{1}{n})^{b-1}$  and  $\beta = 1$ . Therefore:

$$V^* = \bar{p} \frac{m}{n} \left( \frac{1 - (\frac{1}{n})^{b-2}}{1 - (\frac{1}{n})^{b-1}} \right) \equiv \bar{V}(n, b), \quad \delta^* = \frac{n - 1}{n - (\frac{1}{n})^{b-2}} \equiv \bar{\delta}(n, b),$$

where  $\alpha < \alpha^* = \frac{1}{n} \forall b > 2$ .<sup>2</sup>

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<sup>2</sup>Harrington and Skrzypacz (2007) only consider  $T \rightarrow \infty$  and report  $\lim_{T \rightarrow \infty} V^c(T)$ , where  $\lim_{T \rightarrow \infty} V^c(T) < \bar{V}(n, b) \forall \delta > \bar{\delta}(n, b)$  since  $T^* < \infty$ .

### 3 Market Transparency

We now analyse how market transparency affects collusion via the consumer and producer side. Our contribution is to analyse  $\alpha > 0 \forall \beta \leq 1$ , where  $V^* < \pi^c$  due to equilibrium price wars. The analysis replicates Møllgaard and Overgaard (2003) as  $\alpha \rightarrow 0$ , where  $\lim_{\alpha \rightarrow 0} V^* = \pi^c$ , so in contrast to them we analyse the effects on both  $\delta^*$  and  $V^*$ .

#### 3.1 Consumer-side effects

Increased market transparency can make consumers more responsive to prices. We index this by  $\theta \in [0, 1]$ , where all consumers are unresponsive to prices when  $\theta = 0$  and all consumers are willing and able to buy at the cheapest price when  $\theta = 1$ . Following Møllgaard and Overgaard (2003), assume:

**Assumption 1.**  $\frac{\partial \pi^d}{\partial \theta} > 0$ ,  $\frac{\partial \pi^N}{\partial \theta} < 0$ ,  $\frac{\partial \pi^c}{\partial \theta} = 0 \forall 0 \leq \theta \leq 1$

Intuitively, when consumers are more responsive to prices, deviating attracts more demand and competition is more intense. Collusive profits are unaffected because all prices are equal.

**Proposition 2.** *For  $\pi^d > \pi^c > \pi^N$  and  $0 < \alpha < \alpha^* < \beta \leq 1$ , raising  $\theta$  strictly decreases  $V^*$  but has an ambiguous effect on  $\delta^*$ .*

*Proof.* Given  $\pi^d > \pi^c > \pi^N$ ,  $\beta > \alpha > 0$  and Assumption 1,  $\frac{\partial V^*}{\partial \theta} = -\left(\frac{\alpha}{\beta - \alpha}\right) \frac{\partial \pi^d}{\partial \theta} < 0$  and  $\text{sign}\left\{\frac{\partial \delta^*}{\partial \theta}\right\} = \text{sign}\left\{\frac{\partial \pi^d}{\partial \theta}(\pi^c - \pi^N) + \frac{\partial \pi^N}{\partial \theta}(\pi^d - \pi^c)\right\} \gtrless 0$ . ■

This new result on  $V^*$  supports the view that: “measures extending to consumers transparency which already exists among businesses should generally be pro-competitive” (OECD, 2001, p.9). It arises from three effects. First, raising  $\theta$  lowers  $\pi^N$ , which decreases  $V^c(T)$  and  $V^p(T) \forall T$ . Second, the net effect of this loosens (2). Consequently,  $T^*$  shortens, which increases  $V^c(T^*)$ . These two counteracting effects on  $V^c(T^*)$  perfectly offset each other, leaving only the following effect. Third, raising  $\theta$  increases  $\pi^d$ .

This increases the incentive to deviate, which tightens (2). Consequently,  $T^*$  lengthens, which reduces  $V^c(T^*)$ .

The ambiguous effect on  $\delta^*$  is consistent with Møllgaard and Overgaard (2003). Intuitively, as  $\theta$  rises, the short-term incentive to deviate increases but the punishment for  $T \rightarrow \infty$  is harsher.

