THE QUALITY AND QUANTITY OF REGULATORY INFORMATION*

by

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Abstract

This paper explores the consequences of information acquisition by a regulator from a monopolist incumbent when the regulator either uses this information to regulate prices or to decide whether to licence a new entrant. If the incumbent is able to obfuscate some or all of the information it hands to the regulator, we find that the quantity of obfuscation depends on the total amount of information requested, and that the incumbent may obfuscate information in order to affect the beliefs of the regulator about the viability of a second firm.

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1 Introduction

This paper explores firms' reaction to requests for regulatory information when they are able to obfuscate the information provided, the implications for the requests which regulators make and for consequent auditing by regulators. In any regulatory relationship information is asymmetric between regulator and firm. The regulator invariably depends at least partially on information provided by the firm to determine and implement judgements which may not be in the private interests of the firm itself. This raises questions about how much information the regulator should request, whether the firm has incentives to obfuscate the information provided, and how much effort the regulator should apply to remove the obfuscation. These issues arise wherever regulators depend on the firms for information to make regulatory decisions in environmental regulation, health and safety as well as in competition policy, ranging from merger and monopoly investigations to sector-specific regulation.

We use as an example a regulated utility subject to a price cap which is determined, at least in part, by information provided by the regulated firm. Movements toward incentive regulation across the world have changed the nature of the information which regulators request from firms. The model of privatisation developed in the United Kingdom was predicated on a 'hands off' approach in which companies were left to maximise their profits within overall constraints on prices. In this model the regulator needed to verify only that prices were indeed within the caps set, a relatively straightforward process. But as regulation has developed in the UK, and emphasis on incentive regulation has increased in the US and elsewhere, the regulatory process and the relationship between regulator and regulatee has become closer and more complex. Companies are required to provide increasingly detailed information across a broad range of activities.[1] This falls into two broad categories: information which is required directly or indirectly to inform the next price review, and the level of prices or revenue which will be allowed; and information about quality of service, which may or may not feed directly into the price/revenues allowed. Both of these factors are becoming increasingly important in sectors where economies of scale and scope limit the potential for introducing competition, primarily transportation of goods and services along pipes, wires,

[1] Relationships between regulators and firms display varying degrees of frustration over provision of information, with particular difficulties between the (now defunct) gas regulator and the incumbent.

rails and through networks. We observe different regulatory approaches and company responses to the need for such information.

Price caps are generally set to allow sufficient revenue for the firm to recover the costs it would incur if it were efficient (where efficient is defined by the regulator's perception). When the companies were first privatised there was a general belief that they had become inefficient under public ownership, so most price constraints were based on an expectation of increased efficiency (though sometimes prices were allowed to rise to accommodate increased investment, as in the water and rail industries). The government effectively imposed a Vogelsang-Finsinger (1979) price mechanism, using the previous period's prices as an indication of the company's costs. This model was developed precisely to address the problems of information asymmetry in attempting to use the firm's costs to determine prices. In the event, the potential efficiencies were much greater than the initial price caps inferred (price caps were doubtless set at a lax level initially both to encourage the sale of shares and boost government revenue, and to allow for the riskiness of an unknown process). The consequent high profit levels lead to public outcry and a tightening of the caps at the first and subsequent review. Regulators introduced substantial one-off price cuts to reflect achieved efficiencies, and increasingly required detailed information not only about existing costs, but also potential reductions in these costs. But there are limits to how far the regulator wants to maintain downward pressure on costs. In the UK they have a statutory duty to ensure that companies have sufficient revenue to finance (efficient) operations. Companies have used concerns about safety to argue that their efficient 'safe' costs were higher than regulators estimated, and there are issues of short-term versus long term servicing of the network, a classic example of the difficulty of incentives in multi-tasking (Holstrom and Milgrom, 1991). 'Squeezing until the pips squeak' may involve running down the network to a point where rebuilding it is more expensive than maintaining it would have been, and is likely to involve considerably more degradation of service quality. There are thus both economic and political reasons for the regulator to avoid such situations and seek information to avert it.

To try and resolve some of these issues, the energy regulator, Ofgem, has recently introduced an Information and Incentives Project which links the revenue which regional electricity distribution companies can raise to measures of quality of service provided. Ofgem also proposes similar incentive schemes for gas distribution and for the high voltage electricity and high pressure gas transmission. The water regulator has a similar incentive scheme for the water and sewerage companies. But both the energy and water regulators are holding down the proportion of revenue 'at risk' from these schemes because of their doubts about the reliability of the information provided. All these schemes which link revenue directly with quality of service emphasise the increasing importance of information, require carefully defined processes for passing on information and verifying it, and underline the information asymmetry which lies at the heart of regulation.

Given these difficulties, how can the regulator deal with the information asymmetry, and what may the firm do to protect the advantage which such asymmetry bestows? This paper explores the incentives to request, audit, provide and misrepresent information by the interested parties using a simple abstract model of information transmission presented in section 2. The regulator can request signals on costs which, to be useful, have to be audited at some cost. The firm can decide to obfuscate some or all of the requested information, again at a cost. The basic model is solved in section 3, where the incentives to obfuscate are also studied.

The model is extended in section 4 to study the incentives for information acquisition and dissemination when the information may be used for other purposes. In particular, we explore any conflict between providing information for regulation and for assessment of the viability of introducing competition.[2] Regulators themselves might want information about costs of incumbent firms where there is potential competition, perhaps to decide whether or not to allow entry. Such a situation arose in UK telecommunications, where Mercury was allowed protected entry, and the regulator had to allow sufficient 'headroom' for the new company to develop. Thus consumers paid through higher prices in the short term for the privilege of competition. More recently, the energy regulator has published estimates of 'headroom' in the gas and electricity retail markets to inform potential entrants (Ofgem 2001). In section 4 we consider two cases. In the first, we simply look at how the regulator's expectation about the profit of the entrant is affected by the quality and quantity of information provided - if expected profits are positive, entry is allowed. In the second, we assume that the regulator is concerned to avoid making a mistake in one direction and hence will licence the entrant only if the probability of the entrant's failure is below some threshold.

[2] Alternatively a company's signal to the regulator that it is high cost may make it vulnerable to take-over if read and interpreted in the capital market.

In this case obfuscation may increase the probability of failure and hence lead to entry deterrence. Section 5 concludes and all proofs are shown in an appendix.

2 The Basic Model

We model the interaction between an incumbent firm and its regulator, where the latter aims to set a price cap to maximise consumer surplus based on the best possible estimate of the former's costs. Initially, neither firm nor regulator knows the true (constant) marginal costs of the firm, but both know the distribution from which it is drawn. The prior distribution of costs is assumed to be normal, $N(\bar{c}, \sigma_c^2)$.[3] The regulator can demand information from the firm, and the firm must provide the information; it cannot bias the information because such action would be detected and punished. However the incumbent can make the information less useful by introducing noise into the signals which it provides. Obfuscating in this way is costly for the firm.

The regulator, in turn, must assess any piece of information (called a signal) provided by the firm to extract the relevant bits, again at a cost. All information has to be audited on order for its accuracy to be assessed and for it to be useful to the regulator. Audited information which has been obfuscated is less accurate that audited information which has not been obfuscated. The regulator has no prior belief about how accurate each signal is supposed to be and hence cannot tell either before or after auditing whether a particular piece of information has been obfuscated. In particular, the regulator does not make any inference from the accuracy of the information. We will assume that a priori all pieces of information are equally informative and hence the firm has no incentives to obfuscate particular signals.

We consider the following order of moves. The regulator first decides on how much information to request. The firm then decides how many of the signals to obfuscate, and the regulator decides how many to audit. These two latter decisions occur simultaneously in the sense that they are independent and neither party knows the other’s decision when making their own.

