Keep to Sustain or Keep to Exploit? Why Firms Keep Hard Evidence

Panayiotis Agisilaou

CCP Working Paper 12-5

Abstract: We develop a model wherein collusive firm's decisions to keep or to destroy the hard evidence is endogenous. Unlike previous literature, we assume that the administration of the cartel crucially depends on the existence of the hard evidence. Within this framework, we explore the impact of a leniency program on whether firm's incentives are to destroy or keep the hard evidence. Moreover, we examine firm's incentives to report or not to report the hard evidence to the antitrust authority. We show that firms may be wilfully keep the hard evidence, even if a leniency program is not available, in order to enhance the stability of a cartel. Additionally, we prove that firms are more inclined to keep the hard evidence when a leniency program is available. Finally, we demonstrate that firms are more likely to destroy the hard evidence when the collusive profits-fine ratio increases.
March 2012

JEL Classification Codes: L1, L4, K21, K40

Keywords: self-reporting, leniency program, hard evidence, collusion

Acknowledgements: I am in debt to my supervisors Bruce Lyons and Morten Hviid for stimulating discussions and suggestions for improvements to earlier versions of this paper. I am also grateful to comments from seminar participants at the University of East Anglia, Centre for Competition Policy, University of Nottingham and the 7th Royal Economic Society PhD meeting. I am particularly grateful to Peter Dijkstra and Frederick Wandschneider for useful comments and suggestions. The usual disclaimer applies. The support of the Economic and Social Research Council is gratefully acknowledged.

ISSN 1745-9648

Contacts Details:

Panayiotis Agisilaou, School of Economics, University of East Anglia, Norwich Research Park, Norwich, NR4 7TJ, United Kingdom. Email: p.agisilaou@uea.ac.uk .

Keep to sustain or keep to exploit? Why firms keep hard evidence*

Panayiotis Agisilaou†

March 12, 2012

Abstract

We develop a model wherein collusive firms' decisions to keep or to destroy the hard evidence is endogenous. Unlike previous literature, we assume that the administration of the cartel crucially depends on the existence of the hard evidence. Within this framework, we explore the impact of a leniency program on whether firms' incentives are to destroy or to keep the hard evidence. Moreover, we examine firms' incentives to report or not to report the hard evidence to the antitrust authority. We show that firms may willfully keep the hard evidence, even if a leniency program is not available, in order to enhance the stability of the cartel. Additionally, we prove that firms are more inclined to keep the hard evidence when a leniency program is available. Finally, we demonstrate that firms are more likely to destroy the hard evidence when the collusive profits-fine ratio increases.

JEL-Classification: L1, L4, K21, K40
Keywords: self-reporting, leniency program, hard evidence, collusion

*I am indebt to my supervisors Bruce Lyons and Morten Hviid for stimulating discussions and suggestions for improvements to earlier versions of this paper. I am also grateful to comments from seminar participants at the University of East Anglia, Centre for Competition Policy, University of Nottingham and the 7th Royal Economic Society PhD meeting. I am particularly grateful to Peter Dijkstra and Frederick Wandschneider for useful comments and suggestions. The usual disclaimer applies.

†School of Economics, University of East Anglia, Norwich Research Park, Norwich, NR4 7TJ, United Kingdom. Email: p.agisilaou@uea.ac.uk.

1 Introduction

In the last decades several cartels have been dismantled in various jurisdictions either because some of their members have blown the whistle to antitrust authorities[1] (henceforth "AAs") or because AAs' own investigations have exposed incriminating hard evidence.[2] Moreover, numerous cartels have been prosecuted based on indirect evidence, as the AAs investigations failed to disclose hard evidence.[3]

Intuitively the cartels whose members keep hard evidence have a higher probability of detection and, hence, conviction by an AA. For instance, if an AA instigates a dawn raid in a market whereby it detects cogent and irrefutable incriminating hard evidence (i.e. meetings notes/memos/minutes, emails, videos, voice recordings, "*scoresheets*" tracking a cartel's members compliance etc.), it can directly and speedily convict the cartel. On the other hand, if the AA's investigations are not fruitful in tracing hard evidence then presumably more effort is required to substantiate the existence of the cartel.

A question that logically arises is why collusive firms do not destroy the hard evidence of their illegal communication to curtail the likelihood of detection by AAs. A reasonable speculation is that firms keep hard evidence to exchange it with a fine discount on the basis of a leniency program (henceforth "LP"). However, there are several instances in case law where AAs investigations confirmed that firms keep hard self-incriminating evidence even when LPs are not part of the antitrust enforcement policy. This suggests that the existence of a LP may not be the exclusive motive justifying firms' decision to keep hard evidence. Nevertheless, the introduction of a LP, it could sensibly be argued, influences collusive firms' decisions to keep or destroy the hard evidence. Pertinent to this is also the fact that those firms that keep hard evidence do not necessarily exchange it with lenient treatment. The relevant case law illustrates several cases where the AAs' investigations are fruitful in detecting and exposing hard evidence in firms hands.

---

[1] See for instance the following cases: Case COMP/E-1/36 604 - *Citric acid*, 5/12/2001. Case COMP/E-1/37.152 – *Plasterboard*, 27/11/2002. Case COMP/E-1/38.069 - *Copper plumbing tubes*, 3/9/2004. Case COMP/C.38.281/B.2 – *Raw tobacco Italy*, 20/10/2005. Case COMP/F/38.899 – *Gas insulated switchgear*, 24/1/2007. Case COMP/E-1/38.823 - *PO/Elevators and escalators*, 21/2/2007. Case COMP/38.628 - *Nitrile butadiene rubber*, 23/1/2008. Case COMP/38511 – *DRAMs*, 19/5/2010.

[2] See for instance the following cases: Case IV/31.865 - *PVC*, 27/7/1994. COMP/C-38.279 - *French beef*, 2/4/2003. COMP/38.432 - *Professional videotapes*, 20/11/2007. COMP/39165 - *Flat glass*, 28/11/2007. COMP/38.543 - *International removal services*, 11/3/2008. COMP/39125 - *Car glass*, 12/11/2008.

[3] See for instance the following cases: Cases 89/85, 105/85, 114/85, 116-117/85, 125-129/85 - *A. Ahlström Oy v. Commission*, 27//9/1988. Case IV/34.621, 35.059/F-3 - *Irish Sugar*, 24/5/1997. Case IV/33.133 - *Solvay ICI*, 19/12/1990. COMP/E-1/36.756 - *Sodium gluconate II*, 29/9/2004. COMP/B-2/37.666 - *Dutch beer market*, 18/4/2007.

The above discussion brings into the surface several interesting questions in relation to the apparently puzzling behavior of collusive firms. The most critical questions pertain to the factors that influence firms' decisions to keep or destroy the hard incriminating evidence, the impact of a LP on the said decisions, and the underlying incentives of firms to keep nevertheless nor report the hard evidence.

The existing literature remains silent to these puzzling questions as its main focus is on the effects of LPs on cartel formation (deterrence) and/or cartel sustainability (desistance).[4] The major shortcoming of the existing studies relates to the assumption that firms axiomatically keep the hard evidence that is generated by the cartel. In other words, the literature typically assumes that collusive firms have no option to destroy the hard incriminating evidence.

One notable exception to the literature is Aubert et al. [2006], whose study provides some plausible explanations that rationalize firms' decision to keep rather than destroying the hard evidence. One of those contends that firms may keep evidence in order to pay a lower fine in case of detection by the AA. An alternative one is that firms may use the hard evidence as a disciplining device to mitigate firms' incentives to develop opportunistic behavior.[5] A limitation of Aubert et al.'s [2006] analysis is, however, the assumption that the AA can offer positive rewards to self-reporting firms. Although this assumption leads to appealing and desirable theoretical results[6], it has been criticized in the literature primarily because it is politically infeasible and ethically immoral to reward wrongdoers.[7] More importantly their study cannot answer two crucial questions: why firms keep hard evidence in the absence of a LP, and in which respect, if any, does the introduction of a LP, as part of the enforcement policy, affects firms' incentives to keep or destroy the hard evidence.

Another paper that explores firms' incentives to destroy (partially or fully) or even to create additional evidence is Silbye [2010]. This paper highlights the tradeoff associated with firms' decisions to keep or destroy the hard evidence. On the one hand more evidence qualifies a self-reporting firm with a higher fine discount (fine discount is positively related with the quantity of evidence submitted). More evidence on the other hand implies a higher probability of detection and conviction

[4] The existing literature provides answers, among others, as to the optimal level of fine discount that should be offered to self-reporting firms, the number of firms that should be eligible for obtaining leniency and the stage at which leniency should be offered (before or after the commencement of AA's investigation in the market). See Motta and Polo [2003], Spagnolo [2004], Motchenkova [2004] and Harrington [2008]. For a recent literature review see Spagnolo [2008].

[5] A similar argument is put forward by Buccirossi and Spagnolo [2006]. These authors maintain that firms can use the hard evidence as a credible threat towards deviations from the collusive agreement.

[6] See Spagnolo [2004].

[7] For a discussion of the potential costs of rewarding wrongdoers see Aubert et al. [2006] and Spagnolo [2008].

by the AA, as it is easier for the AA to unravel the cartel if more evidence exists. Silbye [2010] concludes that the option to destroy the hard evidence does not affect the design of the LP.