### 3.2 Producer-side effects

Increased market transparency can make monitoring easier for firms. Here, this occurs if it reduces  $\alpha$  and/or raises  $\beta$ .

**Proposition 3.** *For  $\pi^d > \pi^c > \pi^N$  and  $0 < \alpha < \alpha^* < \beta \leq 1$ , reducing  $\alpha$  or raising  $\beta$  strictly increases  $V^*$  and strictly decreases  $\delta^*$ .*

*Proof.* Given  $\pi^d > \pi^c > \pi^N$  and  $\beta > \alpha > 0$ ,  $\frac{\partial V^*}{\partial \alpha} = -\frac{\beta(\pi^d - \pi^c)}{(\beta - \alpha)^2} < 0$ ,  $\frac{\partial V^*}{\partial \beta} = \frac{\alpha(\pi^d - \pi^c)}{(\beta - \alpha)^2} > 0$ ,  $\frac{\partial \delta^*}{\partial \alpha} = \frac{(\pi^d - \pi^c)(\pi^d - \pi^N)}{[\pi^d - \pi^c + \beta(\pi^c - \pi^N) - \alpha(\pi^d - \pi^N)]^2} > 0$  and  $\frac{\partial \delta^*}{\partial \beta} = -\frac{(\pi^d - \pi^c)(\pi^c - \pi^N)}{[\pi^d - \pi^c + \beta(\pi^c - \pi^N) - \alpha(\pi^d - \pi^N)]^2} < 0$ . ■

As standard, improving monitoring for firms facilitates collusion. Decreasing  $\alpha$  reduces the likelihood that firms will enter a punishment phase. This raises  $V^c(T)$  and  $V^p(T) \forall T$ , and the net effect loosens (2). Consequently,  $T^*$  shortens, which increases  $V^c(T^*)$  further. Increasing  $\beta$  raises the likelihood that deviations will be detected. This makes the punishment harsher, which loosens (2). Consequently,  $T^*$  shortens, which increases  $V^c(T^*)$ . Finally, as  $\alpha$  falls or  $\beta$  rises, the punishment for  $T \rightarrow \infty$  becomes relatively harsher than before, so  $\delta^*$  decreases.

### 3.3 Net effects

When increasing market transparency affects both the consumer side (by raising  $\theta$ ) and producer side (by reducing  $\alpha$  and/or raising  $\beta$ ), Proposition 2 and 3 imply that the sign of the net effect on  $V^*$  and on  $\delta^*$  is ambiguous.



## 4 Examples

To explore the counteracting consumer- and producer-side effects further, following Schultz (2005, 2016), we now use Varian (1980) to place more structure on the competition game. In this framework, if  $p_i < p_j \forall i \neq j$ , then firm  $i$  expects to supply  $\theta \in (0, 1)$  of the total  $m$  units and its expected share of the remaining  $m(1 - \theta)$  units that are distributed randomly among the firms. If  $p_i = p \forall i$ , then all units are randomly distributed among the firms. Thus, consistent with Assumption 1:

$$\pi^N = \bar{p} \frac{m}{n} (1 - \theta), \pi^c = \bar{p} \frac{m}{n}, \pi^d = \bar{p} m \left( \theta + (1 - \theta) \frac{1}{n} \right), \quad (5)$$

where  $\pi^N$  is derived from mixed strategies and is equivalent to the minimax payoff a firm receives when charging the reservation price  $\bar{p}$  and supplying its expected share of the  $m(1 - \theta)$  units. Using Proposition 1:

$$V^* = \bar{p} \frac{m}{n} \left( 1 - \theta \frac{\alpha(n-1)}{\beta - \alpha} \right), \quad \delta^* = \frac{n-1}{(1-\alpha)n - (1-\beta)}, \quad (6)$$

where  $\alpha < \alpha^* = \frac{\beta}{n}$ . Our contribution is to analyse  $\alpha > 0$ , and the analysis replicates Schultz (2005, 2016) for homogeneous goods when  $\alpha = 0, \beta \leq 1, n = 2$ .

The following regarding the consumer-side effects is immediate from (6).

