Competitive Nonlinear Pricing and Bundling

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- Two aspects to nonlinear pricing: (i) volume discounts and (ii) bundling discounts
- What is the impact of nonlinear pricing on profit, consumer surplus and welfare?
- Aim to explain and reconcile conflicting results about impact of nonlinear pricing within a tractable, unified model

PATTERNS OF CONSUMER BEHAVIOUR

Single product: “food” from supermarkets; electricity; telephone service

- In static context, natural to model in one-stop shopping framework

Several products:

- air travel in period 1 and period 2
- electricity and gas
- telephone, cable TV, internet
- high “shopping costs” induce one-stop shopping behaviour
- low shopping costs and product differentiation mean some consumers buy wine from one shop and cheese from another
- suppliers may try to increase shopping costs via bundling

SOME LITERATURE

Multiproduct monopoly price discrimination

- McAfee, McMillan, Whinston (1989), Armstrong (1996), Rochet & Choné (1998)
- complicated, though impact on profit obvious

Competitive nonlinear pricing (with one-stop shopping)

- Armstrong & Vickers (2001), Rochet & Stole (2002)
- suggests marginal-cost pricing may emerge in equilibrium
- no explicit profit comparison with linear pricing

Competitive bundling (with unit demands)

- Matutes & Regibeau (1992), Thanasssoulis (2006), Reisinger (2006)
- welfare cost due to “excessive loyalty”
- suggests bundling may reduce all prices compared with linear pricing

Customer poaching (with unit demands)

- Chen (1997), Fudenberg & Tirole (2000)
- welfare cost due to “insufficient loyalty”
- customers often better off

Two sources of welfare problems throughout paper:

1. A consumer buying from a firm may face excessive marginal prices
2. A firm may attract the wrong number of consumers

E.g., **MONOPOLY**

A monopolist faces heterogeneous population of consumers

With two-part tariff used instead of linear prices, we expect

- marginal prices to fall
- number of consumers served to fall

Beneficial “marginal price” effect but damaging “consumer participation” effect

Overall impact on welfare and consumer surplus is ambiguous

What happens with competition?

OUTLINE

Model 1: One-Stop Shopping

- with symmetric firms, no consumers buy from wrong firm under either regime
- welfare problems solely concerned with excessive marginal prices (worse with linear pricing)
- profit higher with nonlinear pricing, consumer surplus typically lower

Model 2: Bundling Model with Unit Demand

- excessive loyalty with nonlinear pricing but not with linear pricing
- no scope for excessive marginal prices
- profit lower with bundling, consumer surplus higher

Model 3: Unified Model with Two-Stop Shopping and Elastic Demand

- both welfare problems are relevant
- excessive marginal prices associated with linear pricing
- excessive loyalty associated with nonlinear pricing
- impact of economic features on [welfare, profit,...] with nonlinear pricing relative to linear pricing is ambiguous, and depends on:

	Welfare	Profit	Consumers
(i) demand elasticity	?	+	–
(ii) consumer heterogeneity	–	+	–
(iii) shopping costs	+	+	–
(iv) brand preference correlation	+	+	–

NONLINEAR PRICING WITH ONE-STOP SHOPPING

Hotelling model with two symmetric firms, A and B

- Each firm offers same $n \geq 1$ products; marginal cost c_i for product i
- Consumers buy all products from one firm or the other
- Consumers are heterogeneous in their location (or firm brand preference) x and their tastes for the products θ; x and θ are independent; x is uniform
- t is lump-sum transport cost per unit of distance
- If type (x, θ) consumer buys quantities q from A in return for payment T, utility is

$$u(\theta, q) - tx - T$$

- If she buys from B utility is

$$u(\theta, q) - t(1 - x) - T$$

- Assume all consumers participate over relevant range of tariffs

Equilibrium nonlinear tariffs

The unique symmetric equilibrium outcome with nonlinear pricing involves efficient consumption, and a consumer who buys quantities q makes payment

$$T(q) = t + \sum_i c_i q_i \ . \qquad (\clubsuit)$$

Welfare is first-best, industry profit is t

[Mainly established in Armstrong & Vickers (2001), Rochet and Tirole (2002)]