To tackle some of the open questions in the relevant literature we develop a model wherein firms' decision to keep or destroy the hard evidence is, as in Aubert et al. [2006], endogenous. A key assumption in our model is that a cartel's sustainability (administration and detection of defections) crucially depends on hard evidence.[8] Specifically we assume that if at least one cartel member destroys the hard evidence, the cartel may, with some probability, collapse, and thus firms are deprived of future collusive profits. The underlying reasoning for this assumption is that hard evidence allows more effective implementation and monitoring of the terms of the collusive agreement. We call this effect the *cartel collapse effect* of hard evidence (henceforth the "*CCE"*). Essentially this effect rationalize firms' decision to keep hard evidence even in absence of a LP.

The decision to keep hard evidence does not, however, come without a cost for the cartel. In accordance with Silbye [2010] we assume that the probability of cartel detection is higher when firms keep hard evidence.[9] If the AA's investigation in the market exposes hard evidence, then the prosecution of the cartel is facilitated and thus the AA does not need to undertake additional effort to validate or further substantiate its case against the cartel. We call this effect the *cartel detection effect* of hard evidence (henceforth the "*CDE*"). Intuitively this effect increases the expected cost of the cartel by increasing the expected fine.

Both effects, the CCE and the CDE, cause a contraction to the net expected collusive profits. On the one hand, when firms destroy the hard evidence cartel's coordination becomes less effective (the cartel collapse with a positive probability) and the gross expected collusive profits decrease. On the other hand, when firms keep the hard evidence the probability of cartel detection is higher, thus the expected cost of the cartel increase. The strength of the two effects will reveal the

[8] It is worth noting that in Aubert et al. [2006] a cartel's sustainability is not influenced directly by the existence of hard evidence. A cartel, in their model, may collapse if a firm's drastic innovation is successful, as the *successful* firm would prefer to compete forever rather than to collude. Within this setup hard evidence may operate as a mechanism to vitiate firms' incentives to defect from the cartel. Essentially, by threatening to denounce the cartel in case of defection from the collusive agreement, the deviant firm's incentive compatibility constraint tightens, as it has to pay the full fine to the AA. Contrarily, in our model the existence of hard evidence directly affects cartel's sustainability by ebableing firms to monitor and react to their collaborators' market conduct.

[9] Aubert et al. [2006] also make this assumption. However, contrary to Aubert et al. [2006] we assume that a cartel can be detected even in absence of hard evidence, based solely on indirect evidence. In our view, this is a more realistic assumption. As already remarked, in real world there are cases where AAs in various jurisdictions have convicted firms for collusion based exclusively on indirect evidence.

conditions under which collusive firms choose to keep or destroy the hard evidence even when the antitrust enforcement policy does not encompass a LP.

In our model the introduction of a LP offers to collusive firms an opportunity to exploit the LP. In particular, firms may keep the hard evidence and subsequently exchange it with a reduced fine within the framework of a LP.[10] We call this effect the *cartel amnesty effect* of hard evidence (henceforth the "*CAE*"). This effect causes a reduction to the expected cost associated with firms' decision to keep hard evidence. Therefore, collusive firms may choose to keep the hard evidence not only to evade the costly breakdown of the cartel but also to exploit the LP. The interplay of the three effects, CCE, CDE and CAE, allows us to explore the impact of a LP on firms' decisions to keep or destroy hard evidence. Moreover, the introduction of a LP allows the investigation of the conditions under which collusive firms keep hard evidence without reporting it to the AA.

The main conclusions of our study is that firms are more likely to keep hard evidence when: *i*) the cartel's sustainability is more sensitive to hard evidence or *ii*) the probability of cartel detection is less depended on hard evidence or *iii*) the collusive profits-fine ratio is high. Moreover, we show that the introduction of a LP reinforces firms' incentives to keep hard evidence. Finally, we demonstrate that firms may keep hard evidence without reporting it to the AA if the cartel's sustainability is very sensitive to hard evidence and the (aggregate) probability of cartel detection is sufficiently low.

The paper is organized as follows. Section 2 outlines the model. Section 3 presents the baseline model without LP. The extended model with LP is presented in Section 4. Section 5 concludes.

2 Outline of the model

Similar to Motta and Polo [2003] we consider two firms playing an infinitely repeated game in the presence of an AA which enforces antitrust law. This is a significant difference to Aubert et al. [2006] who study a dynamic non-repeated game. Essentially these authors assume that the hard evidence is indelible so that firms can use it in any future period, as long as the cartel has not been detected. On the contrary, we assume that there is full information decay after the end of each period. This could be for example the case when the hard evidence consists of "*scoresheets*" tracking cartel members compliance to the individual terms of the agreement. After the end of each period this evidence has no value to firms as it refers only to the past. The repeated structure could also be justified on the ground that the collusive agreement has limited time frame due to changing

[10] This contrasts with Aubert et al. [2006] who show that firms may keep hard evidence in order to discipline defections from the cartel.

market (i.e. demand) or policy conditions (i.e. fine). For instance, firms may shift to other markets which guarantee more lucrative opportunities for trade.

To simplify our analysis we assume markets where cartel formation is always profitable for firms. We call a market with this feature a *pro-collusive* market.[11] Our goal is not to show the deterrence or desistance effects of LPs but to investigate the effects of the introduction of such programs on firms' incentives to keep or destroy the hard evidence and their incentives to reveal or not that evidence to the AA.

We assume that collusion generates and leaves symmetric hard incriminating evidence. In addition, we assume that the existence of a cartel cannot be observed by the AA unless the latter launches an investigation in the market. The AA's investigation is successful only with some probability. This probability crucially depends on whether hard evidence exists. Destroying the hard evidence does not necessarily imply that firms will escape unscathed by the AA's investigations. The cartel can be detected and successfully prosecuted based exclusively on indirect evidence, that is, evidence of facts and circumstances supporting an inference of a cartel.[12] We call this probability of cartel detection, which is independent of hard evidence, the *base probability of detection* and we denote it by p.

If the two collusive firms decide to keep the hard evidence, the probability of detection increases to $p+\Delta p$. Parameter Δp represents the incremental probability of cartel detection when both firms choose to keep rather than destroy the hard evidence. Essentially this parameter reflects the quality of hard evidence or the AA's efficacy in prosecuting cartels when its investigations expose hard evidence. The better the quality of hard evidence is or the more effective the AA is in assessing the hard evidence detected, the higher the incremental probability of detection would be. We assume that Δp is exogenous. If only one firm chooses to keep hard evidence, the additional probability of detection decreases to $\frac{\Delta p}{2}$.[13] The underlying reasoning is that the AA may in that case need to investigate the market more in order to corroborate the quality of the reported evidence. In the extreme case where the AA's competency in detecting cartels is not influenced by the existence of hard evidence or the quality of the hard evidence is unreliable so that the AA cannot rely on it to prosecute the cartel, then $\Delta p = 0$.

To sum up, when both firms destroy the hard evidence the probability of detection is $p \in [0, 1 - \Delta p]$, when both firms keep the hard evidence the probability

[11]Technically the existence of a pro-collusive market is guaranteed by Assumptions 3 and 4, i.e. $\delta \geq \frac{1}{2}$ and $\frac{\pi}{F} \geq 1$, respectively, of our paper.

[12]This evidence could include for example market data on prices or sudden change in firms conduct not justified otherwise by the market conditions.

[13]A similar assumption is made by Aubert et al. [2006]. However, these authors, contrary to this paper, assume that the base probability of detection is zero, that is, a cartel cannot be uncovered without hard evidence.

of detection is $p + \Delta p \in [0, 1]$, and when only one firm keeps the hard evidence the probability of detection is $p + \frac{\Delta p}{2}$.[14] In case of detection by the AA each cartel member must pay a fine F. Both p and F are enforcement policy instruments exogenously fixed by the AA. We define those instruments as the *antitrust enforcement policy*.[15]

When the enforcement policy encompass a LP, the collusive firms that keep hard evidence have an additional decision to make. In particular, they may choose to exchange the hard evidence with lenient treatment or keep the hard evidence without revealing it to the AA. Given that the hard evidence is symmetric only the first self-reporting firm is eligible for leniency. Moreover, we assume that the first self-reporting firms receives full amnesty. When both firms simultaneously apply for leniency then each of them must pay an expected fine of $\frac{F}{2}$.

Hard evidence is of pivotal importance for the success of the cartel. On the one hand it may allow firms to overcome any administration and implementation problems due to the complexity of their agreement. More precisely, we assume that if at least one firm destroys the hard evidence, the management of the cartel is rendered more demanding and challenging, ultimately causing, with some probability, the collapse of the cartel. We denote this probability by $\beta \in [0, 1]$ (henceforth "*the cartel collapse probability*"). Parameter β measures the sensitivity of the cartel to hard evidence. A higher β implies that the cartel agreement is more complex (for instance, in terms of administration, allocation of duties and tasks) and requires the existence of detailed hard evidence to overcome the administration deficit and/or implementation problems. At the extreme case where $\beta = 0$ the cartel's sustainability (administration and implementation) is independent of hard evidence. If one firm destroys while the other keeps the hard evidence then again the cartel may collapse. Although the firm that keeps the hard evidence can administer the agreement the one that destroys may fail and thereby the cartel may, as before, collapse with probability β.

We further assume that if the cartel is implemented, the firms can perfectly monitor the conformity to the agreement (i.e. the market is sufficiently transparent) and thus instantly react to any market defection.

Assumption 1 *Provided that the collusive agreement is implemented firms instantly react to any defections in the market when hard evidence exists.*

[14] The upper bound on p reflects our assumption that a cartel can be detected even if all of its members destroy the hard evidence albeit with a weakly lower probability to the one associated when hard evidence exists.

[15] The probability of detection p is a determinative factor of the deterrence effect of the enforcement of antitrust policy. The limited resources available to AAs condition the level of this probability. For empirical estimation of the probability of detection see Bryant and Eckard [1991], Combe et al. [2008] and Ormosi [2011].