**Corollary 1.** *For  $0 < \alpha < \frac{\beta}{n} < \beta \leq 1$ , raising  $\theta$  strictly decreases  $V^*$  and it has no effect on  $\delta^*$ .*

Under Varian (1980), the effects on  $\delta^*$  in Proposition 2 perfectly offset each other. Consequently, when increasing market transparency affects both the consumer side (by increasing  $\theta$ ) and producer side (by reducing  $\alpha$  and/or raising  $\beta$ ), Corollary 1 and Proposition 3 imply that  $\delta^*$  strictly decreases. Therefore, Schultz's (2005, 2016) result on  $\delta^*$  for homogeneous goods extends to  $\alpha > 0$ . However, the effects via both sides on  $V^*$  still cannot be signed  $\forall \alpha > 0$ , yet there are no such effects in Schultz (2005, 2016). Thus, he concludes that increasing market transparency in homogeneous goods markets

is anti-competitive, because firms can sustain  $\lim_{\alpha \rightarrow 0} V^* = \pi^c = \bar{p} \frac{m}{n}$  for lower  $\delta$ . Yet, when  $\alpha > 0$ , increasing market transparency can be pro-competitive, if it lowers  $V^*$ .

To understand how market transparency affects  $V^*$  in different settings, we now extend the Harrington and Skrzypacz (2007) example in section 2. We consider three new examples, which are described below. Each differs to the original in that the competition game is based on Varian (1980). Thus, (5) gives the expected per-period profits and (6) gives  $V^*$  and  $\delta^*$ , but now  $\alpha$  and/or  $\beta$  are functions of  $\theta$ . Each new example converges to the original as  $\theta \rightarrow 1$ .

Increasing market transparency is anti-competitive in Examples 1 and 3, because  $\delta^*$  strictly decreases and  $V^*$  (weakly) increases. Conversely, increasing market transparency is pro-competitive in Example 2 for  $\delta$  close to 1, because both  $\delta^*$  and  $V^*$  strictly decrease. Figure 1 illustrates the results for  $\bar{p} = 1$ ,  $n = 2$ ,  $b = m = 3$ .

### Example 1

Suppose  $\theta$  is the probability that all  $b \geq 1$  buyers know  $\mathbf{p}_t$  before they each purchase their desired  $\frac{m}{b}$  units, and  $1 - \theta$  is the probability that all buyers do not know  $\mathbf{p}_t$ . If buyers know  $\mathbf{p}_t$ , then they all choose between the firms as in the original example. If buyers do not know  $\mathbf{p}_t$ , then each buyer randomly chooses between the firms and their decisions are i.i.d. Firms never observe whether buyers know  $\mathbf{p}_t$ , but  $\theta$  is common knowledge. Consequently,  $y_\tau = \underline{y}$  if all buyers purchase from one firm, and  $y_\tau = \bar{y}$  otherwise, so  $\alpha = \left(\frac{1}{n}\right)^{b-1}$  and  $\beta = \theta + (1 - \theta) \left(\frac{1}{n}\right)^{b-1}$ . Therefore:

$$V^* = \bar{p} \frac{m}{n} \left( \frac{1 - \left(\frac{1}{n}\right)^{b-2}}{1 - \left(\frac{1}{n}\right)^{b-1}} \right) = \bar{V}(n, b), \quad \delta^* = \frac{n-1}{(n-1+\theta) \left(1 - \left(\frac{1}{n}\right)^{b-1}\right)} > \bar{\delta}(n, b),$$

where  $\alpha < \alpha^*$  if  $\theta > \theta^* \equiv \frac{(n-1)}{n^{b-1}-1}$  and  $\theta^* \in (0, 1) \forall b > 2$ . Thus,  $\frac{\partial V^*}{\partial \theta} = 0$ , so the effect of Proposition 2 perfectly offsets the effect of Proposition 3 through only  $\beta$ .