The formal model is as follows. The firm’s marginal costs are initially unknown to the firm and the regulator, but it is common knowledge that they are normally distributed with mean

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[3] Thus c could have been obtained from an OLS regression. Note that the qualitative results do not depend on the assumption that the prior distribution is normal, see eg. Ericson (1967), Li (1985).

$\bar{c}$ and variance, σ_c^2. The regulator asks for n signals on the incumbent's costs. The ith signal, S_i, measures the true value of c with some error, ε_i, and is given by:

$$S_i = c + \varepsilon_i$$

where ε_i is assumed to have a normal distribution with zero mean $N(0, \sigma_i^2)$. We assume that these errors are uncorrelated across observations[4] and that there is no correlation between c and ε, possibly as the result of the auditing process. The quality of the signal is measured by the variance of ε_i so that the smaller is the variance of ε_i, the more precise and valuable is the signal.

For each signal requested by the regulator, the firm can either leave the signal alone or obfuscate it. The cost of obfuscation is c_f per signal. The firm decides to obfuscate m (m≤n) signals. This decision is made before the firm learns the true value of its costs, c. The regulator has to audit a signal in order to learn the value of σ_i^2 and chooses to audit[5] a sample of k (k≤n) signals at a cost c_r per signal. Those signals that are audited and have not been obfuscated have variance $\sigma_i^2 = \sigma_\varepsilon^2$, those that are audited and have been obfuscated have a larger variance, $\sigma_i^2 = \mu \cdot \sigma_\varepsilon^2$, where $\mu \geq 1$.[6] Finally, signals that are not audited have no informational value, signified by an infinite variance, $\sigma_i^2 = \infty$. The regulator thus requests a set of I signals (with n elements); a subset , I_o, of these with $k \cdot m/n$ elements are audited and obfuscated; a subset, I_c, with $k \cdot (n-m)/n$ elements are audited and unobfuscated. Finally a subset, $I \setminus I_o \cap I_c$, with n - k elements, are not audited.

Note that each signal is normally distributed with mean $\bar{c}$ and variance $V_i = \sigma_c^2 + \sigma_i^2$. We can summarise the *n* signals (or observations) by a single composite signal S given by[7]

$$S = \sum_{i=1}^{n} \delta_i \cdot S_i$$

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[4] While this is restrictive and will limit us in what we can do because, for example, we rule out any strategic thinking about the order in which you obfuscate and audit signals, it greatly simplifies the analysis.
[5] Auditing refers here to the general process by which the regulator verifies information, rather than the specific arrangements made for auditing by external bodies at the firms' expense, which are used in some UK regulated industries.
[6] An alternative is to assume that the auditing process is precise but that it costs more to audit obfuscated signals. This approach would require us to work with a regulatory budget for auditing services and is hence more cumbersome.
[7] The derivations are relegated to an appendix. S is a sufficient statistic for the n signals.

where $\delta_i = \frac{1}{\sigma_i^2} \Big/ \sum_{k=1}^{n} \frac{1}{\sigma_k^2}$; i = 1, ...,n. Given our assumptions, we can write δ_i as:

$$\delta_i = \begin{cases} \frac{n}{\mu \cdot n - (\mu - 1) \cdot m} \cdot \frac{1}{k} & \text{if} \quad i \in I_o \\ \frac{\mu \cdot n}{\mu \cdot n - (\mu - 1) \cdot m} \cdot \frac{1}{k} & \text{if} \quad i \in I_c \\ 0 & \text{if} \quad i \in I \setminus I_o \cap I_c \end{cases}$$

Note that S is normal with mean $\bar{c}$ and variance

$$\sigma_S^2 = \sigma_c^2 + \frac{\mu \cdot n}{\mu \cdot n - (\mu - 1) \cdot m} \cdot \frac{1}{k} \cdot \sigma_\varepsilon^2 .$$

Note that in the special case where obfuscation has no effect, $\mu = 1$, this reduces to $\sigma_S^2 = \sigma_c^2 + \frac{1}{k} \cdot \sigma_\varepsilon^2$, as it does in the case where no signal is obfuscated, m = 0. Finally, if all signals are obfuscated, m = n, the variance expression reduces to $\sigma_S^2 = \sigma_c^2 + \frac{1}{k} \cdot \mu \cdot \sigma_\varepsilon^2$.

2.1 Posterior beliefs

Based on the available signals, the regulator can update its beliefs about the costs, *c*. Given our assumption, both the prior distribution and the signal on c are normal and hence the posterior $c|S$ is normal $N(\bar{c} + t(S - \bar{c}), (1-t)\sigma_c^2)$, where

$$t(n, m, k) = \frac{\sigma_c^2}{\sigma_c^2 + \frac{\mu \cdot n}{\mu \cdot n - (\mu - 1) \cdot m} \cdot \frac{1}{k} \cdot \sigma_\varepsilon^2}$$

Note that this implies that the conditional expectation of costs, given the signal, is a linear combination of the prior expectations and the new information.[8]

$$E(c|S) = \bar{c} + t(S - \bar{c})$$

Note that t measures the quality of the signal S, such that the closer is t to 1, the better is the signal.[9] Consider the two extreme cases. Firstly, if the signal is perfect, $\sigma_\varepsilon^2 = 0$, in which

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[8] The linear updating rule of the prior expected value, $E(c|S) = \bar{c} + t(S - \bar{c})$, is more general and, for example, also holds when the prior is beta distributed and the signal is binomial and when the prior is gamma distributed and the signal is Poisson. See Ericson (1967), DeGroot (1982) or Li (1985).

[9] Allowing for correlation would not add very much. If we assumed that $\text{cov}(c, \varepsilon) = \rho \cdot \sigma_c \cdot \sigma_\varepsilon$, the precision of the signal becomes $t = (\sigma_c^2 + \rho \cdot \sigma_c \cdot \sigma_\varepsilon)/(\sigma_c^2 + \sigma_\varepsilon^2 + 2\rho \cdot \sigma_c \cdot \sigma_\varepsilon)$. Note that you only want to reduce the correlation ρ if $\sigma_c > \sigma_\varepsilon$, i.e. if the signal is already reasonable informative.

case the regulator should rely entirely on the signal, t should be unity. This is indeed the case. Secondly, if the signal is useless, $\sigma_\varepsilon^2 = \infty$, in which case it should not be used, t should be zero. Again this is the case. More generally, the more useful observations are included in the composite signal, S, the more precise is the signal, the higher is t, and the more reliance is placed on S in the construction of the regulator's conditional expectation.

Because of the importance of t, we present some of the comparative statics in lemma 1 below:

Lemma 1. $t(n, m, k)$ is increasing and concave in *k* and *n* and decreasing and concave in *m*.

$$\frac{\partial t}{\partial n} > 0;\ \frac{\partial t}{\partial m} < 0;\ \frac{\partial t}{\partial k} > 0;$$

$$\frac{\partial^2 t}{\partial n^2} < 0\ ;\frac{\partial^2 t}{\partial m^2} < 0;\ \frac{\partial^2 t}{\partial k^2} < 0;$$

$$\frac{\partial^2 t}{\partial m \partial n} > 0;\ \frac{\partial^2 t}{\partial m \partial k} \leq (\geq) 0 \text{ if } t \leq (\geq) \frac{1}{2};\ \frac{\partial^2 t}{\partial k \partial n} \geq (\leq) 0 \text{ if } t \leq (\geq) \frac{1}{2}.$$

Proof: See appendix.