Aubert et al. [2006] neglect this value associated with hard evidence. Although we concede that this is a very strong assumption to be made it allows us, at the same time, to simplify the analysis and focus our attention on deviations from the individual terms of the agreement other than those referred to firms' market conduct.

Given that we assume a pro-collusive market, when the enforcement policy does not include a LP, firms have only one decision to make: either to keep or destroy the hard evidence. When a LP is available, collusive firms have two sequential decisions to make: either to keep or destroy, and if they keep, either to report or not the hard evidence to the AA. We analyze these strategy profiles and the corresponding payoffs in Sections 3 and 4.

To simplify the exposition we normalize non-collusive profits to zero. Per-firm collusive profits are denoted by $\pi \in (0, \pi^M]$. The upper bound denotes the per-firm monopoly profits. Moreover, we assume that firms have the same discount factor, denoted by δ, and adopt standard grim trigger strategies to sustain their agreement.

Another critical assumption we made is that a cartel that collapses due to administration problems, this could be for instance the case where at least one firm destroys the hard evidence, avoids prosecution.

Assumption 2 *An unsuccessful cartel avoids prosecution.*

3 Baseline Model: without Leniency Program

In this section we present the baseline model wherein the antitrust enforcement policy does not include a LP. Given that we assume a pro-collusive market the two firms have only one decision to make, namely, to keep or destroy the hard incriminating evidence generated by the cartel. The analysis of this setup enables us to rationalize firms' decisions to keep hard evidence even though a LP is not available. Moreover, it provides a benchmark to analyze the impact on the said incentives of firms when a LP is available. Before deriving the subgame perfect equilibrium (henceforth "SPE") we formally present the game, that is, the timing, strategies and payoffs.

3.1 The timing of the stage game

In each period the timing of the game is as follows:

- Stage 0 (Policy Design): The AA commits and announces a certain enforcement policy (p and F).

- Stage 1 (Cartel Formation): Having observed the policy parameters and having full and complete information about parameters Δp and β, firms decide whether to communicate and form a cartel. Hard evidence is produced.
- Stage 2 (Keep or Destroy): Firms simultaneously decide whether to keep or destroy the hard evidence of their illegal agreement.
- Stage 3 (Policy Implementation): The AA launches an investigation into the market. The success of its investigation hinges on the existence or not of hard evidence by collusive firms.

3.2 Firms' strategies and payoffs

For the purposes of our analysis we consider only the two symmetric strategy profiles: (Keep, Keep) and (Destroy, Destroy).[16]

First, consider the strategy profile (Keep, Keep). According to this both firms agree to keep the hard evidence of their illegal communication. With probability $p + \Delta p$ firms obtain the collusive profits π but have to pay a fine F. With the complement probability, that is, with $1 - p - \Delta p$, firms evade detection and obtain the collusive profits π. If no deviation occurs, the game is repeated forever and the collusive agreement is stable. The expected discounted value of this strategy profile is equal to $V_K = \frac{\pi - (p+\Delta p)F}{1-\delta}$. V_K is non-negative if $\frac{\pi}{F} \geq p + \Delta p$.

If a deviation occurs, that is, if one firm destroys the hard evidence instead, the cartel will encounter administration problems and as a consequence with probability β collapse. At the same time, given that only one firm keeps the hard evidence the probability of cartel detection decreases to $p + \frac{\Delta p}{2}$. Thus, the short-run gain associated with a deviation from the strategy prescription is the lower expected fine, given that the probability of detection decreases. This gain will be realized by both firms only if the cartel does not collapse, that is, with probability $1 - \beta$. With probability β the cartel collapses and both firms obtain zero profits. Given that a deviation occurs firms revert to a permanent punishment phase wherein they obtain zero profits forever. Therefore, the expected payoff for the deviant firm is equal to $V_K^d = (1-\beta)[\pi - (p + \frac{\Delta p}{2})F]$ (the superscript stands for deviation).

[16] There is also an asymmetric strategy profile that firms may coordinate on. In particular, firms may agree that one of them will destroy, while the other will keep the hard evidence, i.e. coordinate on (Destroy, Keep). The expected discounted payoff associate with this profile is $V_{KD} = \frac{(1-\beta)}{1-\delta}[\pi - (p + \frac{\Delta p}{2})F]$. However, this strategy profile is always Pareto dominated by the symmetric ones. Notice that $V_K = \frac{\pi-(p+\Delta p)F}{1-\delta} > \frac{(1-\beta)}{1-\delta}[\Pi - (p + \frac{\Delta p}{2})F] = V_{KD}$ and $V_D = \frac{(1-\beta)(\Pi - pF)}{1-\delta} > \frac{(1-\beta)}{1-\delta}[\Pi - (p + \frac{\Delta p}{2})F] = V_{KD}$. Therefore, to simplify our analysis we discard the possibility that firms coordinate on asymmetric strategies.

By Assumption 1, and given that the cartel is implemented with certainty, any defection in the market is instantly detected and punished. Such deviation is therefore inferior to the one described above.

Consider now the strategy profile (DESTROY, DESTROY). According to this both firms agree to destroy the hard evidence of their illegal communication. Thus, the cartel collapse with probability β. Moreover, the cartel is detected with probability p, since no hard evidence exists, in which case the firms must pay a fine F. If no deviation occurs then the game is repeated forever and the collusive agreement is stable. The expected discounted payoff of this strategy profile is $V_D = \frac{(1-\beta)(\pi - pF)}{1-\delta}$. V_D is non-negative if $\frac{\pi}{F} \geq p$.

If a firm instead keeps the hard evidence, then the probability of cartel detection increases to $p + \frac{\Delta p}{2}$. The cartel remains fragile and collapse with probability β since one firm, the one that conforms to the strategy prescription, destroys the hard evidence. Given that a deviation occurs firms revert to a permanent punishment phase wherein they obtain zero profits forever. Therefore, the expected payoff for the deviant firm is equal to $V_D^d = (1-\beta)[\pi - (p + \frac{\Delta p}{2})F]$.

By the same line of reasoning, as elucidated before, a simultaneous deviation in the market, provided that the cartel is implemented, is inferior to the one described above.

At the beginning of this section we emphasize that we consider a pro-collusive market. To guarantee this we need to make two assumptions. Firstly, that the cartel is profitable to be formed and is sustainable even in absence of antitrust law enforcement. Given our assumption that firms sustain collusion by trigger strategies this requires that $\delta \geq \frac{1}{2}$.[17]

Assumption 3 $\delta \geq \frac{1}{2}$.

Secondly, the aforementioned strategies yield a non-negative payoff to firms. For this we need to restrict the fine so that it is bounded above by collusive profits.[18]

Assumption 4 $\frac{\pi}{F} \geq 1$.

Effectively, this assumption implies that firms always find it profitable to collude irrespective of the strictness of antitrust policy. This assumption could also be justified on the basis that the fine level must not jeopardize firms' financial stability.

---

[17] See Friedman [1971] and Tirole [1988].

[18] Notice that the condition required so that (KEEP, KEEP) yields a non-negative payoff is more stringent than the one required for (DESTROY, DESTROY).

3.3 Equilibrium Analysis

The game we have to analyze is a repeated sequential move game with imperfect information. Thus, the appropriate solution concept is subgame perfect equilibrium (henceforth "SPE").[19] We only consider pure strategies. When multiple equilibria exist we apply the Pareto dominance criterion.[20] Given that in our scenario firms communicate with the aim to reach a collusive agreement it is not unrealistic to assume that firms agree to orcherstrate their actions by coordinating to a particular collusive strategy.

3.3.1 Solution of the baseline game

Suppose that the two firms agree to keep the hard evidence of their illegal communication, i.e. coordinate on (KEEP, KEEP). This strategy profile is a SPE if and only if (henceforth "iff") $V_K > V_K^d$. The latter inequality holds iff $p \leq p_1 \equiv \frac{\pi}{F} - \frac{(\delta+\beta-\beta\delta+1)\Delta p}{2(\delta+\beta-\beta\delta)}$. Observe that $p_1 > 0$ iff $\Delta p < \Delta p_1 \equiv \frac{\pi}{F}\frac{2(\delta+\beta-\beta\delta)}{(\delta+\beta-\beta\delta+1)}$ and $p_1 < 1 - \Delta p$ iff $\Delta p > \Delta p_2 \equiv (\frac{\pi}{F} - 1)\frac{2(\delta+\beta-\beta\delta)}{(1-\beta)(1-\delta)}$. Hence, if the additional probability of detection is sufficiently low ($\Delta p < \Delta p_2$) then (KEEP, KEEP) is a strict SPE. If this probability is sufficiently high ($\Delta p > \Delta p_1$) then (KEEP, KEEP) is not a SPE.

Lemma 1 *(KEEP, KEEP) is a SPE of the Keep-Destroy game if* $p \in [0, \min\{p_1, 1 - \Delta p\}]$.

Notice that p_1 increases in β and $\frac{\pi}{F}$, while it decreases in Δp. The intuition is the following. As the cartel becomes less sensitive to hard evidence then, *ceteris paribus*, the deviation gain from the strategy profile (KEEP, KEEP) rise. Moreover, the expected foregone collusive profits due to the collapse of the cartel increase, *ceteris paribus*, in $\frac{\pi}{F}$. Thus, an increase either in β or $\frac{\pi}{F}$ makes the CCE sharper. In the case where $\frac{\pi}{F}$ is sufficiently high, and in particular if $\frac{\pi}{F} > \frac{(\beta+\delta-\beta\delta+1)\Delta p}{2(\beta+\delta-\beta\delta)}$, then (KEEP, KEEP) is a strict NE. On the contrary, an increase in the additional probability of detection, causes, *ceteris paribus*, the deviation profits to shrink as the expected fine increases. Therefore, the CDE intensifies with a higher Δp.