Sketch: Suppose firm B offers tariff ($\clubsuit$)

- Suppose A can observe each consumer's θ (but not x)
- For each θ, most profitable way to deliver utility to consumer is marginal-cost pricing with some fixed charge
- But when B offers ($\clubsuit$), A's best fixed charge is t which does not depend on θ
- Firm A's best response to ($\clubsuit$) is ($\clubsuit$)

Equilibrium linear prices

For presentation, assume just single product with marginal cost c

With linear prices p_A and p_B, A's profit is

$$\pi_A = E_\theta \left[\underbrace{\left(\frac{1}{2} + \frac{v(\theta, p_A) - v(\theta, p_B)}{2t} \right)}_{\text{market share}} \times \underbrace{q(\theta, p_A)(p_A - c)}_{\text{profit}} \right]$$

$q(\theta, p)$ is demand function and $v(\theta, p)$ is consumer surplus function

Symmetric equilibrium linear price satisfies

$$t \times E_\theta[q + (p-c)q_p] = (p-c)E_\theta[q^2]$$

so industry profit is

$$\pi = (p-c)E_\theta[q] < t$$

There is "elasticity effect" ($q_p < 0$) and "heterogeneity effect" ($E_\theta[q^2] > (E_\theta[q])^2$)

Nonlinear tariff, and profit, doesn't depend on elasticity or heterogeneity

Linear prices, and profit, decrease with elasticity and heterogeneity

If welfare is concave in linear prices, consumer surplus falls with nonlinear pricing

BUNDLING, TWO-STOP SHOPPING AND UNIT DEMANDS

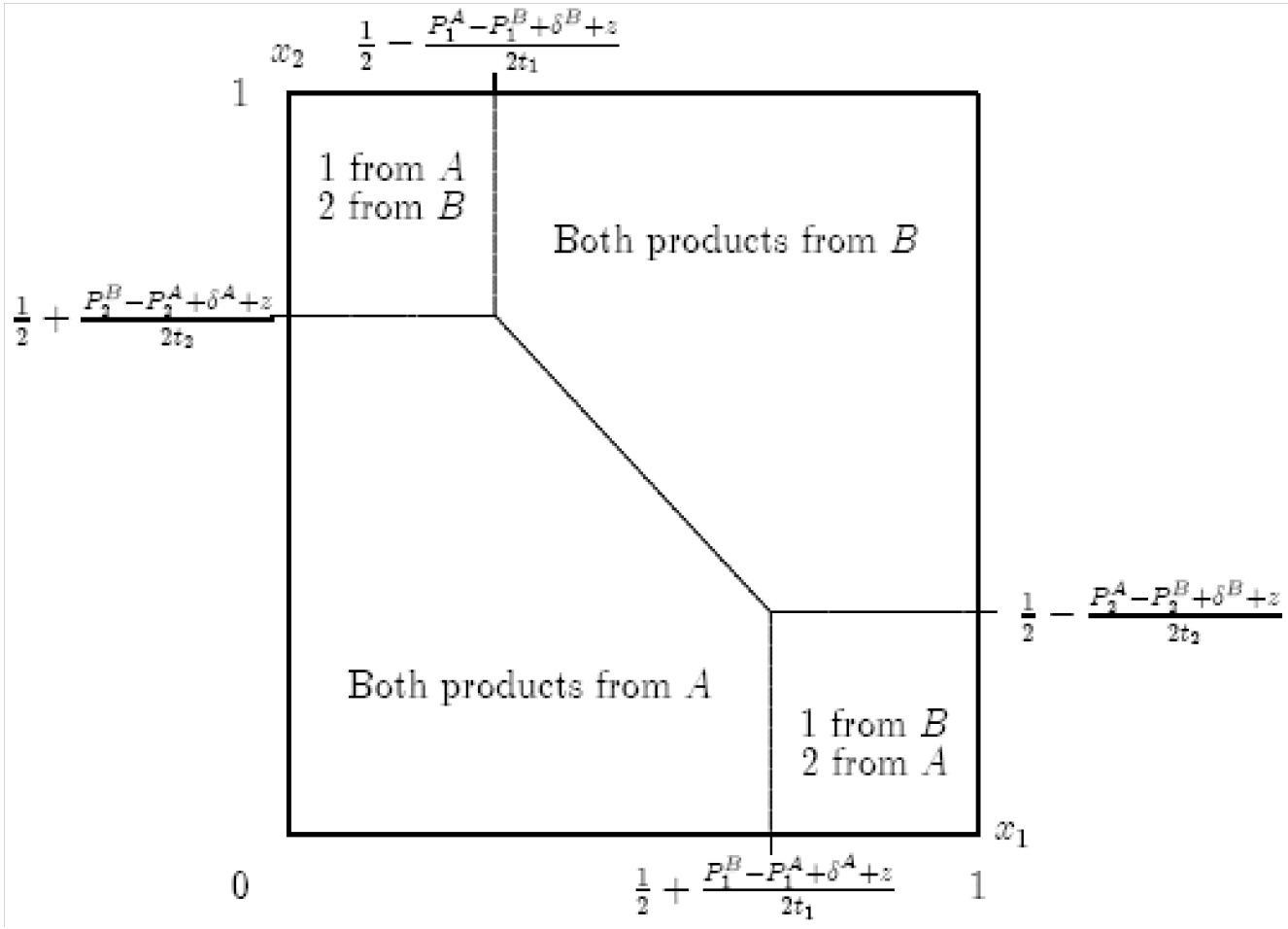
Consider "square" Hotelling model with two products and locations (x_1, x_2)