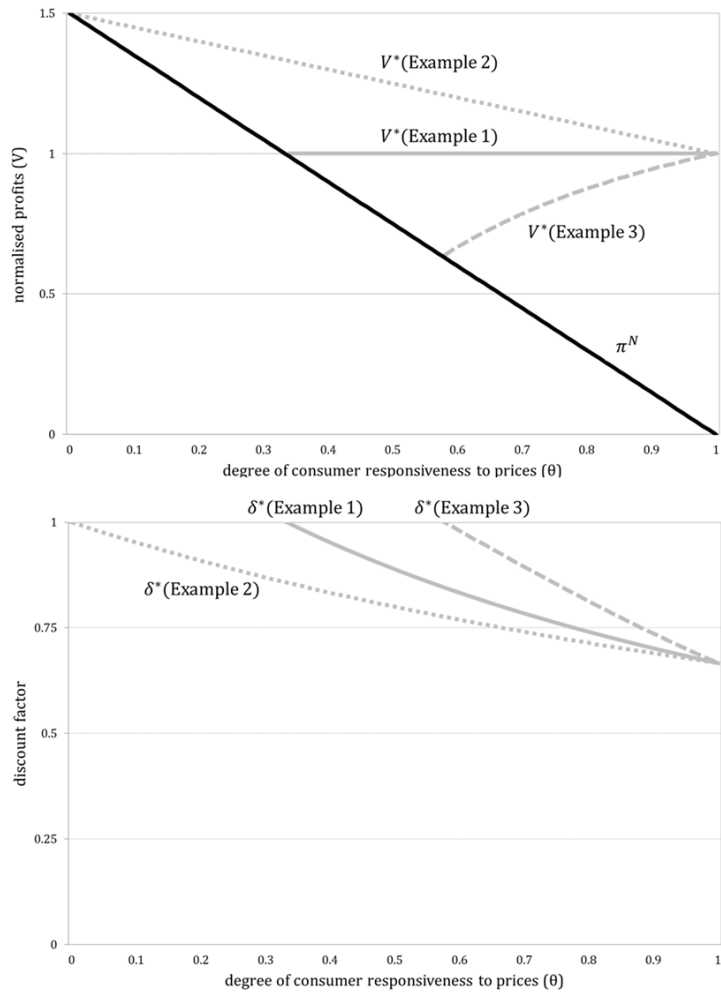


Figure 1:  $\bar{p} = 1, n = 2, b = m = 3$

### Example 2

Next, consider the setting of Example 1 except that now, after choosing prices in period  $t$ , firms observe whether buyers know  $\mathbf{p}_t$ . Here, if all buyers purchase from one firm *and* if buyers know  $\mathbf{p}_t$ , then rivals are uncertain whether this results from chance or a deviation. Consequently,  $y_\tau = \underline{y}$  if all buyers purchase from one firm and if buyers know  $\mathbf{p}_t$ , and  $y_\tau = \bar{y}$  otherwise, so  $\alpha = \theta \left(\frac{1}{n}\right)^{b-1}$  and  $\beta = \theta$ . Therefore:

$$V^* = \bar{p} \frac{m}{n} \left( 1 - \theta \frac{(n-1) \left(\frac{1}{n}\right)^{b-1}}{1 - \left(\frac{1}{n}\right)^{b-1}} \right) > \bar{V}(n, b), \quad \delta^* = \frac{n-1}{n-1 + \theta \left(1 - \left(\frac{1}{n}\right)^{b-2}\right)} > \bar{\delta}(n, b),$$

where  $\alpha < \alpha^* = \frac{\theta}{n} \forall b > 2$ . Thus,  $\frac{\partial V^*}{\partial \theta} = -\bar{p} \frac{m}{n} \left( \frac{(n-1) \left(\frac{1}{n}\right)^{b-1}}{1 - \left(\frac{1}{n}\right)^{b-1}} \right) < 0$ , where the effect of Proposition 2 dominates the effects of Proposition 3 through both  $\alpha$  and  $\beta$ .