Suppose now that the two firms agree to destroy the hard evidence, that is, coordinate on (DESTROY, DESTROY). A unilateral deviation from this strategy profile is not profitable if $V_D \geq V_D^d$. The latter inequality holds iff $p \leq p_2 \equiv \frac{\pi}{F} + \frac{(1-\delta)\Delta p}{2\delta}$.

[19] A strategy is optimal in the sense of SPE if it maximizes players' payoff for every period and for every history of the game.

[20] A strategy satisfies this criterion if there is no other strategy for which each player of the game has a strictly higher payoff. Such a strategy is also called payoff-dominant. The notion of Pareto-dominant equilibrium is well established in the literature. See [Fudenberg and Tirole, 1991, pp. 20-22].

Assumption 4 implies that $p_2 \geq 1$, and hence (Destroy, Destroy) is a strict SPE. This is due to the fact that if a firm keeps hard evidence, rather than destroying it, the probability of detection increases by $\frac{\Delta p}{2}$, while at the same time the stability of the collusive agreement remains intact. Given that one of the two firms complies with the prescriptions of the agreed strategy, and accordingly destroys the hard evidence, the cartel continues to collapse with probability β. Thus, such deviation does not enhance the sustainability of the cartel. On the contrary, it increases the expected fine, and thus the expected cost, of the cartel. Moreover, given that a deviation from the agreed strategies occurs firms are deprived of future collusive profits, as they enter in an eternal punishment phase.

Lemma 2 *(Destroy, Destroy) is a strict SPE of the Keep-Destroy game.*

For the set of parameter values of p and Δp where the *Keep-Destroy* game has two SPE we apply the Pareto criterion. Thus, (Keep, Keep) Pareto dominates (Destroy, Destroy) iff $V_K > V_D$. The latter inequality holds iff $p < p_3 \equiv \frac{\pi}{F} - \frac{\Delta p}{\beta}$.

Lemma 3 *(Keep, Keep) Pareto dominates (Destroy, Destroy) iff $p < p_3$, where $p_3 \equiv \frac{\pi}{F} - \frac{\Delta p}{\beta}$.*

By taking into consideration that (i) $p_3 > 0$ iff $\Delta p < \Delta p_3 \equiv \beta \frac{\pi}{F}$ and (ii) $p_3 < 1 - \Delta p$ iff $\Delta p > \Delta p_4 \equiv \frac{\beta}{1-\beta}(\frac{\pi}{F} - 1)$ and Lemmata 1, 2 and 3 we can state the main result of this section.

Proposition 1 *For given policy and other parameters (p, Δp, F, β, π) the Pareto dominant SPE of the baseline model without LP is:*

1. *(Keep, Keep) iff:*
   (a) $\frac{\pi}{F} \in [1, \frac{1}{\beta}]$ *and*
      i. $\Delta p \in [0, \Delta p_4]$ *and* $p \in [0, 1 - \Delta p]$ *or*
      ii. $\Delta p \in [\Delta p_4, \Delta p_3]$ *and* $p \in [0, p_3)$ *or*
   (b) $\frac{\pi}{F} > \frac{1}{\beta}$, $\Delta p \in [0, 1)$ *and* $p \in [0, 1 - \Delta p]$
2. *(Destroy, Destroy) iff* $\frac{\pi}{F} \in [1, \frac{1}{\beta}]$ *and*
   (a) $\Delta p \in [\Delta p_4, \Delta p_3]$ *and* $p \in [p_3, 1 - \Delta p]$ *or*
   (b) $\Delta p \in (\Delta p_3, 1)$ *and* $p \in [0, 1 - \Delta p]$

where $p_3 \equiv \frac{\pi}{F} - \frac{\Delta p}{\beta}$, $\Delta p_3 \equiv \beta\frac{\pi}{F}$ *and* $\Delta p_4 \equiv \frac{\beta}{1-\beta}(\frac{\pi}{F} - 1)$.

Proof. See Appendix A. ■

Proposition 1 can be intuitively explained as follows. Firms keep hard evidence, rather than destroying it, if the base probability of detection is relatively low, and in particular lower than the threshold value p_3. The latter balances the CCE, that is, the expected foregone collusive profits if firms destroy the hard evidence ($\beta\pi$) and the CDE, that is, the incremental expected fine due to keeping hard evidence (ΔpF).[21] The threshold value p_3 is positively related to the collusive profits-fine ratio and the cartel collapse probability and negatively related to the additional probability of detection due to retaining hard evidence. Thus, an increase either in the collusive profits-fine ratio or in the cartel collapse probability makes it more likely that firms would refrain from destroying the hard evidence.

Consider the effects of an increase in the cartel collapse probability. If firms destroy the hard evidence the cartel's expected profits shrinks, given that with a higher probability the cartel collapses. At the same time if firms keep the hard evidence the expected collusive profits are intact. In other words, the CCE is reinforced, while the CDE remains unchanged. An increase in collusive profits also strengthens the CCE, since firms forego higher future profits when the cartel collapses. Moreover, the CDE is alleviated since the net gain from keeping hard evidence increases. A similar argument holds when the fine decreases. Thus, firms are more inclined to keep hard evidence both with an increase in the cartel collapse probability and with an increase in the collusive profits-fine ratio. These effects are illustrated in Figure 1 below.

When the cartel collapse probability is zero, that is, cartel's sustainability is insensitive to hard evidence (i.e. $\beta = 0$) , firms always destroy the hard evidence. Retaining evidence does not bring about any benefit in that case. On the contrary, when the cartel collapse probability in absence of hard evidence is sufficiently high (i.e. $\beta > \frac{1}{2}$), the foregone profits linked to firms' decision to destroy the hard evidence are considerably high. In that case the CCE dominates the CDE. Thus, firms keep hard evidence to sustain the cartel, although they run the risk of a higher probability of detection.

While an increase in the additional probability of detection does not influence the CCE, such an increase bolsters the CDE. Thus, firms are more likely to destroy the hard evidence. When this probability is sufficiently low ($\Delta p < \Delta p_5$) then the incremental fine associated with firms decision to keep hard evidence is very low, the CDE dominates the CCE, making firms keep the hard evidence to enhance the stability of the cartel. Conversely, when Δp is sufficiently high ($\Delta p > \Delta p_3$)

[21] Notice that p_3 can can be expressed as $p_3 = \frac{1}{\beta F}(\beta\pi - \Delta pF)$. The first term in the parenthesis captures the CCE, while the second term captures the CDE.

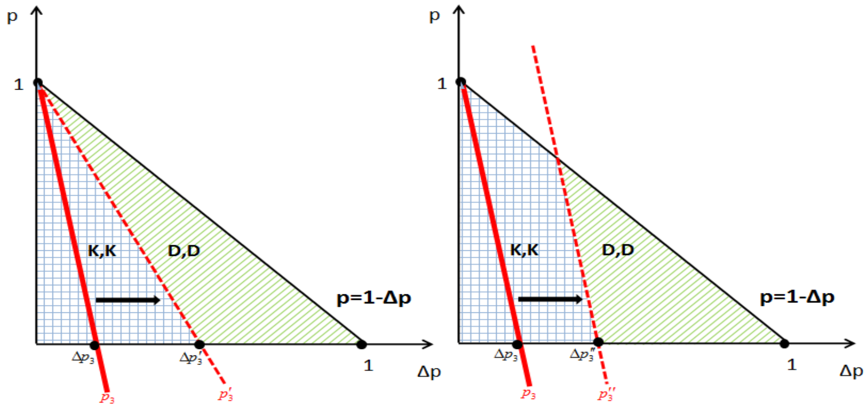

Figure 1: Without LP. Graph A: An increase in β causes a non-parallel shift of p_3 to the right to p_3'. Graph B: An increase in $\frac{\pi}{F}$ causes a parallel shift of p_3 to the right to p_3''. Both graphs are drawn with the initial assumption that $\frac{\pi}{F} = 1$.

then the incremental fine is very high, the CDE dominates the CCE, making firms destroy the hard evidence (see Figure 1, Graph B).

Corollary 1 *Firms are more likely to keep the hard evidence with a higher β and a lower Δp. In the extreme case where $\beta = 0$, firms always destroy the hard evidence. If $\beta > \frac{1}{2}$ firms always keep the hard evidence.*

4 Extended model: with Leniency Program

In this section we extend the baseline model so that a LP is part of the enforcement policy. This extension allows to shed light on the influence of a LP on firms' decision to keep or destroy hard evidence. Moreover, it enables to investigate the conditions under which firms keep hard evidence albeit not report it to the AA.

Within this richer framework the cartel may be detected either by an AA's own investigations or by a cartel's member self-reporting to the AA, before an investigation is initiated in the market.

4.1 The timing of the stage game

In each period the timing of the game is as follows:

- Stage 0 (Policy Design): The AA commits and announces a certain enforcement policy which includes a LP.

- Stage 1 (Cartel Formation): Having observed the enforcement policy and having full and complete information about parameters Δp and β, firms decide whether to communicate and form a cartel. Hard evidence is produced.

- Stage 2 (Keep or Destroy): Firms simultaneously decide whether to keep or destroy the hard evidence of their illegal agreement. If firms choose to keep the hard evidence then move to Stage 3. If firms destroy the hard evidence then move to Stage 3 with probability $1-\beta$. With probability β the cartel collapse and firms obtain zero profits. The game restarts from Stage 0.