- x_i is distance of consumer to firm A's product i; $f(x_1, x_2)$ is symmetric density
- t_i is "transport cost" for product i
- consumers incur extra shopping cost $z < \min\{t_1, t_2\}$ if they buy from two firms
- consumers want one unit of each product
- production is costless

With (mixed) bundling, firm i's tariff comprises:

- P_1^i is price for product 1 alone
- P_2^i is price for product 2 alone
- $P_1^i + P_2^i - \delta^i$ is price for both products

Pattern of Demand with Mixed Bundling



It is straightforward to calculate the equilibrium linear prices (i.e., without bundling)

- prices decrease with the shopping cost z
- more "doubly profitable" one-stop consumers which intensifies competition

Suppose firms initially offer the equilibrium linear prices. A firm's profit increases if it unilaterally introduces small discount $\delta > 0$ for joint purchase

[cf. McAfee, McMillan & Whinston 1989]

Sketch:

- Envelope theorem implies firm is roughly indifferent if it reduces each linear price by (small) $\frac{1}{2}\delta$
- If instead firm offers discount δ to one-stop shoppers
  - this brings in the same gain in custom for each product
  - but price reduction enjoyed only by one-stop shoppers not all its customers
  - must be strictly profitable for firm

Important points:

- each firm unilaterally wants to offer bundling discount δ
- when both firms offer discount δ profits fall relative to linear pricing
- prisoner's dilemma nature of bundling
- welfare falls relative to linear pricing since there is excessive loyalty (first-best welfare requires $\delta = 0$)

What is equilibrium discount δ?

Define (decreasing) function

$$\Phi(\delta) \equiv \text{number of two-stop shoppers with symmetric tariffs and discount } \delta$$