3 Regulator and Incumbent Firm

Assume that the regulator requests information (perhaps through regulatory accounts), which gives rise to a composite signal S as described in section 2 above. Thus in the first stage, the regulator sets n, the number of observations requested. In the second stage, having observed n, the firm and regulator simultaneously set m and k respectively. Finally, based on the updated beliefs about the marginal costs of the firm, the regulator imposes a simple price rule on the incumbent:[10]

$$P_I^R = \lambda \cdot E(c|S) = \lambda \cdot (\bar{c} + t(S - \bar{c}))$$

where λ >1 represents a mark-up which the regulator allows the firm on expected costs. Demand for the good produced by the incumbent is given by $Q = a - b \cdot P$.

[10] We ignore that fact that there exist both realisations of S such that the incumbent would refuse to produce and levels such that the incumbent would choose a lower (monopoly) price. λ could be set such that the probability of observing an S such that either case occurs is less than α%.

To solve the second stage, we compute the best replies of the firm and regulator. Given the pricing rule, the expected profits of the incumbent is:

$$E[\Pi(m,k)] = E[(a - b\cdot\lambda\cdot E(c|S))(\lambda\cdot E(c|S) - c)] - m\cdot c_f$$
$$= E[(a - b\cdot\lambda\cdot(\bar{c} + t(S-\bar{c})))\cdot(\lambda\cdot(\bar{c} + t(S-\bar{c})) - c)] - m\cdot c_f$$

Since $E\left[(S-\bar{c})^2\right] = \frac{1}{t}\cdot\sigma_c^2$ and $E[(S-\bar{c})\cdot(C-\bar{c})] = \sigma_c^2$ this reduces to

$$E[\Pi(m,k)] = (a - b\cdot\lambda\cdot\bar{c})\cdot(\lambda-1)\cdot\bar{c} - b\cdot\lambda\cdot t(n,m,k)\cdot(\lambda-1)\cdot\sigma_c^2 - m\cdot c_f$$

For $\lambda > 1$, expected profits depends on t and for $\mu > 1$, t depends on m. From lemma 1 we know that t is concave in m and hence the expected profit of the incumbent firm is convex in m, the number of signals which it obfuscates. The implication is that the incumbent chooses an either-or strategy. Either it obfuscate all the data requested by the regulator, m = n, or it obfuscates none, m = 0.

Which of the two strategies is chosen depends on the values of c_f, n and k. Note that m = n if

$$b\cdot\lambda\cdot(\lambda-1)\cdot\sigma_c^2\cdot[t(n,0,k) - t(n,n,k)] > n\cdot c_f$$

which we can rewrite as

$$\Psi(k,n) \equiv b\cdot\lambda\cdot(\lambda-1)\cdot(\mu-1)\cdot\frac{k\cdot\sigma_c^4\cdot\sigma_\varepsilon^2}{(k\cdot\sigma_c^2+\sigma_\varepsilon^2)(k\cdot\sigma_c^2+\mu\cdot\sigma_\varepsilon^2)} - n\cdot c_f \geq 0$$

Note that

$$\frac{\partial\Psi(k,n)}{\partial k} = -\frac{2\cdot b\cdot\lambda\cdot(\lambda-1)\cdot(\mu-1)\cdot\sigma_c^2}{(k\cdot\sigma_c^2+\sigma_\varepsilon^2)(k\cdot\sigma_c^2+\mu\cdot\sigma_\varepsilon^2)}\cdot\left(t(n,n,k) - \frac{1}{2}\right)$$

and $\Psi(0,n) = -n\cdot c_f < 0$. Hence a necessary but not sufficient condition for m = n to be a best reply for some k is t(n,n,k) < 1/2, that is the information has to be relatively uninformative. Intuitively, if the information is already very precise, less is gained from obfuscation. If t(n,n,k) < 1/2 there may exist a $\hat{k} < n$ solving $\Psi(\hat{k},n) = 0$ such that for $k \leq \hat{k}$, m = 0 is a best reply, while for $k \geq \hat{k}$, m = n is a best reply. In addition to this, we require that n and the cost of obfuscation, c_f, be small. We summarise this in lemma 2.

Lemma 2: The incumbent monopolist chooses either to obfuscate none or all of the signals for which it is asked. In particular,

i. If $t(n,n,k) \geq 1/2$, m = 0 is the dominant strategy.

ii. If $t(n,n,k) \geq 1/2$ and $\hat{k} > n$ where $\hat{k}$ solves $\Psi(\hat{k}, n) = 0$, m = 0 is a dominant strategy.

iii. If $t(n,n,k) < 1/2$ and $\hat{k} \leq n$, m = 0 is a best reply for $k \leq \hat{k}$ and m = n is a best reply for $k \geq \hat{k}$.

Obfuscation is more likely to be a best reply if the regulator asks for a small sample, the cost of obfuscation is small, obfuscation has a large impact in the variance of the signal and if the allowed mark up is high.

Turning to the regulator, expected consumer surplus, given the information contained in S, can be written as:

$$E[CS(k,m)] = E\left[\frac{1}{2b}\left(a - b \cdot \lambda \cdot E(c|S)\right)^2\right] = \frac{1}{2b} E\left[\left(a - b \cdot \lambda \cdot \bar{c} - b \cdot \lambda \cdot t \cdot (S - \bar{c})\right)^2\right]$$
$$= \frac{1}{2b}\left(a - b \cdot \lambda \cdot \bar{c}\right)^2 + \frac{b}{2} \cdot \lambda^2 \cdot t \cdot \sigma_c^2$$

so that consumer surplus is increasing and concave in n, the number of observations, and in k, the amount of information audited. Assuming that the regulator maximises expected consumer surplus net of auditing costs,

$$E[CS(k,m)] = \frac{1}{2b}\left(a - b \cdot \lambda \cdot \bar{c}\right)^2 + \frac{b}{2} \cdot \lambda^2 \cdot t(n,m,k) \cdot \sigma_c^2 - k \cdot c_r$$

which, from lemma 1, is concave in k. The best reply for the regulator, given n, is found by maximising this with respect to k. The first order condition is

$$\frac{b}{2} \cdot \lambda^2 \cdot t(1-t) \cdot \frac{1}{k} \cdot \sigma_c^2 - c_r = 0$$

Defining $\varphi(m) \equiv \frac{\mu \cdot n}{\mu \cdot n - (\mu - 1) \cdot m}$, the best reply is given by

$$k^*(m) = \sqrt{\varphi(m)} \cdot \sqrt{\frac{b}{2c_r} \cdot \lambda^2 \cdot \sigma_\varepsilon^2} - \varphi(m) \cdot \frac{\sigma_\varepsilon^2}{\sigma_c^2}$$

Differentiating $k^*(m)$ with respect to m yield the following:

Lemma 3:

i. For $\sqrt{\frac{b}{2c_r}\cdot\lambda^2\cdot\sigma_\varepsilon^2} < 2\cdot\frac{\sigma_\varepsilon^2}{\sigma_c^2}$, $\frac{\partial k^*(m)}{\partial k} < 0$ for all m.

ii. For $2\cdot\frac{\sigma_\varepsilon^2}{\sigma_c^2} < \sqrt{\frac{b}{2c_r}\cdot\lambda^2\cdot\sigma_\varepsilon^2} < \left(1+\sqrt{\mu}\right)\cdot\frac{\sigma_\varepsilon^2}{\sigma_c^2}$, $\frac{\partial k^*(m)}{\partial k} < (>) 0$ for $m > (<) \hat{m}$ where $0 < \hat{m} < m$ <u>and</u> $k^*(0) > k^*(n)$.

iii. For $\left(1+\sqrt{\mu}\right)\cdot\frac{\sigma_\varepsilon^2}{\sigma_c^2} < \sqrt{\frac{b}{2c_r}\cdot\lambda^2\cdot\sigma_\varepsilon^2} < 2\cdot\sqrt{\mu}\cdot\frac{\sigma_\varepsilon^2}{\sigma_c^2}$, $\frac{\partial k^*(m)}{\partial k} < (>) 0$ for $m > (<) \hat{m}$ where $0 < \hat{m} < m$ <u>and</u> $k^*(0) < k^*(n)$.

iv. For $2\cdot\sqrt{\mu}\cdot\frac{\sigma_\varepsilon^2}{\sigma_c^2} < \sqrt{\frac{b}{2c_r}\cdot\lambda^2\cdot\sigma_\varepsilon^2}$, $\frac{\partial k^*(m)}{\partial k} > 0$ for all m.