### Example 3

Finally, suppose  $\theta$  is the probability that a single buyer knows  $\mathbf{p}_t$  before they purchase, and  $1-\theta$  is the probability that this buyer does not know  $\mathbf{p}_t$ . Each buyer chooses between the firms as in Example 1. For simplicity, suppose  $n = 2$  and  $b = 3$ . Consequently, if  $p_i < p_j \forall j \neq i$ , then the probability that all buyers purchase from one firm is  $(\theta + \left(\frac{1-\theta}{2}\right))^3 + \left(\frac{1-\theta}{2}\right)^3 = \frac{1}{4}(1 + 3\theta^2)$ ; however, if  $p_i = p^c \forall i$ , then this probability is  $2\left(\frac{1}{2}\right)^3 = \frac{1}{4}$ . Firms never observe whether buyers know  $\mathbf{p}_t$ , but  $\theta$  is common knowledge. Then,  $y_\tau = \underline{y}$  if all 3 buyers purchase from one firm, and  $y_\tau = \bar{y}$  otherwise, so  $\alpha = \frac{1}{4}$  and  $\beta = \frac{1}{4}(1 + 3\theta^2)$ . Therefore:

$$V^* = \bar{p} \frac{m}{2} \left( 1 - \frac{1}{3\theta} \right) < \bar{V}(2, 3), \quad \delta^* = \frac{4}{3(1 + \theta^2)} > \bar{\delta}(2, 3),$$

where  $\alpha < \alpha^*$  if  $\theta > \sqrt{\frac{1}{3}} > 0$ . Thus,  $\frac{\partial V^*}{\partial \theta} = \bar{p} \frac{m}{2} \left( \frac{1}{3\theta^2} \right) > 0$ , where the effect of Proposition 2 is dominated by the effect of Proposition 3 through only  $\beta$ .

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## Appendix: Optimal Perfect Public Equilibria

One possible limitation of the analysis in the main paper is that restricting attention to trigger strategies leaves open the question of whether there are other PPE with higher payoffs. Consequently, to check the robustness of our results, we now use the techniques of Abreu *et al.* (1986, 1990) to find the set of perfect public equilibria in the Varian (1980) framework. This analysis shows that trigger strategies are a strategy profile that supports the maximal PPE payoffs, and it also shows that trigger strategies generate the lowest critical discount factor. To do so, we make the common assumption that, after observing  $y_t$ , the firms observe the realisation of a publically observable randomisation device, which allows them to select among the continuation equilibria. This ensures that the set of PPE payoffs is convex. For a similar analysis to the below, see Athey and Bagwell (2001) and the online appendix of Garrod and Olczak (2016).

Following Abreu *et al.* (1986, 1990), we define an operator  $B(W)$  that, for any set  $W \subseteq \mathbf{R}$ , yields the set of PPE payoffs as the largest invariant set. The operator is defined as follows:

$$\begin{aligned} B(W) &\equiv \{V : \exists p \in [0, \bar{p}] \text{ and } V^p, V^c \in \text{co}(W), \text{ such that} \\ &V = (1 - \delta) \frac{m}{n} p + \delta [\alpha V^p + (1 - \alpha) V^c] \text{ and} \\ &V \geq (1 - \delta) \pi_i(p_i^*, p) + \delta [\beta V^p + (1 - \beta) V^c]\} \cup \pi^N, \end{aligned}$$

where  $\pi^N = \bar{p} \frac{m}{n} (1 - \theta)$  is the expected static Nash equilibrium profits, which may be used as an off-the-equilibrium path punishment. This operator decomposes play on the equilibrium path into a current period price,  $p$ , and continuation payoffs,  $V^p$  and  $V^c$ , that are drawn from the convex hull of the set  $W$ . The inequality is the ICC that ensures that no firm is able to gain by a (one-stage) deviation from the public strategy, where firm  $i$ 's optimal deviation profit from any price  $p \in [0, \bar{p}]$  is:

$$\pi_i(p_i^*, p) \equiv \begin{cases} pm \left( \theta + \frac{1-\theta}{n} \right) & \text{if } p \geq \bar{p} \left( \frac{1-\theta}{1+\theta(n-1)} \right) \\ \pi^N & \text{otherwise} \end{cases}$$

This says that firm  $i$ 's optimal deviation from  $p$  is to undercut  $p$  marginally, if  $p$  is sufficiently high, otherwise firm  $i$  should supply the residual demand at the reservation price. This implies that  $\pi_i(p_i^*, p)$  is (weakly) increasing in  $p$ .