In a symmetric bundling equilibrium

$$2\Phi(\delta) + \delta\Phi'(\delta) = 0$$

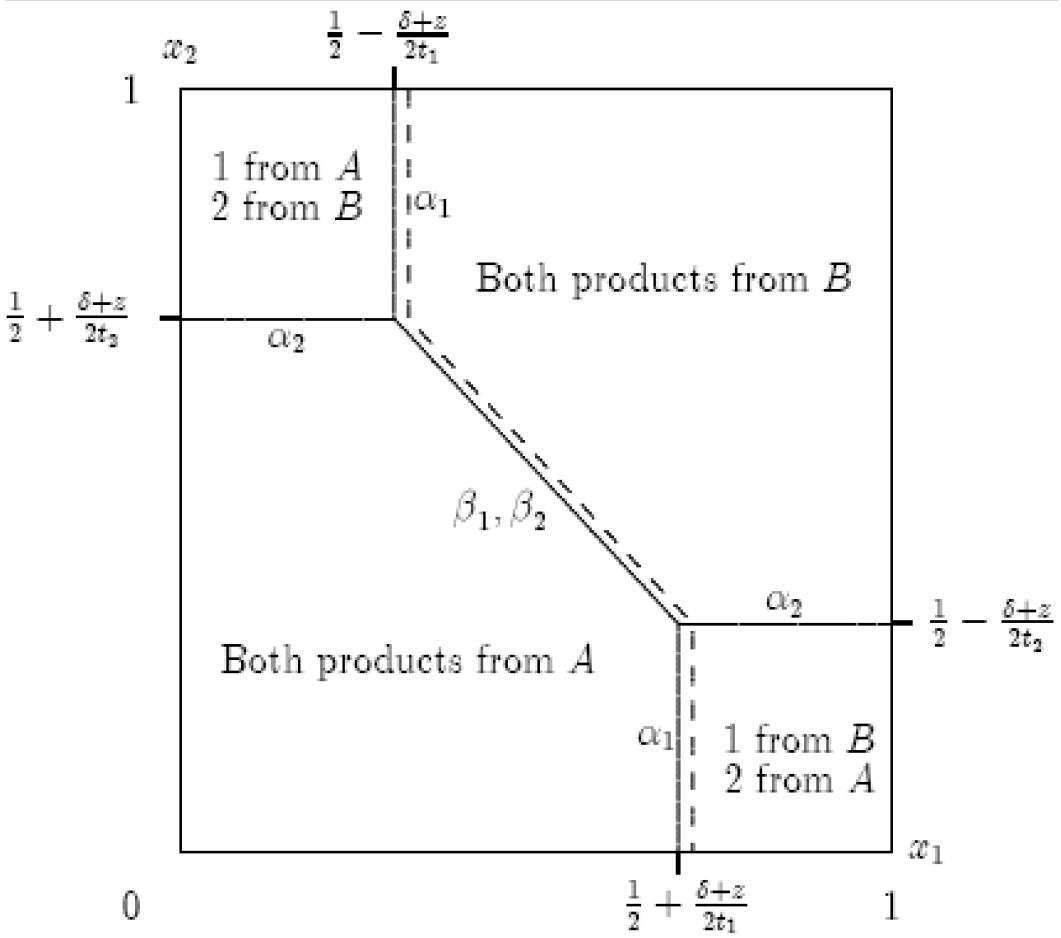
Sketch:

- Suppose a firm raises each stand-alone price by $\frac{1}{2}\varepsilon$ and raises its discount δ by ε
- total charge to its one-stop shoppers unchanged but charge to two-stop shoppers rises by $\frac{1}{2}\varepsilon$
- impact on firm's profit proportional to $2\Phi(\delta) + \delta\Phi'(\delta)$

What are the equilibrium stand-alone prices (given δ)?

Closed form (but tedious) expressions, given δ

Effect of Reducing P_1



UNIFORM EXAMPLE: $f(x_1, x_2) \equiv 1,\ t_1 = t_2 = t$

[Assume $z < t$ to have some two-stop shoppers]

Linear tariff:

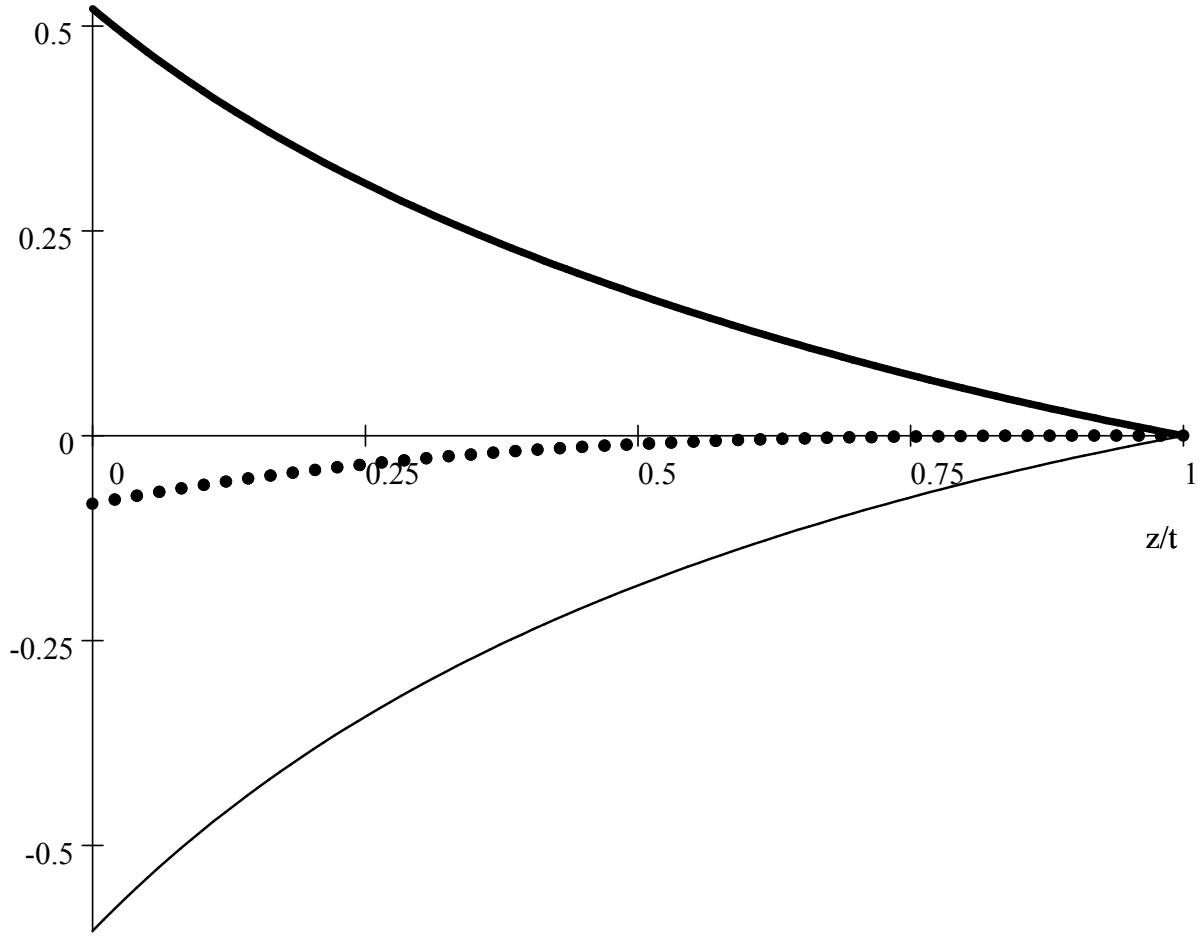
$$P_1 = P_2 = \frac{t^2}{t+z}\ ;\ \text{welfare} = [\text{first-best}]$$

Nonlinear tariff:

$$P_1 = P_2 = \tfrac{1}{4}(t-z) + \frac{2t^2}{3t+z}\ ;\ \delta = \tfrac{1}{2}(t-z)\ ;\ \text{welfare } = [\text{first-best}] - \tfrac{1}{12}\frac{(t-z)^3}{t^2}$$

- Shopping cost reduces all prices (with or without bundling)
- For relatively small z, all prices fall with bundling relative to linear pricing
- Linear and nonlinear pricing regimes converge as $z \to t$