Thus only if $\sqrt{\frac{b}{2c_r}\cdot\lambda^2\cdot\sigma_\varepsilon^2}$ is small relative to $\frac{\sigma_\varepsilon^2}{\sigma_c^2}$ does more obfuscation lead to more auditing.

Combining lemma 2 and 3, we can show that

Proposition 1. If m = 0 is not a dominant strategy, equilibria where m* = n and k = k*(n) may exist. In particular,

i. for $\sqrt{\frac{b}{2c_r}\cdot\lambda^2\cdot\sigma_\varepsilon^2} < \left(1+\sqrt{\mu}\right)\cdot\frac{\sigma_\varepsilon^2}{\sigma_c^2}$, the equilibrium is unique, but may be in either pure or mixed strategies.

ii. for $\left(1+\sqrt{\mu}\right)\cdot\frac{\sigma_\varepsilon^2}{\sigma_c^2} < \sqrt{\frac{b}{2c_r}\cdot\lambda^2\cdot\sigma_\varepsilon^2}$, the equilibrium need not be unique, but is always in pure strategies.

The results are illustrated in the figures below, where the solid line is the best reply of the firm and the dashed lines show different possible best replies of the regulator. Figures 1 and 2 illustrate part (i) and figures 3 and 4 illustrate part (ii).

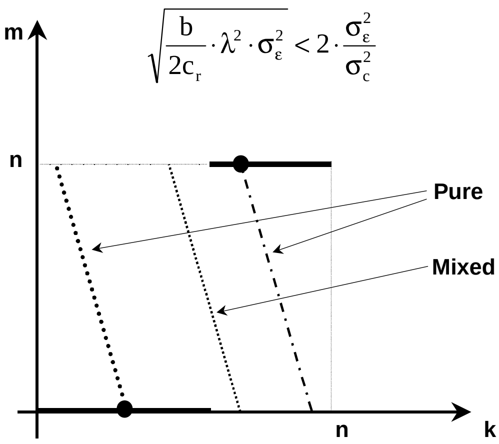

Figure 1: k(m) monotone decreasing.

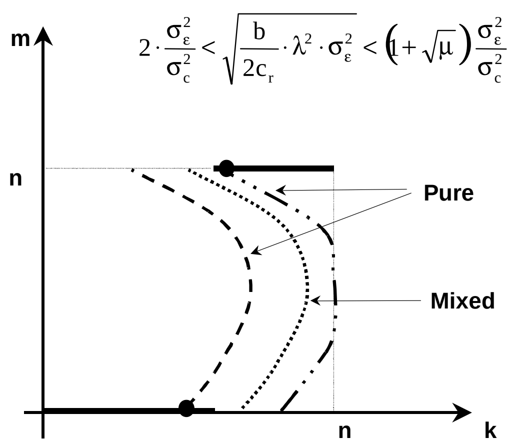

Figure 2: k(m) non-monotone; k(0)>k(n).

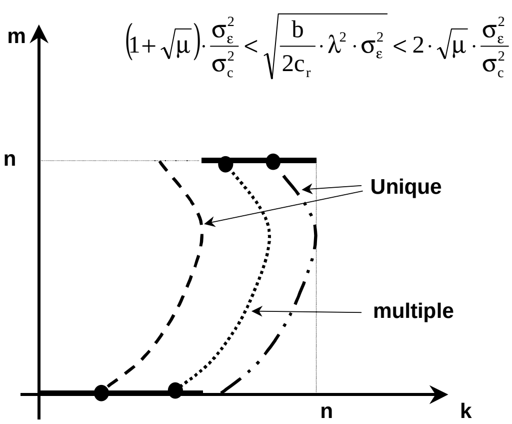

Figure 3: k(m) non-monotone; k(0)<k(n).

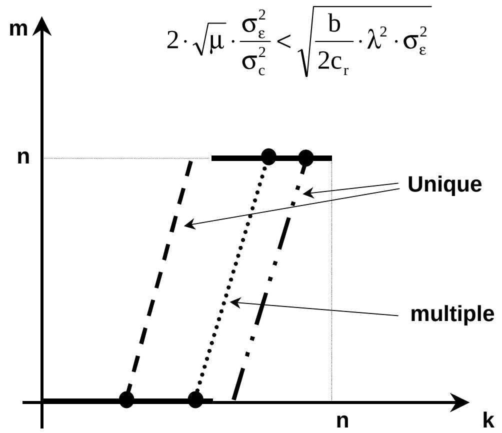

Figure 4: k(m) monotone increasing.

Although in most cases the firm does not obfuscate, it is interesting to see that for some n, equilibria exist in which the firm obfuscates all the information which it is asked for and where the regulator audits only a subsample of the information requested.

This raises the question of whether the regulator always sets n such that m = 0 is a dominant strategy. Welfare is higher at m = n if:

$$\frac{b}{2}\cdot\lambda^2\cdot t\left(n,n,k^*(n)\right)\cdot\sigma_c^2 - k^*(n)\cdot c_r > \frac{b}{2}\cdot\lambda^2\cdot t\left(n,0,k^*(0)\right)\cdot\sigma_c^2 - k^*(0)\cdot c_r$$

which reduces to:

$$\frac{b}{2}\cdot\lambda^2\cdot\sigma_c^2\cdot\left(t\left(n,n,k^*(n)\right) - t\left(n,0,k^*(0)\right)\right) > \left(k^*(n) - k^*(0)\right)\cdot c_r$$

and further to:

$$\left(\sqrt{\mu}+1\right)\cdot\sqrt{\sigma_{\varepsilon}^{2}} > \sqrt{c_{r}}\cdot\sqrt{\frac{b}{2}}\cdot\lambda\cdot\left(1+\sigma_{c}^{2}\right)$$

Thus there are parameter values such that the regulator would prefer to set n so low that the firm obfuscates all signals. In other words, even though the regulator could ensure that m = 0 by setting n high enough, the regulator may choose not to do so.

4 Licensing an entrant

The regulator might also use the information about the incumbent's costs to decide whether or not to licence an entrant. We consider two different licensing rules. In section 4.1 we assume that licensing depends on whether or not the entrant's expected profit is non-negative. In section 4.2 we consider that entry is allowed if the probability that the entrant fails to make a profit is below a threshold θ.