Next, we establish that the operator  $B(W)$  maps compact sets to compact sets, which is the critical property for applying the techniques of Abreu *et al.* (1990).

**Lemma 1.** *For any given  $n \geq 2$  and  $0 \leq \theta \leq 1$ ,  $B(W)$  maps compact sets to compact sets.*

*Proof.* Notice that the feasible set of payoffs is real-valued, bounded and closed, e.g.  $[0, \frac{m}{n}]$ , hence it is compact. Then  $B(W)$  is bounded and closed, because the constraints entail weak inequalities and each component of the value function and the constraints is real-valued, continuous, and bounded. Thus,  $B(W)$  is compact. ■

Given the operator is compact, we can use the following algorithm to compute the set of PPE. Let  $W_0 = [0, \frac{m}{n}]$  such that it is compact and it contains all feasible payoffs, and let  $W_{z+1} = B(W_z)$ , for any  $z > 0$ . Then  $B(W)$  implies that  $B(W_z) = W_{z+1} \subseteq W_z$  such that we have a monotonic sequence. It follows from this, and the fact that  $W_z$  is non-empty (since  $\pi^N$  is in every  $W_z$ ), that  $W^* = \lim_{z \rightarrow \infty} W_z$  is a non-empty, compact set. Following the arguments in Abreu *et al.* (1990), we can conclude that  $W^*$  is the largest invariant set of  $B(W)$  and hence it is the set of PPE payoffs. Thus, it can be represented by the interval  $[\underline{V}, \bar{V}]$  and solving for this set reduces to the problem of finding the minimal  $\underline{V}$  and the maximal  $\bar{V}$  that satisfy  $[\underline{V}, \bar{V}] = B([\underline{V}, \bar{V}])$ .

The value of the minimal PPE payoff is  $\underline{V} = \pi^N$ . The reason is that this is equivalent to each firm's minimax payoff. Consequently, there exists no PPE with a payoff  $V < \pi^N$ , because firms could deviate by playing their minimax strategy in every period to receive  $\pi^N$ . Furthermore,  $V = \pi^N$  is a PPE payoff as this can be achieved by firms playing the static Nash equilibrium in every period. So, it remains to solve for the maximal PPE payoffs. Proposition 4 solves for  $\bar{V}$ .

**Proposition 4.** For  $0 < \alpha < \frac{\beta}{n} < \beta \leq 1$ , if  $\delta \geq \frac{n-1}{(1-\alpha)n-(1-\beta)} = \delta^*$ , then  $\bar{V} = \bar{p} \frac{m}{n} \left(1 - \theta \frac{\alpha(n-1)}{\beta-\alpha}\right)$ .

*Proof.* We find  $\bar{V}$  by solving the following constrained maximisation problem:

$$\bar{V} = \max_{p, V^p} V^c$$

subject to:

$$V^c = (1 - \delta) \frac{m}{n} p + \delta [\alpha V^p + (1 - \alpha) V^c] \quad (7)$$

$$V^c \geq (1 - \delta) \pi_i(p_i^*, p) + \delta [\beta V^p + (1 - \beta) V^c] \quad (8)$$

$$p \in [0, \bar{p}] \text{ and } V^p \in [\underline{V}, \bar{V}]$$

The first constraint (7) is just an identity that says that the target continuation payoff,  $V^c$ , can be decomposed into the profit of the stage game and a continuation payoff function, where if  $y$ , then each firm gets  $V^p$ , but if  $\bar{y}$ , they each get  $V^c$ . The second constraint (8) is the ICC for each firm.