- Stage 3 (Revelation of hard evidence): Firms simultaneously decide whether to report or not the hard evidence to the AA.

- Stage 4 (Policy Implementation): If at least one firm applies for leniency then the cartel is detected. If no firm applies for leniency then the AA initiates an investigation into the market. The success of its investigation hinges on the existence of hard convicting evidence.

4.2 Firms' strategies and payoffs

For the purposes of our analysis we consider only the three symmetric strategy profiles: (Destroy, Destroy), (Keep and Not Report, Keep and Not Report) and (Keep and Report, Keep and Report).[22]

Consider first the strategy profile (Destroy, Destroy). The two colluding firms agree to destroy the hard evidence. As shown in Section 3.2 the expected discounted payoff associated with this strategy profile is $V_D = \frac{(1-\beta)(\pi-pF)}{1-\delta}$. A deviant firm that keeps the hard evidence, rather than destroying it, has two options. Either to report the evidence to the AA and obtain full amnesty or not report the hard evidence. In the former case the deviant firm receives an expected payoff equal to $V_D^{(d,r)} = (1-\beta)\pi$, while in the latter case its expected payoff is $V_D^{(d,nr)} = (1-\beta)[\pi - (p + \frac{\Delta p}{2})F]$. Clearly a deviant firm will never keep hard

[22] There are also other possible (asymmetric) strategy profiles that firms may coordinate on: (Destroy, Keep and Report), (Destroy, Keep and Not Report) and (Keep and Report, Keep and Not Report). (Destroy, Keep and Not Report) is Pareto dominated by (Destroy, Destroy), as $V_D = \frac{(1-\beta)(\Pi-pF)}{1-\delta} > \frac{(1-\beta)[\Pi-(p+\frac{\Delta p}{2})F]}{1-\delta} = V_{D,KNR}$. (Destroy, Keep and Report) is Pareto dominated by (Keep and Report, Keep and Report), as $V_{KR} = \frac{\pi-\frac{F}{2}}{1-\delta} > \frac{(1-\beta)(2\pi-F)}{2(1-\delta)} = V_{D,KR}$. Note also that the strategy profile (Keep and Report, Keep and Not Report) yields the same payoff as the strategy profile (Keep and Report, Keep and Report). This is intuituve given that only one firm is eligible to receive amnesty.

evidence without reporting it to the AA, as $V_D^d > V_D^{d'}$. An analogous argument to the baseline model holds with regards to deviations in the market.

Remark 1 *A firm that deviates from (*Destroy, Destroy*) will never keep hard evidence without reporting it to the AA.*

Consider now the strategy profile (Keep and Not Report, Keep and Not Report). The two colluding firms agree to keep the hard evidence, nonetheless, not report it to the AA.[23] Given that both firms keep hard evidence the cartel is detected with probability $p + \Delta p$, in which case firms must pay a fine F. The expected discounted payoff of this strategy profile is $V_{KNR} = \frac{\pi-(p+\Delta p)F}{1-\delta}$. Two types of deviation from this strategy profile are possible. A firm may deviate at the *revelation* stage and report the hard evidence to the AA. In that case the deviant firm receives full amnesty. Given that the cartel does not collapse, since the two firms continue to keep hard evidence, the deviation payoff is $V_{KNR}^{(d,r)} = \pi$. If instead a deviation occurs at the *keep-destroy* stage, whereby the deviant firm destroys the hard evidence, the probability of detection decreases to $p + \frac{\Delta p}{2}$. However, the cartel may in that case collapse with probability β. Thus, the expected deviation payoff is $V_{KNR}^{(d,nr)} = (1-\beta)[\pi - (p+\frac{\Delta p}{2})F]$. Clearly, a deviant firm will never destroy the hard evidence, given that $V_{KNR}^{(d,r)} > V_{KNR}^{(d,nr)}$.

A similar argument for the deviation in the market as in the benchmark model without LP holds. Therefore, a simultaneous deviation in the *revelation* stage or in the *keep-destroy* stage with deviation in the market is inferior, in terms of profitability, to the deviations described above.

Remark 2 *A firm that deviates from (*Keep and Not Report, Keep and Not Report*) will never destroy the hard evidence.*

Lastly, consider the strategy profile (Keep and Report, Keep and Report). The two collusive firms agree to keep hard evidence and subsequently report it to the AA. The cartel is detected with certainty and firms pay an expected fine $\frac{F}{2}$. The expected discounted payoff of this strategy profile is $V_{KR} = \frac{\pi - \frac{F}{2}}{1-\delta}$. Again, two types of deviations may occur from this strategy profile. A firm may deviate at the *keep-destroy* stage and instead of keeping the hard evidence destroy it. If this is the case then the cartel collapse with probability β. At the same time, the cartel is detected with certainty given that the compliant firm keeps and reports the hard evidence to the AA. The deviant firm's payoff is $V_{KR}^d = (1-\beta)(\pi - F)$.

[23] (Keep, Keep) from Section 3 is equivalent to (Keep and Not Report, Keep and Not Report). However, the deviations from this strategy profile are different. A firm that deviates from (Keep and Not Report, Keep and Not Report) can now report the hard evidence. This option is not available to firms when a LP does not exist.

Indeed, deviating from this strategy does not bring about any benefit, as such deviation radically increases the probability of paying a fine and at the same time it deprives the deviator, as well as the compliant firm, all future collusive profits. The second type of deviation may occur at the *revelation* stage. Trivially, any deviation at this stage will be unprofitable as the deviant firm pays the full fine F with certainty, rather than an expected fine of $\frac{F}{2}$. Besides, firms are deprived of future collusive profits as they enter a permanent punishment phase. The same line of reasoning for the simultaneous deviation in the market holds as for the strategy profiles discussed above.

4.3 Equilibrium Analysis

The game we have to analyze in this section is a repeated sequential move game with imperfect information. Contrary to the game in Section 3 the firm(s) that choose to keep the hard evidence at the *Keep-Destroy* stage have an additional decision to make at the *revelation* stage: either to report the hard evidence to the AA or not. This option for firms that keep hard evidence transpires as a result of the availability of the LP.

As in Section 3 the appropriate solution concept is SPE. We only consider pure strategies. When multiple equilibria exist we apply the Pareto dominance selection criterion.

4.3.1 *Revelation subgame* - Report v. Not Report

This subgame is reached if firms have already chosen to keep the hard evidence at the *Keep-Destroy* subgame. Firms' decision is either to report the hard evidence to the AA or keep the hard evidence without reporting to the AA.

To begin with suppose that the two firms agree to keep the hard evidence and subsequently report it to the AA, i.e. they coordinate on (KEEP AND REPORT, KEEP AND REPORT). The latter is a NE iff $V_{KR} \geq V_{KR}^{d}$. Given Assumption 4 this inequality always holds.

Lemma 4 *(*KEEP AND REPORT, KEEP AND REPORT*) is a strict NE of the Revelation subgame.*

Suppose now that firms agree to keep the hard evidence, nevertheless not report it to the AA, i.e. coordinate on (KEEP AND NOT REPORT, KEEP AND NOT REPORT). The latter is a NE iff $V_{KNR} \geq V_{KNR}^{d}$ which holds iff $p + \Delta p \leq \delta \frac{\pi}{F}$.

Lemma 5 *(*KEEP AND NOT REPORT, KEEP AND NOT REPORT*) is a NE of the Revelation subgame iff $p + \Delta p \leq \delta \frac{\pi}{F}$.*

Notice that when $\delta\frac{\pi}{F} > 1$, (Keep and Not Report, Keep and Not Report) is a strict NE.

Given Lemmata 4 and 5 we apply the Pareto criterion. Thus, (Keep and Report, Keep and Report) Pareto dominates (Keep and Not Report, Keep and Not Report) iff $V_{KR} > V_{KNR}$. The latter inequality holds iff $p + \Delta p > \frac{1}{2}$.

Lemma 6 *The Pareto dominant NE of the Revelation subgame is:*

1. *(*Keep and Not Report, Keep and Not Report*) iff* $p + \Delta p \leq \frac{1}{2}$
2. *(*Keep and Report, Keep and Report*) iff* $p + \Delta p > \frac{1}{2}$

Proof. See Appendix B. ■

In words, if the (aggregate) probability of detection when firms keep hard evidence is relatively high, in particular $p+\Delta p > \frac{1}{2}$, firms report the hard evidence. The underlying reasoning is that when the probability of detection with hard evidence is greater (lower) than $\frac{1}{2}$, the fine that firms expect to pay, in case of detection by the AA, is greater (lower) than the one associated with self-reporting.

We now proceed backwards to the *Keep-Destroy* subgame. Depending on which strategy dominates at the *revelation* subgame we distinguish, by Lemma 6, two cases, which we examine below.

4.3.2 Keep-Destroy subgame for $p + \Delta p > \frac{1}{2}$

If $p + \Delta p > \frac{1}{2}$ then by Lemma 6 at the *revelation* stage the firms self-report to the AA, provided that they keep hard evidence. Thus, for those parameters' values firms' decisions is either to destroy the hard evidence or keep the hard evidence and then report it to the AA.

To begin with, suppose that the two firms agree to keep the hard evidence (and subsequently report it to the AA). This forms a NE iff $V_{KR} \geq V_{KR}^d$. The latter inequality always holds given that $\beta \in [0, 1]$.

Lemma 7 *(*Keep and Report, Keep and Report*) is a strict NE of the Keep-Destroy subgame for* $p + \Delta p > \frac{1}{2}$.