The entrant has known marginal costs c_E. While the incumbent knows its costs, the entrant only observes the signal from the regulatory accounts, *S*.[11] Assume that we are looking at a differentiated goods Bertrand case, with demand for good i given by

$$q_i = \alpha - \beta\cdot P_i + \gamma\cdot P_j$$

The first order condition when the entrant observes *S* is:

(1) $$\alpha - 2\beta\cdot P_E + \gamma\cdot E\left(P_I \middle| S\right) - c_E = 0$$

The entrant expects the best reply of the incumbent to be:

(2) $$\alpha - 2\beta\cdot E\left(P_I \middle| S\right) + \gamma\cdot P_E - E\left(c \middle| S\right) = 0$$

Solving (1) and (2) yields:

$$P_E^* = \frac{2\beta+\gamma}{4\beta^2-\gamma^2}\cdot\alpha + \frac{2\beta^2}{4\beta^2-\gamma^2}\cdot c_E + \frac{\beta\gamma}{4\beta^2-\gamma^2}\cdot E\left(c \middle| S\right)$$

$$P_I^* = \frac{2\beta+\gamma}{4\beta^2-\gamma^2}\cdot\alpha + \frac{1}{2}\cdot c + \frac{\gamma\beta}{4\beta^2-\gamma^2}\cdot c_E + \frac{1}{2}\cdot\frac{\gamma^2}{4\beta^2-\gamma^2}\cdot E\left(c \middle| S\right)$$

Note that, as $E\left(P_E^*\right) = \frac{(2\beta+\gamma)\cdot\alpha + 2\beta^2\cdot c_E + \beta\gamma\cdot\bar{c}}{4\beta^2-\gamma^2}$ and $E\left(P_I^*\right) = \frac{(2\beta+\gamma)\cdot\alpha + 2\beta^2\cdot\bar{c} + \beta\gamma\cdot c_E}{4\beta^2-\gamma^2}$,

we can write these as

$$P_E^* = E(P_E^*) + \frac{\beta\gamma}{4\beta^2 - \gamma^2} \cdot t \cdot (S - \bar{c})$$

$$P_I^* = E(P_I^*) + \frac{1}{2} \cdot (c - \bar{c}) + \frac{1}{2} \cdot \frac{\gamma^2}{4\beta^2 - \gamma^2} \cdot t \cdot (S - \bar{c})$$

In the appendix we show:

Lemma 4: The expected profits of the incumbent and entrant when the entrant observes the information requested by the regulator are given by:

$$E[\Pi_I] = \beta \cdot \left(E(P_I^*) - \bar{c}\right)^2 + \frac{\beta}{4} \cdot \left(1 - \frac{(8\beta^2 - 3\gamma^2)\gamma^2}{(4\beta^2 - \gamma^2)^2} \cdot t\right) \cdot \sigma_c^2$$

$$E[\Pi_E] = \beta \cdot \left(E(P_E^*) - c_E\right)^2 + \frac{2\beta^3\gamma}{(4\beta^2 - \gamma^2)^2} \cdot \frac{\gamma}{2} \cdot t \cdot \sigma_C^2$$

4.1 The entrant's expected profit

Note that for both firms, the second term is positive. Additionally, while the profit of the incumbent is decreasing in the precision of the composite signal, *S*, the profit of the entrant is increasing. Hence there are two reasons why the incumbent would be willing to obfuscate in this case. Firstly, if entry occurs, the bigger is *m*, the smaller is *t* and the higher is expected profit. Secondly, there exists a range of parameter values such that by obfuscating m signals, t is such that the expected profits of the entrant net of entry costs, F_E, is negative. Thus obfuscation may deter entry. If the entry test is solely based on expected profits, then the incumbent may use obfuscation to deter entry so that the regulator is unwilling to grant a licence.

The regulator has an incentive to make its information available to a potential entrant when this makes entry more likely. However, if entry does occur, the more precise the information, the higher will be the prices of both the incumbent and entrant. The overall effect on consumer surplus and hence the welfare of the regulator is thus not clear.

[11] The implicit assumption is that the regulator discloses such information.

4.2 The entrant's probability of failure

An alternative story, with similar implications, is this. The regulator focuses not on the entrant's expected profit *per se*, but is concerned if the probability of the entrant making negative profits is above a critical value, because of the likelihood of bankruptcy. The regulator's aversion to the entrant's failure could arise either from adverse publicity or from the direct costs of bailing out or compensating a failed entrant. To examine this case we analyse the following sequence of events. An entrant approaches the regulator for a licence; the regulator demands information from the incumbent firm to which the incumbent responds with information which may or may not have been obfuscated.

For the entrant to be profitable, the incumbent must be sufficiently "inefficient" relative to the entrant. Entry failure is defined by an ad-hoc rule that the entrant's marginal cost exceeds that of the entrant by a particular amount, i.e. $c_E - c \geq \xi$.[12] Given the distribution of the signal *S*, the regulator can compute the probability that the true value of the incumbent's costs *c* is below the threshold, given the observed realisation of the signal. The regulator only allows entry if such probability of failure is small enough. That is, if

$$\Pr(c \leq c_E - \xi | S, t) \leq \theta^{13} \qquad \textit{(licensing rule)}$$

As $\frac{c - \bar{c} - t(S - \bar{c})}{\sigma_c^2(1-t)} \sim N(0,1)$, then $\Pr\left(c \leq c_E - \xi | S, t\right) = F\left(\frac{c_E - \xi - \bar{c} - t(S - \bar{c})}{\sigma_c^2(1-t)}\right)$, where *F* is the normal distribution function. The question is therefore how does *t*, the precision of information, affect the likelihood of entry once the regulator is informed about *S* (ex post decision)[14]. We obtain

$$\frac{\partial \Pr\left(c \leq c_E - \xi | S, t\right)}{\partial t} = -\frac{F'}{(1-t)} \cdot \left(S - \left(c_E - \xi\right)\right)$$

Note that the sign of this derivative depends on the sign of $S - (c_E - \xi)$. If $c_E - \xi \geq (\leq) S$, then, $\frac{\partial \Pr\left(c \leq c_E - \xi | S, t\right)}{\partial t} \geq (\leq) 0$.

[12] where ξ is chosen by the regulator but a plausible ξ would be for example the maximum difference between costs that would allow profitable entry when costs were known.

[13] Where α is also chosen by the regulator. We assume that θ is smaller that ½.

[14] One must distinguish the ex post decision undertaken by the regulator from the ex ante decision taken by the incumbent, who must choose *t* taking into consideration how the precision of information will affect the regulators licensing decision via *t* and via the resulting *S*.

If the composite signal is small relative to the threshold, the regulator has bad news on the success of entry. In this case it will only allow entry if it is convinced that this news is not very informative, i.e. whenever *t* is small. Alternatively if the signal on the incumbent's marginal cost is large, the regulator is more likely to license the entrant if the precision of information is good. In the latter case, the regulator licenses the entrant thanks to the good information. In the former case the regulator licenses the entrant despite the fact that the incumbent looks efficient because it believes that the information is not good.

Note that the size of *S* drives the ex post incentives of the incumbent with respect to *t* in different directions. If the aim of the incumbent is to prevent entry, a low *S* will imply that it favours a large *t* and wished it had not obfuscated. On the contrary, a small *S* results in the incumbent wishing it had obfuscated. However, the decision on how many signals to obfuscate is taken ex ante, before the realisation of *S*. In influencing *t*, the incumbent will take into consideration not only how the precision of information will affect the likelihood that the signal *S* exceeds or not the threshold, but also how it will indirectly affect the threshold *t**. The incentives for obfuscation will depend on α, the threshold $c_E - \xi$ and the ex ante distribution of incumbent's costs. We will now check these ex ante incentives.

We can rewrite the regulator's choice to allow entry as: *license whenever* $c_E - \xi - \bar{c} - \sigma_c^2 \cdot F^{-1}(\theta) \leq t \cdot (S - \bar{c} - \sigma_c^2 \cdot F^{-1}(\theta))$.

Define $t^* = \frac{A}{B} = \frac{c_E - \xi - \bar{c} - \sigma_c^2 \cdot F^{-1}(\theta)}{(S - \bar{c} - \sigma_c^2 \cdot F^{-1}(\theta))}$ and consider four possible cases.