The Lagrangian function for this constrained maximisation problem is:

$$L = V^c + \lambda^c \xi^c,$$

where  $\xi^c$  denotes the slack in the ICC, such that  $\xi^c = V^c - (1 - \delta) \pi_i(p_i^*, p) - \delta [\beta V^p + (1 - \beta) V^c]$ ,  $\lambda^c$  represents the Lagrange multiplier, and:

$$V^c = \frac{(1 - \delta) \frac{m}{n} p + \delta \alpha V^p}{1 - \delta(1 - \alpha)},$$

from rearranging (7). It is helpful to solve the constrained maximum for a given  $p \in [0, \bar{p}]$ , and then take this solution to its maximum with respect to  $p$ . Thus, the necessary Kuhn-Tucker conditions for a maximum are:

$$\frac{\partial L}{\partial V^p} = \frac{\partial V^c}{\partial V^p} + \lambda^c \left( \frac{\partial V^c}{\partial V^p} - \delta \left[ \beta + (1 - \beta) \frac{\partial V^c}{\partial V^p} \right] \right) = 0$$



$$\frac{\partial L}{\partial \lambda^c} = \xi^c \geq 0, \quad \lambda^c \geq 0, \quad \lambda^c \xi^c = 0.$$

These conditions are necessary and sufficient to determine a maximum, because the Lagrangian is concave.

We begin the solution by noting that the Kuhn-Tucker conditions are not satisfied if  $\xi^c > 0$  such that  $\lambda^c = 0$ , because then  $\frac{\partial L}{\partial V^p} > 0$ , which is a contradiction. So, consider the case of  $\xi^c = 0$  such that  $\lambda^c \geq 0$ . Rearranging  $\frac{\partial L}{\partial V^p} = 0$  yields  $\lambda^c = -\frac{\partial V^c}{\partial V^p} \left( \frac{1}{\frac{\partial V^c}{\partial V^p} - \delta[\beta + (1-\beta)\frac{\partial V^c}{\partial V^p}]} \right) > 0$  for any  $\beta > \alpha$ . Thus, the solution to the problem sets  $\xi^c = 0$ , where:

$$V^p = V^c - \frac{1-\delta}{\delta\beta} (\pi_i(p_i^*, p) - V^c).$$

Substituting the above into (7) yields  $V^c = \left( \frac{1}{\beta-\alpha} \right) (\beta \frac{m}{n} p - \alpha \pi_i(p_i^*, p))$ . Given this is strictly increasing in  $p$ , for all  $p \in [0, \bar{p}]$ , it follows that  $\bar{V}$  has  $p = \bar{p}$  such that  $\pi_i(p_i^*, p) = \bar{p}m \left( \theta + \frac{1-\theta}{n} \right)$  and  $\bar{V} = \bar{p} \frac{m}{n} \left( 1 - \theta \frac{\alpha(n-1)}{\beta-\alpha} \right)$ .

The above conditions imply that the strategy profiles that support the maximal PPE payoffs can be chosen in the following form: for  $t = 1$ , each firm sets  $p = 1$ ; for every  $t > 1$ , each firm sets  $p = 1$  if  $y_{t-1} = \bar{y}$ ; otherwise firms adopt any PPE strategy profile that supports  $V^p = \bar{V} - \frac{1-\delta}{\delta\beta} (\bar{p}m \left( \theta + \frac{1-\theta}{n} \right) - \bar{V})$ . Thus, it remains to construct such a PPE profile. One way to achieve this is to use the public randomisation device, where firms choose some probability  $\gamma \in [0, 1]$  such that  $V^p = (1-\gamma)\bar{V} + \gamma\underline{V}$ . Substituting in for  $V^p$ ,  $\bar{V}$  and  $\underline{V} = \pi^N$  yields:

$$\gamma = \frac{(1-\delta)(n-1)}{\delta[\beta - \alpha n]}, \quad (9)$$

where  $\gamma > 0$  if  $\alpha < \frac{\beta}{n} = \alpha^*$  and  $\gamma \leq 1$  if  $\delta \geq \frac{n-1}{(1-\alpha)n - (1-\beta)} = \delta^*$ . ■

This and Proposition 1 imply that the maximal PPE payoffs are the same as the optimal PPE payoffs under trigger strategies in the Varian (1980) framework,  $\bar{V} = V^*$ . Furthermore, the critical discount factor,  $\delta^*$ , and the necessary condition  $\alpha < \alpha^*$  are also the same.