Suppose now that firms agree to destroy the hard evidence, i.e. coordinate on (Destroy, Destroy). This strategy profile forms a NE iff $V_D \geq V_D^d$. The latter inequality holds iff $p \leq \delta\frac{\pi}{F}$.

Lemma 8 *(*Destroy, Destroy*) is a NE of the Keep-Destroy subgame if* $p \leq \delta\frac{\pi}{F}$.

Considering that $p \in [0, 1-\Delta p]$ from Lemma 8 we can distinguish two cases: either $i)$ $\delta\frac{\pi}{F} \leq 1-\Delta p$ or $ii)$ $\delta\frac{\pi}{F} > 1-\Delta p$. If the former condition holds then for $p > \delta\frac{\pi}{F}$ (Keep and Report, Keep and Report) is the unique NE of the *Keep-Destroy* subgame. Otherwise, both (Destroy, Destroy) and (Keep and Report, Keep and Report), are NE of the *Keep-Destroy*.

Lemma 9 *For $p + \Delta p > \frac{1}{2}$ the Keep-Destroy subgame has two NE iff:*

1. $\delta\frac{\pi}{F} \leq 1-\Delta p$ *and* $p \in [\frac{1}{2}-\Delta p, \delta\frac{\pi}{F}]$ *or*
2. $\delta\frac{\pi}{F} > 1-\Delta p$ *and* $p \in [\max\{0, \frac{1}{2}-\Delta p\}, 1-\Delta p]$

Given Lemma 9 we apply the Pareto criterion. Thus, (Keep and Report, Keep and Report) Pareto dominates (Destroy, Destroy) iff $V_{KR} > V_D$. The latter inequality holds iff $p > p_4 \equiv \frac{F-2\beta\pi}{2(1-\beta)F}$.

Lemma 10 *(*Keep and Report, Keep and Report*) Pareto dominates (*Destroy, Destroy*) iff $p > p_4 \equiv \frac{F-2\beta\pi}{2(1-\beta)F}$.*

By taking into consideration that $i)$ $p_4 < 0$ iff $\frac{\pi}{F} > \frac{1}{2\beta}$, $ii)$ $p_4 < \frac{1}{2}-\Delta p$ iff $\Delta p < \Delta p_5 \equiv \frac{\beta}{1-\beta}\frac{\pi}{F} - \frac{\beta}{2(1-\beta)}$ and $iii)$ $p_4 > 1-\Delta p$ iff $\Delta p < \Delta p_6 \equiv \frac{1-2\beta}{2(1-\beta)} + \frac{\beta}{1-\beta}\frac{\pi}{F}$, and Lemma 10 we establish the Pareto dominant SPE of the game when $p+\Delta p > \frac{1}{2}$.

Lemma 11 *The Pareto dominant SPE of the model with LP for $p + \Delta p > \frac{1}{2}$ is:*

1. *(*Keep and Report, Keep and Report*) iff:*
   - *(a)* $\beta < \frac{1}{2}$ *and*
     - *i.* $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p < \Delta p_5$ *and* $p \in [\frac{1}{2}-\Delta p, 1-\Delta p]$ *or*
     - *ii.* $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_6]$ *and* $p \in (p_4, 1-\Delta p]$ *or*
     - *iii.* $\frac{\pi}{F} > \frac{1}{2\beta}$ *and* $p \in [\max\{0, \frac{1}{2}-\Delta p\}, 1-\Delta p]$ *or*
   - *(b)* $\beta > \frac{1}{2}$, $\frac{\pi}{F} \geq 1$ *and* $p \in [\max\{0, \frac{1}{2}-\Delta p\}, 1-\Delta p]$
2. *(*Destroy, Destroy*) iff* $\beta < \frac{1}{2}$, $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$ *and*
   - *(a)* $\Delta p \in [\Delta p_5, \Delta p_6]$ *and* $p \in [\max\{0, \frac{1}{2}-\Delta p\}, p_4]$ *or*
   - *(b)* $\Delta p > \Delta p_6$ *and* $p \in [0, 1-\Delta p]$

where $p_4 = \frac{F-2\beta\pi}{2(1-\beta)F}$, $\Delta p_5 = \frac{\beta}{1-\beta}\frac{\pi}{F} - \frac{\beta}{2(1-\beta)}$ *and* $\Delta p_6 = \frac{1-2\beta}{2(1-\beta)} + \frac{\beta}{1-\beta}\frac{\pi}{F}$.

Proof. See Appendix B. ■

The intuition underlying Lemma 11 is the following. Firms keep the hard evidence and subsequently report it to the AA, rather than destroying it, if p is relatively high, and in particular greater than the threshold value p_4. The latter balances the CCE ($\beta\pi$) and the CAE ($\frac{F}{2}$).[24] The threshold value p_4 is negatively related both, to β and $\frac{\pi}{F}$. Thus, with an increase either in β or in $\frac{\pi}{F}$ firms are less inclined to destroy the hard evidence. An increase in β makes the CCE sharper, as the foregone profits in case of the collapse of the cartel are greater, leaving the CDE intact. When β is sufficiently high ($\beta > \frac{1}{2}$), the CCE outweights the CAE. The foregone profits in the case where the cartel collapse are greater than the expected fine with self-reporting. Thus, firms keep and report the hard evidence to the AA. An increase in $\frac{\pi}{F}$ makes the CCE sharper, while at the same time it mitigates the CDE. Moreover, when $\frac{\pi}{F}$ is sufficiently high ($\frac{\pi}{F} > \frac{1}{2\beta}$) then firms always keep the hard evidence.

Suppose that β is relatively low ($\beta < \frac{1}{2}$) and that p, the base probability of detection, is also relatively low, while Δp is sufficiently high ($\Delta p > \Delta p_6$), but at the same time the aggregate probability of detection is relatively high ($p + \Delta p > \frac{1}{2}$). Then, the expected cost associated with firms' decision to destroy the hard evidence, that is, the sum of the fine firms have to pay in case of detection by the AA on the basis of indirect evidence and the expected foregone profits in case of the collapse of the cartel, is lower than the fine associated with self-reporting to the AA. Thus, firms destroy the hard evidence. On the contrary, if Δp is sufficiently low ($\Delta p < \Delta p_5$) then the expected fine with self-reporting ($\frac{F}{2}$) is lower than the cost associated with firms' decision to destroy the hard evidence (expected fine and expected foregone profits). Thus firms keep the hard evidence to exploit the LP.

Corollary 2 *Firms are less likely to destroy the hard evidence with a higher β and $\frac{\pi}{F}$ and lower Δp.*

4.3.3 Keep-Destroy subgame for $p + \Delta p \leq \frac{1}{2}$

If $p + \Delta p \leq \frac{1}{2}$ then by Lemma 6 firms do not self-report at the *revelation* stage, provided that they indeed keep hard evidence. Thus, for those parameters' values firms' decision is between destroying and keeping the hard evidence without disclosing it to the AA.

To begin with, suppose that the two firms agree to destroy the hard evidence, i.e. coordinate on (Destroy, Destroy). This strategy profile is a NE if no profitable unilateral deviation exists, i.e. iff $V_D \geq V_D^d$. The latter inequality always holds.

[24] Notice that p$_4$ can be expressed as: $p_4 = \frac{1}{(1-\beta)F}(\frac{F}{2} - \beta\pi)$.

Lemma 12 *(*DESTROY, DESTROY*) is a strict NE of the keep-Destroy subgame for* $p + \Delta p \leq \frac{1}{2}$.

Suppose now that firms agree to keep the hard evidence that, however, they do not report to the AA at the *revelation* stage, i.e. coordinate on (KEEP AND NOT REPORT, KEEP AND NOT REPORT). This strategy profile is a NE iff $V_{KNR} \geq V^d_{KNR}$. This inequality holds iff $p \leq p_5 \equiv \frac{\pi}{F} - \frac{(\delta+\beta-\beta\delta+1)\Delta p}{2(\delta+\beta-\beta\delta)}$.

Lemma 13 *(*KEEP AND NOT REPORT, KEEP AND NOT REPORT*) is a NE of the Keep-Destroy subgame iff* $p \leq p_5$.

Given Lemmata 12 and 13 for $p \leq p_5$ there are two NE. Taking into account that *i)* $p_5 > \frac{1}{2} - \Delta p$ and *ii)* $p_5 > 0$ iff $\Delta p < \Delta p_7 \equiv \frac{\pi}{F}\frac{2(\beta+\delta-\beta\delta)}{\beta+\delta-\beta\delta+1}$, we conclude that (KEEP AND NOT REPORT, KEEP AND NOT REPORT) Pareto dominates (DESTROY, DESTROY) if $V_{KNR} > V_D$. The latter inequality holds iff $p < p_3 \equiv \frac{\pi}{F} - \frac{\Delta p}{\beta}$. Notice that this is the same condition as in the baseline model, however here it holds only for $p + \Delta p \leq \frac{1}{2}$.

Lemma 14 KEEP AND NOT REPORT, KEEP AND NOT REPORT*) Pareto dominates (*DESTROY, DESTROY*) iff* $p < p_3 \equiv \frac{\pi}{F} - \frac{\Delta p}{\beta}$.

By taking into consideration that: *i)* $p_3 > 0$ iff $\Delta p < \Delta p_3 \equiv \beta\frac{\pi}{F}$ and $p_3 < \frac{1}{2} - \Delta p$ iff $\Delta p \geq \Delta p_5 \equiv \frac{\beta}{1-\beta}(\frac{\pi}{F} - \frac{1}{2})$ and Lemma 14 we establish the Pareto dominant SPE of the game for $p + \Delta p \leq \frac{1}{2}$.