(i) If A > 0 and B > 0, the regulator licences when t* < t.

(ii) If A > 0 and B < 0, t* < 0; since the regulator licence only when t* > t, licences are never granted.

(iii) If A < 0 and B > 0, the regulator licences when t* > t.

(iv) If A < 0 and B < 0, t* < 0; since the regulator licence when t* < t, licences are granted for all t.

Note that if $c_E - \xi \geq S$, then $|A/B| > 1$ and, in consequence, $|t^*| > 1$. In this case if A>0 and B>0 the regulator will not license, and if A<0 and B<0 the regulator will license.

From this comparison two cases emerge. If the entrant is inefficient, that is $c_E > \xi + \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta)$, we obtain the following situation:

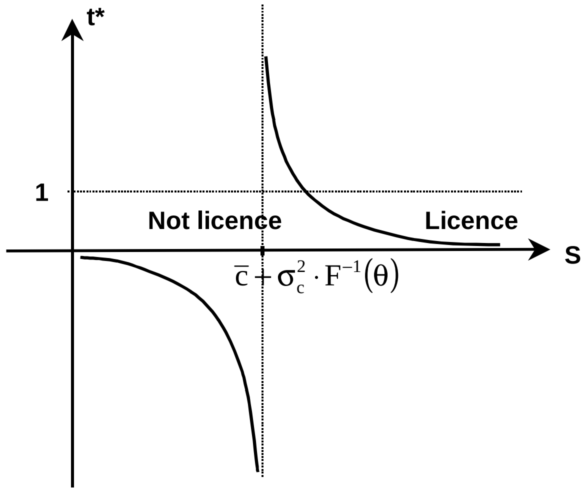

Figure 5: The licensing decision with an inefficient entrant.

If the entrant is efficient, that is $c_E < \xi + \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta)$, we obtain the following situation:

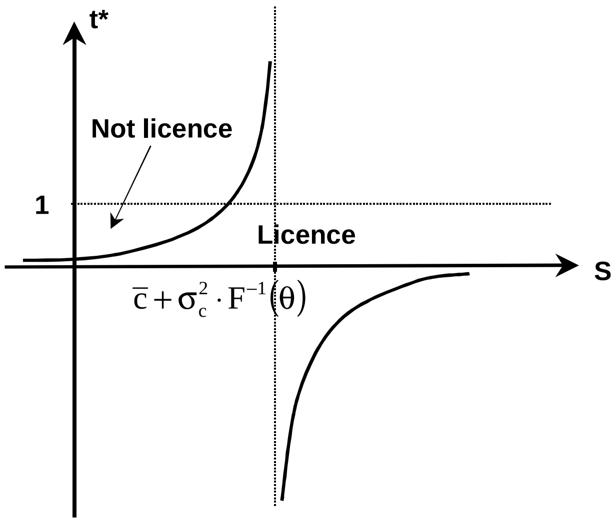

Figure 6: The licensing decision with an efficient entrant.

These graphs help understand the ex ante probability of licensing (P_L). This probability is considered by the incumbent when choosing how much information to obfuscate (m). Note that the obfuscation decision preceeds and affects the value of the realised signal S. We distinguish between the efficient and inefficient entrant cases[15].

[15] Note that the condition on the efficiency of the entrant is public knowledge, hence the regulator and the incumbent will understand which is the relevant regime.

With an efficient entrant: $P_L = \Pr\left(S > \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta)\right) + \Pr\left(S < \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta) \cap t < t^*\right)$.

With an inefficient entrant:

$$\begin{aligned} P_L &= 1 - \left(\Pr\left(S < \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta)\right) + \Pr\left(S > \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta) \cap t < t^*\right)\right) \\ &= \Pr\left(S > \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta)\right) - \Pr\left(S > \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta) \cap t < t^*\right) \end{aligned}$$

Given the distribution of *S*:

$$\Pr(S > \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta)) = 1 - F(t \cdot F^{-1}(\theta))$$

$$\begin{aligned} &\Pr\left(S < \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta) \cap t < t^*\right) \\ &= \Pr\left(c_E - \xi - \bar{c} - \sigma_c^2 \cdot F^{-1}(\theta) + t \cdot \bar{c} + t \cdot \sigma_c^2 \cdot F^{-1}(\theta) < S < \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta)\right) \\ &= F\left(t \cdot F^{-1}(\theta)\right) - F\left(\frac{c_E - \xi - \bar{c} - \sigma_c^2 \cdot (t-1) \cdot F^{-1}(\theta)}{\sigma_c^2}\right) \end{aligned}$$

and

$$\Pr\left(S > \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta) \cap t < t^*\right) = -\Pr\left(S < \bar{c} + \sigma_c^2 \cdot F^{-1}(\theta) \cap t < t^*\right)$$

Lemma 5: *For both efficient and inefficient entrant regimes the probability of licensing is:*

$$P_L = 1 - F\left(\frac{c_E - \xi - \bar{c} + \sigma_c^2 \cdot (t-1) \cdot F^{-1}(\theta)}{\sigma_c^2}\right)$$

Although the expression is the same for both the efficient and inefficient cases, the value of c_E is larger when entry is relatively inefficient and hence the probability of licensing will be smaller.

We can show the following three comparative statics results. Firstly, $\frac{\partial P_L}{\partial c_E - \xi - \bar{c}} < 0$ so that the more inefficient is the entrant, the lower is the probability that entry will be licensed. Secondly, $\frac{\partial P_L}{\partial \theta} > 0$, i.e. that the more tolerant is the regulator of failure by the entrant, the higher is the probability that entry will be licensed. Finally, iff $\theta < 1/2$, so the regulator dislikes failure, $\frac{\partial P_L}{\partial t} = -F'\left(\frac{c_E - \xi - \bar{c} + \sigma_c^2 \cdot (t-1) \cdot F^{-1}(\theta)}{\sigma_c^2}\right) \cdot F^{-1}(\theta) > 0$. In other words, the more precise is the information on the incumbent's marginal cost, the more likely it is that the regulator will license the entrant. Recall that θ is the threshold for the probability that the entrant will fail. We can think of $\theta < 1/2$ as the case where the regulator fears failure. In this case, if the incumbent wished to reduce the chances of entry, it would need to reduce *t*, and would ex ante need to obfuscate the signals[16].

4.3 The incumbent's choice: duopoly or regulation?

We will now consider the incentives of the incumbent when it understands that the information revealed may be used either to regulate its price or allow an entrant in the market. Recall that a regulated incumbent prefers the information of the regulator to be imprecise when it is used to set a price cap. However it may prefer more precise information to deter entry. Thus the regulator might be able to use the threat of licensing an entrant in order to oblige the efficient incumbent to provide more precise information. From the comparative static results above, it is clear that this could be true if $\theta > 1/2$. However, if $\theta < 1/2$ obfuscation not only makes it harder for the regulator to regulate, it also makes entry less likely (see our first result in this section).

Initially, we must rewrite the expected profit of each regime (entry and regulated monopoly) and establish some assumptions, which allow the comparability of demand and profit functions of both regimes.

With a monopoly, demand was defined as: $Q = a - b \cdot P$. With a duopoly $q_i = \alpha - \beta \cdot P_i + \gamma \cdot P_j$. We assume that a$> \alpha + \gamma \cdot P_j$ for any plausible P_j and that $\beta = b$.