Effect of z on relative consumer surplus (thick), welfare (dots) and profit (thin) of nonlinear vs. linear pricing:



This example generalizes:

Suppose that x_1 and x_2 are independently distributed, with density functions $f_1(x_1)$ and $f_2(x_2)$ and distribution functions $F_1(x_1)$ and $F_2(x_2)$. If

$$\frac{d}{dx}\frac{F_1(x)}{f_1(x)} \geq \frac{1}{4}\ ;\ \frac{d}{dx}\frac{F_2(x)}{f_2(x)} \geq \frac{1}{4}$$

then compared to linear pricing, profit and welfare fall and consumer surplus rises when bundling is used

These are precisely the opposite comparative statics to one-stop shopping model

Firms have unilateral incentive to offer bundling discounts

- this expands boundary of one-stop shoppers indifferent between firms ("doubly profitable" consumers)
- this intensifies competition for these one-stop shoppers

Correlation in Product Preferences

- proportion $1-\rho$ of consumers are uniformly distributed on the square
- proportion ρ of consumers are uniformly distributed on the diagonal $x_1 = x_2$

Linear tariff:

$$P_1 = P_2 = \frac{t^2}{t + (1-\rho)z}$$

Nonlinear tariff:

$$P_1 = P_2 = \tfrac{1}{4}(t-z) + \frac{2t^2}{2t + (1-\rho)(t+z)} \; ; \; \delta = \tfrac{1}{2}(t-z)$$

- Correlation increases all prices (with or without bundling)
- Profit and welfare lower with bundling, consumer surplus higher
- Regimes converge as $\rho \to 1$ (as proportion of two-stop shoppers is zero)

Summary of first two models

Two kinds of welfare problems:

1. *excessive marginal price effect*
2. *excessive loyalty effect*

In one-stop shopping model:

- problem 1 seen only with linear pricing
- problem 2 irrelevant

In bundling model with unit demands:

- problem 1 irrelevant
- problem 2 only seen with nonlinear pricing

UNIFIED MODEL: BUNDLING AND ELASTIC DEMAND

Type-θ consumer has gross utility $u(\theta, q_1, q_2)$ if q_i of product $i = 1, 2$ consumed

Marginal costs of supply are c_1 and c_2

Transport costs as with unit demand bundling model

θ and (x_1, x_2) are independent

Assume all consumers buy both products over relevant range of tariffs

General nonlinear tariff from firm i:

- $T_1^i(q_1)$ is its tariff for consuming product 1 alone
- $T_2^i(q_2)$ is its tariff for consuming product 2 alone
- $T_{12}^i(q_1, q_2)$ is its tariff for consuming both products

Equilibrium linear prices

Excessive marginal prices but no excessive loyalty

LINEAR UNIFORM EXAMPLE: $f(x_1, x_2) \equiv 1,\ t_1 = t_2 = t, c_1 = c_2 = 0$ and

$$q_i(\theta_1, \theta_2, p_1, p_2) = \theta_i(1 - bp_i)$$

θ_i has mean 1 and variance σ^2 ; Covariance of θ_1 and θ_2 is $\kappa\sigma^2$

Equilibrium linear price is

- increasing in t
- decreasing in z, b, σ^2 and κ

Price decreases with σ^2 since a price cut attracts proportionally more high demand

(and high profit) consumers, and improves the *mix* of consumers

Price decreases with κ since that increases the variance of one-stop shoppers

Equilibrium nonlinear tariffs

It is an equilibrium for each firm to offer the tariff

$$T_1(q_1) = P_1 + c_1 q_1 \; ; \; T_2(q_2) = P_2 + c_2 q_2 \; ; \; T_{12}(q_1, q_2) = T_1(q_1) + T_2(q_2) - \delta$$