Lemma 15 *The Pareto dominant SPE of the model with LP for* $p + \Delta p \leq \frac{1}{2}$ *is:*

1. (KEEP AND NOT REPORT, KEEP AND NOT REPORT) *iff:*
   (a) $\beta < \frac{1}{2}$ *and*
      i. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p < \Delta p_5$ *and* $p \in [0, \frac{1}{2} - \Delta p]$ *or*
      ii. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_3]$ *and* $p \in [0, p_3]$ *or*
      iii. $\frac{\pi}{F} \in [\frac{1}{2\beta}, \frac{1}{\beta}]$, $\Delta p < \frac{1}{2}$ *and* $p \in [0, \frac{1}{2} - \Delta p]$ *or*
      iv. $\frac{\pi}{F} > \frac{1}{\beta}$, $\Delta p < \frac{1}{2}$ *and* $p \in [0, \frac{1}{2} - \Delta p]$ *or*
   (b) $\beta > \frac{1}{2}$, $\frac{\pi}{F} \geq 1$, $\Delta p < \frac{1}{2}$ *and* $p \in [0, \frac{1}{2} - \Delta p]$
2. (DESTROY, DESTROY*) iff* $\beta < \frac{1}{2}$ *and*
   (a) $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_3]$ *and* $p \in [p_3, \frac{1}{2} - \Delta p]$ *or*

(b) $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $[\Delta p_3, \frac{1}{2}]$ and $p \in [0, \frac{1}{2} - \Delta p]$

where $p_3 = \frac{\pi}{F} - \frac{\Delta p}{\beta}$, $\Delta p_3 = \beta \frac{\pi}{F}$ *and* $\Delta p_5 = \frac{\beta}{1-\beta}\frac{\pi}{F} - \frac{\beta}{2(1-\beta)}$.

Proof. See Appendix B. ■

The intuition of Lemma 15 is the following. Firms keep hard evidence without reporting it to the AA, rather than destroying, if p is relatively low, and in particular lower than the threshold value p_3. The latter balances the CCE and the CDE. The threshold value of p_3 is positively related to $\frac{\pi}{F}$ and β and negatively related to Δp. Thus, an increase either in $\frac{\pi}{F}$ or in β makes firms less prone to destroy the hard evidence. Equivalently, it makes it more likely that firms will keep nevertheless not report the hard evidence. Such changes make the CCE more acute, while at the same time they alleviate the CDE. When β is sufficiently high ($\beta > \frac{1}{2}$), the CCE outweights the CDE so that the firms keep hard evidence without reporting it to the AA, rather than destroying it.

The CCE is immuned to a change in Δp. Such a change, on the contrary, markedly affects the CDE. Specifically, an increase (decrease) Δp makes the CDE more (less) acute. Thus, an increase in Δp makes firms' decision to destroy the hard evidence more likely. When Δp is sufficiently low ($\Delta p < \Delta p_5$) then the CCE dominates the CDE (p_3 becomes greater than 1) and thus firms always keep the hard evidence. Conversely, when Δp is sufficiently high ($\Delta p > \Delta p_3$) then the CDE dominates the CCE (p_3 becomes negative) and thus firms always destroy the hard evidence.

Corollary 3 *Firms are less prone to destroy the hard evidence with a higher* β *or* $\frac{\pi}{F}$. *They are less likely to keep the hard evidence with a higher* Δp.

By Lemmata 11 and 15 we can state the main proposition of this section.

Proposition 2 *For given policy and other parameters* (p, Δp, F, β, π) *the Pareto dominant SPE of the extended game with LP is:*

1. (Destroy, Destroy) *iff* $\beta < \frac{1}{2}, \frac{\pi}{F} \in [1, \frac{1}{2\beta}]$ *and*
   (a) $\Delta p \in [\Delta p_5, \Delta p_3]$ and $p \in [p_3, \frac{1}{2} - \Delta p]$ or
   (b) $[\Delta p_3, \frac{1}{2})$ and $p \in [0, \frac{1}{2} - \Delta p]$ or
   (c) $\Delta p \in [\Delta p_5, \Delta p_6]$ and $p \in [\max\{0, \frac{1}{2} - \Delta p\}, p_4]$ or
   (d) $\Delta p > \Delta p_6$ and $p \in [0, 1 - \Delta p]$
2. (Keep and Not Report, Keep and Not Report) *iff:*

(a) $\beta < \frac{1}{2}$ *and*

i. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p < \Delta p_5$ *and* $p \in [0, \frac{1}{2} - \Delta p]$ *or*

ii. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_3]$ *and* $p \in [0, p_3)$ *or*

iii. $\frac{\pi}{F} > \frac{1}{2\beta}$, $\Delta p < \frac{1}{2}$ *and* $p \in [0, \frac{1}{2} - \Delta p]$ *or*

(b) $\beta > \frac{1}{2}, \frac{\pi}{F} \geq 1$, $\Delta p < \frac{1}{2}$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

3. *(*Keep and Report, Keep and Report*) iff:*

(a) $\beta < \frac{1}{2}$ *and*

i. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p < \Delta p_5$ *and* $p \in [\frac{1}{2} - \Delta p, 1 - \Delta p]$ *or*

ii. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_6]$ *and* $p \in (p_4, 1 - \Delta p]$ *or*

iii. $\frac{\pi}{F} > \frac{1}{2\beta}$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$ *or*

(b) $\beta > \frac{1}{2}$, $\frac{\pi}{F} \geq 1$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$

where $p_3 = \frac{\pi}{F} - \frac{\Delta p}{\beta}$, $p_4 = \frac{F - 2\beta\pi}{2(1-\beta)F}$, $\Delta p_3 = \beta\frac{\pi}{F}$, $\Delta p_5 = \frac{\beta}{1-\beta}\frac{\pi}{F} - \frac{\beta}{2(1-\beta)}$ *and* $\Delta p_6 = \frac{1-2\beta}{2(1-\beta)} + \frac{\beta}{1-\beta}\frac{\pi}{F}$.

Proof. See Appendix B. ■

Depending on the parameters' values of the game (p, Δp, β, π , F), all three strategies could emerge as Pareto dominant SPE. If the base probability of detection is relatively low, and in particular lower than the threshold value p_3, firms agree to keep, nevertheless not report the hard evidence to the AA. In that case firms keep the hard evidence to enhance cartel's sustainability rather than to exploit the LP. Conversely, if the base probability is sufficiently high, and in particular greater than the threshold value p_3, firms agree to keep the hard evidence and then report to the AA. In that case firms keep the hard evidence to exploit the LP rather than to enhance cartel's sustainability. For intermediate values of the base probability of detection, that is, for $p \in [p_3, p_4]$, the CDE and the CAE prevail over the CCE. Thus, firms destroy the hard evidence in order to minimize the probability of cartel detection.

Firms are more prone to keep hard evidence and either report not with a higher β or $\frac{\pi}{F}$. Their decision on whether to self-report or not depends on the aggregate probability of detection. Firms are less prone to keep without reporting if p or Δp increase.

Corollary 4 *Firms are less likely to destroy the hard evidence when β or $\frac{\pi}{F}$ increase. They are more likely to destroy hard evidence when Δp decrease.*

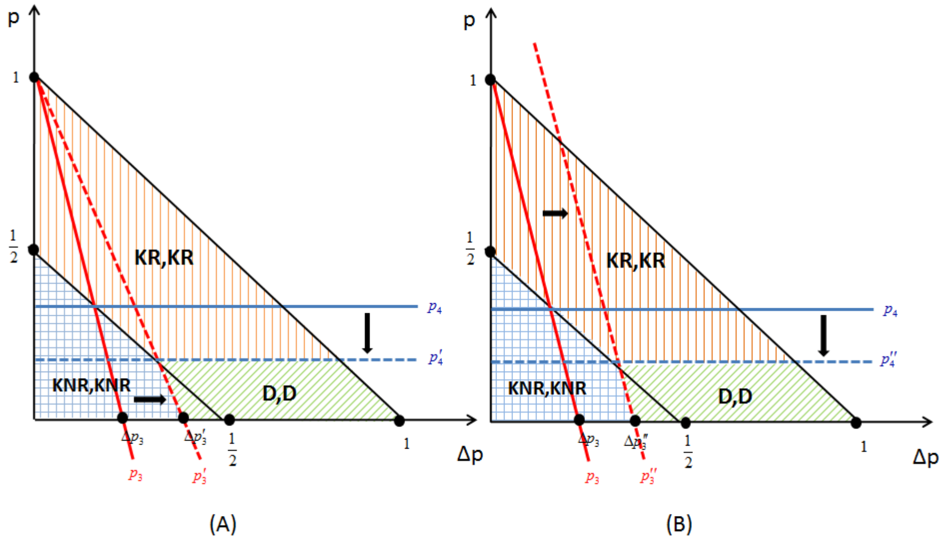

Figure 2: With LP. Graph A shows the effect of an increase in β. Graph B shows the effect of an increase in $\frac{\pi}{F}$. Both graphs are drawn with the initial assumption that $\frac{\pi}{F} = 1$.

We have to underline though that when β is sufficiently high ($\beta > \frac{1}{2}$), firms never destroy the hard evidence. With a sufficiently high β the expected foregone profits are higher than the expected fine associated with firms decision to keep the hard evidence. Moreover, if Δp is sufficiently high ($\Delta p > \frac{1}{2}$) then firms always keep and exploit the LP by reporting the evidence to the AA. In this latter case the expected fine with self-reporting is lower than the one without self-reporting. The same conclusion is reached if $\frac{\pi}{F}$ is sufficiently high ($\frac{\pi}{F} > \frac{1}{2\beta}$).