The expression for the incumbent's ex ante profits is:

$$\pi_I = P_L \cdot \pi_I^E + (1 - P_L) \cdot \pi_I^R - m \cdot c_f = P_L \cdot (\pi_I^E - \pi_I^R) + (\pi_I^R - m \cdot c_f)$$

Hence $\frac{\partial \pi_I}{\partial m} = \left(\frac{\partial P_L}{\partial t} \cdot (\pi_I^E - \pi_I^R) + P_L \cdot \frac{\partial (\pi_I^E - \pi_I^R)}{\partial t} \right) \cdot \frac{\partial t}{\partial m} + \frac{\partial (\pi_I^R - m \cdot c_f)}{\partial m}$. From sections 3 and 4 we know that, $\frac{\partial t}{\partial m} < 0$, $\frac{\partial P_L}{\partial t} > 0$ iff $\theta < 1/2$, $\frac{\partial \pi_I^E}{\partial t} < 0$, $\frac{\partial \pi_I^R}{\partial t} < 0$, and finally that $\pi_I^R - m \cdot c_f$ is convex in m. We can use these to characterise the conditions under which a "full obfuscation" or "no obfuscation" result exists and will discuss when each of these conditions

[16] Note that the opposite is true when θ > 1/2, that is, when the regulator us relatively unconcerned with failure.

is more reasonable. Define $\Delta_0 \equiv \left(\pi_I^R - m \cdot c_f\right)\Big|_{m=0}$ and $\Delta_n \equiv \left(\pi_I^R - m \cdot c_f\right)\Big|_{m=n}$. Note that the size of Δ_0 and Δ_n determine the value of *m* that maximises the value of $(\pi_I^R - m \cdot c_f)$.

Proposition 2: The incumbent will fully obfuscate (*m=n*) if $\Delta_n > \Delta_0$ and

(i) $\theta < 1/2$, $\pi_I^E < \pi_I^R$ and $\frac{\partial(\pi_I^E - \pi_I^R)}{\partial t} < 0$ or

(ii) $\theta > 1/2$, $\pi_I^E > \pi_I^R$ and $\frac{\partial(\pi_I^E - \pi_I^R)}{\partial t} < 0$

The incumbent will not obfuscate at all (*m=0*) if $\Delta_n < \Delta_0$ and

(iii) $\theta < 1/2$, $\pi_I^E > \pi_I^R$ and $\frac{\partial(\pi_I^E - \pi_I^R)}{\partial t} > 0$ or

(iv) $\theta > 1/2$, $\pi_I^E < \pi_I^R$ and $\frac{\partial(\pi_I^E - \pi_I^R)}{\partial t} > 0$

All this information can be summarised in the following tables:

$\Delta_n > \Delta_0$ Precision affects entry profit more: $\frac{\partial(\pi_I^E - \pi_I^R)}{\partial t} < 0$	
Reg not. pro entry ($\theta < 1/2$) Soft cap and efficient entrant ($\pi_I^E < \pi_I^R$)	Full obfuscation
Reg. pro entry ($\theta > 1/2$) Tough cap and inefficient entrant ($\pi_I^E > \pi_I^R$)	Full obfuscation

Table 1a: Conditions leading to full obfuscation

$\Delta_n < \Delta_0$ Precision affects reg. profit more: $\frac{\partial(\pi_I^E - \pi_I^R)}{\partial t} > 0$	
Reg. not pro entry ($\theta < 1/2$) Tough cap and inefficient entrant ($\pi_I^E > \pi_I^R$)	No obfuscation
Reg. pro entry ($\theta > 1/2$) Soft cap and efficient entrant ($\pi_I^E < \pi_I^R$)	No obfuscation

Table 1b: Conditions leading to no obfuscation

5 Conclusion

We have developed a general model for analysing the process of information request, obfuscation and auditing to explain the increasing tension over exchange of information in the regulatory relationship. By treating obfuscation as an increase in the variance of the information rather than a change in its mean value, we focus on the quality of the information, rather than on any attempt to bias the cost estimates. We have shown that the firm will sometimes want to obfuscate all, and sometimes none, of the information it is required to provide. The chances of obfuscation fall as the regulator asks for more information, but the regulator does not always ask for so much than none of it is obscured. When information is used both to govern the price cap and to determine whether entry is allowed, we again find that the firm will either choose complete obfuscation or none.

Doubts about the reliability of information is curbing the regulators' introduction of incentive schemes based on quality; while information demands from regulators, and frustration on the part of firms and regulators over the exchange of such information are both increasing. For example, the following exchange between the UK gas regulator and British Gas illustrates a typical regulatory argument. "Ofgas … was concerned at the paucity of information supplied by BGT. However, BGT has supplied no additional cost information" (Ofgas 1996). The company's view was that "In our ABC analysis we draw the line at publishing cost drivers. We feel that there comes a point where under intensive regulation the managerial responsibility to improve the business should have some advantage at least in terms of understanding the business" (Copley, 1995). At this time British Gas was defending its position both against pressure for additional entry from the regulator and against more stringent price caps (the price review of the gas pipeline business in 1996 was referred to the Monopolies and Mergers Commission). The model developed in this paper helps explain the variation in requests for information made by different regulations, and the differing extent of co-operation from companies.

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Appendix

Computing the composite signal: Let the i'th signal be $S_i = \alpha + \varepsilon_i$, where we assume that $Cov(\varepsilon_i, \varepsilon_j) = 0$ to avoid too many problems. Let $Var(S_i) = V_i^2$ and $Cov(S_i, S_j) = \sigma_{ij}$. A sufficient statistic for the n observations is

$$S = \sum_{i=1}^{n} \delta_i \cdot S_i \quad \text{where} \quad \sum_{i=1}^{n} \delta_i = 1 .$$

To find the values of the coefficients, minimise the mean squared prediction error $E(S - \overline{\alpha})^2$.

$$E(S - \overline{\alpha}) = E\left[\sum_{i=1}^{n} \delta_i \cdot (S_i - \overline{\alpha})\right]^2$$

recalling that $\delta_n = 1 - \sum_{i=1}^{n-1} \delta_i$, the first order conditions are

$$2 \cdot \delta_i \cdot V_i^2 - 2 \cdot \delta_n \cdot V_n^2 + 2 \sum_{j \neq i}^{n} \delta_j \sigma_{1j} - 2 \sum_{k=1}^{n-1} \delta_k \sigma_{kn} = 0 \qquad i = 1,..., n-1$$

(1)

Pair wise subtraction from i=1 yields

$$\delta_1 \cdot V_1^2 - \delta_i \cdot V_i^2 + \delta_i \sigma_{1i} - \delta_1 \sigma_{i1} = 0 \qquad i = 2,..., n-1$$

(2)

so that, as $\sigma_{ij} = \sigma_\alpha^2$,

$$\delta_i = \delta_1 \cdot \frac{V_1^2 - \sigma_\alpha^2}{V_i^2 - \sigma_\alpha^2} \qquad i = 2,..., n-1$$

Using this in (1) and simplifying yields

$$\delta_1 \cdot V_1^2 - \left(1 - \sum_{k=1}^{n-1} \delta_k\right) \cdot V_n^2 + \sigma_\alpha^2 \cdot (1 - \delta_1) - \sigma_\alpha^2 \sum_{k=1}^{n-1} \delta_k = 0$$

and eventually

$$\delta_i = \frac{\frac{1}{V_i^2 - \sigma_\alpha^2}}{\sum_{k=1}^{n} \frac{1}{V_k^2 - \sigma_\alpha^2}} \qquad i = 1,..., n$$

Note that if S_i is very noisy, in the limit infinitely so, then $\delta_i = 0$.