where P_1, P_2 and δ define the mixed bundling tariff with unit demand

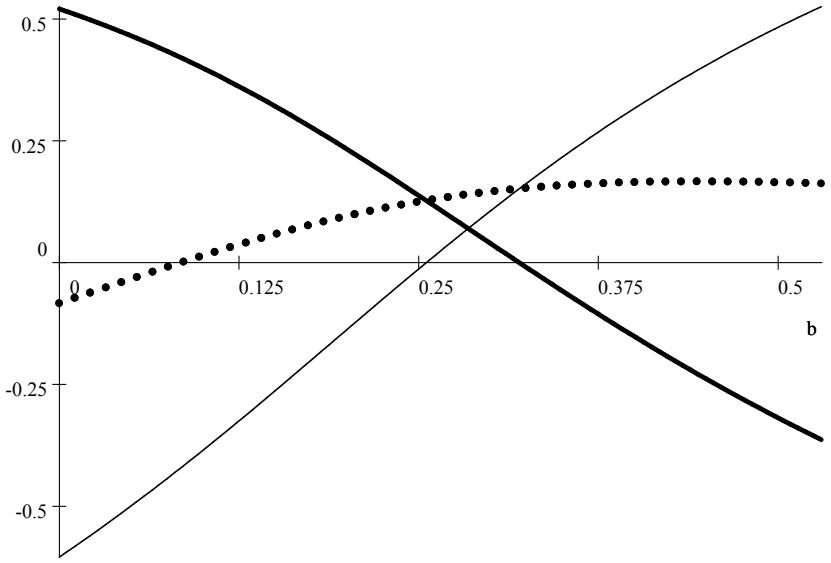
Therefore, excessive loyalty but not excessive marginal pricing

Profit does not depend on the shape or variance of demands (b, σ^2, κ, etc.)

Effect of Demand Elasticity

One reason why linear pricing yielded lower profit in one-stop shop model

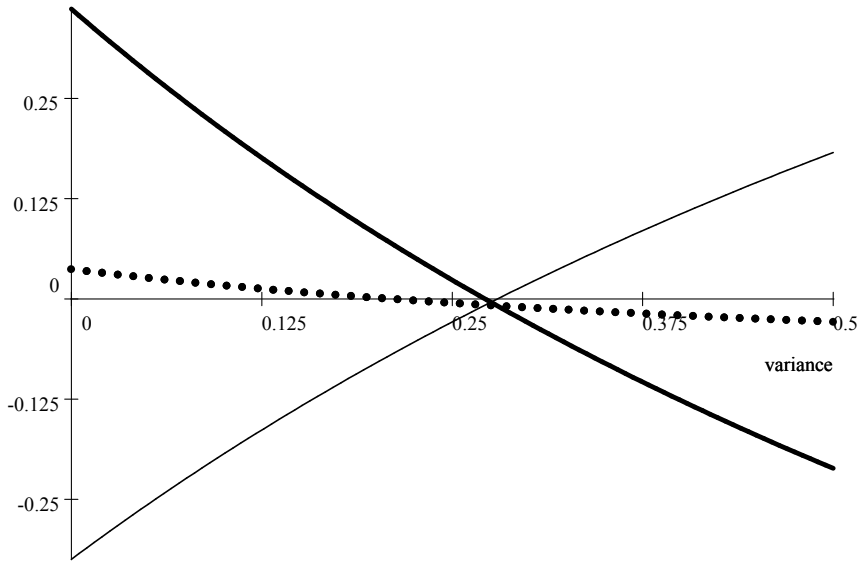
Effect of b on relative consumer surplus (thick), welfare (dots) and profit (thin) of nonlinear vs. linear pricing ($t = 1,\ z = 0, \sigma^2 = 0$) :