Proof of lemma 1. Write t as $t = \sigma_c^2 \cdot \left(\sigma_c^2 + \frac{\mu \cdot n}{\mu \cdot n - (\mu - 1) \cdot m} \cdot \frac{1}{k} \cdot \sigma_\varepsilon^2\right)^{-1}$. Then:

$$\frac{\partial t}{\partial k} = -\sigma_c^2 \cdot \left(\sigma_c^2 + \frac{\mu \cdot n}{\mu \cdot n - (\mu - 1) \cdot m} \cdot \frac{1}{k} \cdot \sigma_\varepsilon^2\right)^{-2} \frac{\mu \cdot n}{\mu \cdot n - (\mu - 1) \cdot m} \cdot \frac{-1}{k^2} \cdot \sigma_\varepsilon^2 = t \cdot (1 - t) \cdot \frac{1}{k} > 0$$

$$\frac{\partial^2 t}{\partial k^2} = -t \cdot (1 - t) \cdot \frac{1}{k^2} + (1 - 2t) \cdot \frac{1}{k} \cdot \frac{\partial t}{\partial k} = -t \cdot (1 - t) \cdot \frac{1}{k^2} 2t < 0$$

from which t is increasing and concave in k.

$$\frac{\partial t}{\partial m} = -\sigma_c^2 \cdot \left(\sigma_c^2 + \frac{\mu \cdot n}{\mu \cdot n - (\mu - 1) \cdot m} \cdot \frac{1}{k} \cdot \sigma_\varepsilon^2\right)^{-2} \frac{(\mu - 1)}{(\mu \cdot n - (\mu - 1) \cdot m)^2} \cdot \frac{\mu \cdot n}{k} \cdot \sigma_\varepsilon^2$$

$$= -t \cdot (1 - t) \cdot \frac{(\mu - 1)}{(\mu \cdot n - (\mu - 1) \cdot m)} < 0$$

$$\frac{\partial^2 t}{\partial m^2} = -t \cdot (1 - t) \cdot \frac{(\mu - 1)^2}{(\mu \cdot n - (\mu - 1) \cdot m)^2} - (1 - 2t) \cdot \frac{(\mu - 1)}{(\mu \cdot n - (\mu - 1) \cdot m)} \cdot \frac{\partial t}{\partial m}$$

$$= -2 \cdot t^2 \cdot (1 - t) \cdot \frac{(\mu - 1)^2}{(\mu \cdot n - (\mu - 1) \cdot m)^2} < 0$$

from which t is decreasing and concave in m.

$$\frac{\partial t}{\partial n} = -\sigma_c^2 \cdot \left(\sigma_c^2 + \frac{\mu \cdot n}{\mu \cdot n - (\mu - 1) \cdot m} \cdot \frac{1}{k} \cdot \sigma_\varepsilon^2\right)^{-2} \frac{-\mu \cdot (\mu - 1) \cdot m}{(\mu \cdot n - (\mu - 1) \cdot m)^2} \cdot \frac{1}{k} \cdot \sigma_\varepsilon^2$$

$$= t \cdot (1 - t) \cdot \frac{(\mu - 1) \cdot m}{(\mu \cdot n - (\mu - 1) \cdot m) \cdot n} > 0$$

$$\frac{\partial^2 t}{\partial n^2} = -t \cdot (1 - t) \cdot \frac{(\mu - 1) m (2\mu \cdot n - (\mu - 1) \cdot m)}{(\mu \cdot n - (\mu - 1) \cdot m)^2 n^2} + (1 - 2t) \cdot \frac{(\mu - 1) \cdot m}{(\mu \cdot n - (\mu - 1) \cdot m) \cdot n} \cdot \frac{\partial t}{\partial m}$$

$$= -t \cdot (1 - t) \cdot \frac{(\mu - 1) m}{(\mu \cdot n - (\mu - 1) \cdot m)^2} \cdot 2 \cdot (\mu \cdot n - (1 - t) \cdot (\mu - 1) \cdot m) < 0$$

from which is increasing and concave in n.

Finally, the cross derivatives are:

$$\frac{\partial^2 t}{\partial m \partial n} = -(1 - 2t) \cdot t \cdot (1 - t) \cdot \frac{(\mu - 1) \cdot m}{(\mu \cdot n - (\mu - 1) \cdot m) \cdot n} \frac{(\mu - 1)}{(\mu \cdot n - (\mu - 1) \cdot m)} + t \cdot (1 - t) \cdot \frac{\mu \cdot (\mu - 1)}{(\mu \cdot n - (\mu - 1) \cdot m)^2}$$

$$= t \cdot (1 - t) \cdot \frac{(\mu - 1)}{(\mu \cdot n - (\mu - 1) \cdot m)^2} \left(\mu \cdot \left(1 - \frac{m}{n}\right) + \frac{m}{n} + 2 \cdot t \cdot \frac{(\mu - 1) \cdot m}{n}\right) > 0$$

$$\frac{\partial^2 t}{\partial m \partial k} = -(1-2t)\cdot t\cdot(1-t)\cdot\frac{1}{k}\cdot\frac{(\mu-1)}{(\mu\cdot n-(\mu-1)\cdot m)} < 0 \text{ if } t < \frac{1}{2}.$$

$$\frac{\partial^2 t}{\partial k \partial n} = (1-2t)\cdot\frac{1}{k}\cdot t\cdot(1-t)\cdot\frac{(\mu-1)\cdot m}{(\mu\cdot n-(\mu-1)\cdot m)\cdot n} > 0 \text{ if } t < \frac{1}{2}.$$

Proof of lemma 2: Write profits as

$$E[\Pi_I] = E\left[\beta(P_I^* - c)^2\right] = E\left[\beta\cdot\left(E(P_I^*) + \frac{1}{2}\cdot(c-\bar{c}) + \frac{1}{2}\cdot\frac{\gamma^2}{4\beta^2-\gamma^2}\cdot t\cdot(S-\bar{c}) - c\right)^2\right]$$

Then

$$E[\Pi_I] = E\left[\beta\cdot\left(E(P_I^*) - \bar{c} - \frac{1}{2}\cdot(c-\bar{c}) + \frac{1}{2}\cdot\frac{\gamma^2}{4\beta^2-\gamma^2}\cdot t(S-\bar{c})\right)^2\right]$$

$$= \beta\cdot\left(E(P_I^*) - \bar{c}\right)^2 + \frac{\beta}{4}\cdot\left(1 + \left(\frac{\gamma^2}{4\beta^2-\gamma^2}\right)^2\cdot t - \frac{2\gamma^2}{4\beta^2-\gamma^2}\cdot t\right)\cdot\sigma_c^2$$

which is decreasing in t. Thus the Incumbent wants the precision to be as poor as possible.

The profit of the entrant is

$$E[\Pi_E] = E\left[(P_E^* - c_E)\cdot(\alpha - \beta\cdot P_E^* + \gamma\cdot P_I^*)\right]$$

Using that $\alpha - \beta\cdot P_E^* + \gamma\cdot P_I^* = \beta\cdot\left(E(P_E^*) - c_E\right) + \frac{\gamma}{2}\cdot(c-\bar{c}) - \frac{\gamma}{2}\cdot\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}\cdot t\cdot(S-\bar{c})$, we can write profits as

$$E[\Pi_E] = E\left[\left(E(P_E^*) - c_E + \frac{\beta\gamma}{4\beta^2-\gamma^2}t(S-\bar{c})\right)\left(\beta\left(E(P_E^*) - c_E\right) + \frac{\gamma}{2}(c-\bar{c}) - \frac{\gamma}{2}\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}t(S-\bar{c})\right)\right]$$

which reduces to

$$E[\Pi_E] = \beta\cdot\left(E(P_E^*) - c_E\right)^2 + \frac{2\beta^3\gamma}{(4\beta^2-\gamma^2)^2}\cdot\frac{\gamma}{2}\cdot t\cdot\sigma_C^2$$