Effect of Consumer Heterogeneity

Second reason why linear pricing yielded lower profit in one-stop shop model

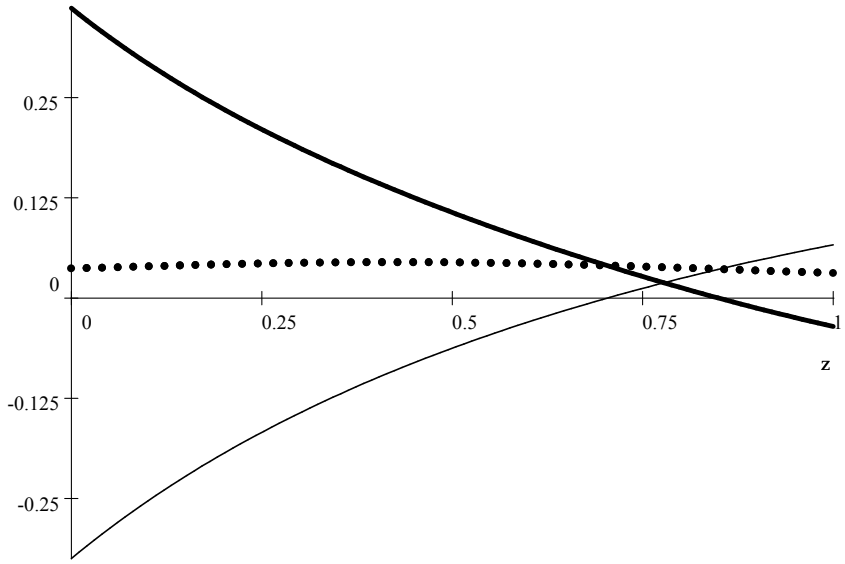
Effect of σ^2 on relative consumer surplus (thick), welfare (dots) and profit (thin) of nonlinear vs. linear pricing ($t = 1,\ z = 0, b = \frac{1}{8}$) :



Effect of Shopping Cost

When z is large δ is small and excessive marginal price effect dominates

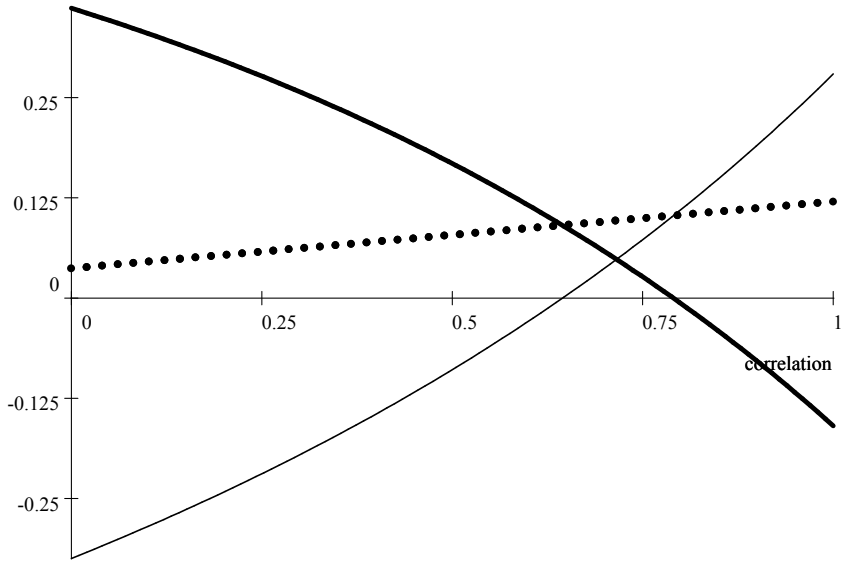
Effect of z on relative consumer surplus (thick), welfare (dots) and profit (thin) of nonlinear vs. linear pricing ($t = 1,\ \sigma^2 = 0, b = \frac{1}{8}$) :



Effect of Correlation in Brand Preferences

Similar to the effect of shopping costs

Effect of ρ on relative consumer surplus (thick), welfare (dots) and profit (thin) of nonlinear vs. linear pricing ($t = 1,\ z = 0,\ \sigma^2 = 0,\ b = \frac{1}{8}$) :



Future Work

The unified model is general in some respects but scope to extend beyond:

- symmetric firms
- two-product firms
- duopoly
- all consumers buy some of each product
- brand preferences are “lump-sum” rather than “per-unit”
- comparison between linear pricing and “fully nonlinear” pricing
- static interaction