Corollary 5 *If $\beta < \frac{1}{2}$ and $\frac{\pi}{F} > \frac{1}{2\beta}$ or $\beta > \frac{1}{2}$ then firms always keep hard evidence. If in addition $\Delta p < \frac{1}{2}$ then the Pareto dominant SPE is (*Keep and Report, Keep and Report*) if $p \in [\frac{1}{2} - \Delta p, 1 - \Delta p]$ and (*Keep and Not Report, Keep and Not Report*) if $p \in [0, \frac{1}{2} - \Delta p]$. However, if $\Delta p \geq \frac{1}{2}$ the unique Pareto dominant SPE is (*Keep and Report, Keep and Report*) $\forall p \in [0, 1 - \Delta p]$.

If $\beta = 0$ and $\Delta p > \frac{1}{2}$ (implying that $p < \frac{1}{2}$) then firms always destroy the hard evidence.[25] In words, if the cartel's sustainability is independent of hard evidence and if the additional probability of detection is sufficiently high (i.e. $\Delta p > \frac{1}{2}$), the unique SPE is (Destroy, Destroy). Given that $\beta = 0$ the CCE evaporates. Moreover, for these parameters' values the CDE dominates the

[25] If $\beta = 0$ then $\Delta p_3 = 0$, $\Delta p_5 = 0$ and $\Delta p_6 = \frac{1}{2}$.

CAE, i.e. the expected fine with self-reporting is lower than the one without self-reporting. Given that the expected fine with self-reporting is higher than the cost associated with firms' decision to destroy the hard evidence (foregone profits and expected fine), firms destroy the evidence. On the other hand, if the additional probability of detection is relatively low ($\Delta p < \frac{1}{2}$) firms destroy the hard evidence as long as the base probability of detection is relatively low ($p < \frac{1}{2}$). In that case the CDE dominates both the CAE and the CCE. The expected fine firms have to pay if convicted on the basis of indirect evidence is lower than the one firms have to pay if they decide to keep the hard evidence regardless of reporting or not to the AA. On the contrary, if the base probability of detection is sufficiently high ($p > \frac{1}{2}$) then firms keep the hard evidence and exploit the LP. In the latter case the CAE dominates the CDE. For a graphical illustration see Figure 3 below (Graphs A and B).

Corollary 6 *If $\beta = 0$ then for any $p < \frac{1}{2}$ ($p > \frac{1}{2}$) firms always destroy (keep) the hard evidence.*

If $\beta = 1$, that is, if hard evidence is imperative for the survival of the cartel then firms always keep the hard evidence. The decision on whether to apply for leniency depends on the aggregate probability of detection. If it is relatively high ($p + \Delta p > \frac{1}{2}$) then firms always find it advantageous to exploit the LP as the expected fine with self-reporting is lower than that without self-reporting. If it is relatively low ($p + \Delta p < \frac{1}{2}$) then firms keep nevertheless not report. The expected fine without self-reporting is lower than the one associated with firms' decision to self-report. For a graphical illustration see Graph C in Figure 3.

Corollary 7 *When $\beta = 1$ firms always keep hard evidence. If in addition $p + \Delta p > \frac{1}{2}$ then they exploit the LP. Otherwise, they keep the hard evidence only to sustain the cartel.*

4.4 Comparison with the baseline model without LP

To simplify the comparison of the two settings, with and without LP, let without loss of generality $\pi = F$ so that $\frac{\pi}{F} = 1$. Then, a simple contrast of Propositions 1 and 2 reveal that the introduction of the LP as part of the enforcement policy makes firms more prone to keep the hard evidence. The reason is that the introduction of the LP provides an *option value* to those firms that keep hard evidence. Firms keep hard evidence not only to enhance the administration of their agreement, and sustain the cartel, but also to exploit the LP, when such an exploitation is advantageous for them.

Figure 4 below illustrates the effects of the introduction of a LP. Notice that for $p \in [\frac{1}{2} - \Delta p, p_3]$ firms continue to keep the hard evidence, however, they exploit

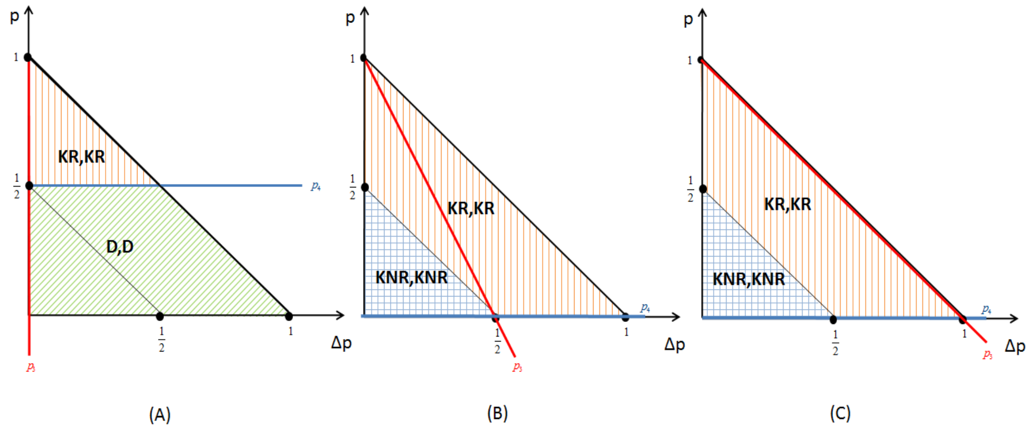

Figure 3: Some extreme cases. Graph A: $\frac{\pi}{F} = 1$ and $\beta = 0$. Graph B: $\frac{\pi}{F} = 1$ and $\beta = \frac{1}{2}$. Graph C: $\frac{\pi}{F} = 1$ and $\beta = 1$.

the LP by reporting to the AA (indicated by $NR \rightarrow R$). The primal objective for keeping hard evidence is to enhance the administration of their unlawful agreement and not to exploit the LP. However, firms also find it profitable to exploit the LP and pay a lower fine in case of conviction. For $p > max\{p_3, p_4\}$ firms' decisions at the keep-destroy stage is reversed. Now firms keep the hard evidence primarily to exploit the LP (indicated by $D \rightarrow KR$ in Figure 4 below). The expected fine linked to firms' decisions to keep and report the hard evidence is lower than the cost associated with firms' decision to destroy the evidence (foregone profits and expected fine). Therefore, by keeping hard evidence the firms enhance the administration of their agreement, and thus sustain the cartel, and simultaneously exploit the LP by self-reporting.

Corollary 8 *For $p \in [\frac{1}{2} - \Delta p, p_3]$ firms' decisions to keep the hard evidence, when a LP is available, is intact.*[26] *However, firms exploit the LP by reporting the hard evidence to the AA. On the contrary, for $p > max\{p_3, p_4\}$ firms' decisions to destroy the hard evidence is reversed when a LP is available.*[27] *Firms keep the hard evidence primarily to exploit the LP.*

[26] Notice that $p \in [\frac{1}{2} - \Delta p, p_3]$ if $\Delta p < \Delta p_5$.
[27] Equivalently, if (i) $\Delta p > \Delta p_5$ and $p \in [p_3, 1 - \Delta p]$ or (ii) $\Delta p > p_5$.

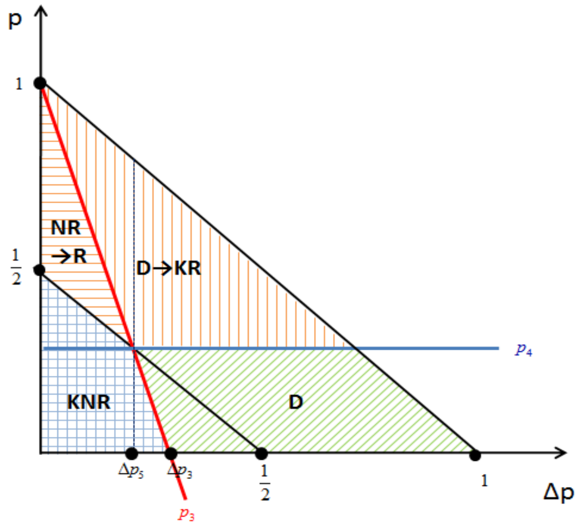

Figure 4: The impact of a LP on firms' decision.

5 Conclusions

The motivation of this paper derives from the empirical observation that many collusive firms keep hard evidence, even when the enforcement policy does not encompass a LP. We provide plausible answers to three relevant questions: 1) why firms keep hard evidence when a LP is not available, 2) what is the impact of a LP on firms' decisions' to keep or destroy hard evidence and 3) why firms keep hard evidence, nevertheless not report to the AA.

In absence of a LP we have showed that firms' decisions to keep or destroy the hard incriminating evidence of their illegal communication hinges upon *i*) the sensitivity of cartel's administration and implementation to hard evidence, *ii*) the increase in the probability of detection when firms keep the hard evidence and *iii*) the collusive profits-fine ratio. Reflecting on these theoretical results our paper propose three alternative, but not exclusionary, explanations for the empirical observation that many collusive firms keep hard evidence even when a LP is not part of the enforcement policy. First, cartel's administration and implementation is very sensitive to hard evidence, so that firms keep hard evidence to enhance the stability of their illicit agreement. Second, the efficiency of the AA in detecting and prosecuting cartels with hard evidence is relatively low, or the quality of hard evidence is poor, so that the expected incremental fine, if firms keep the hard evidence, is lower than the expected forgone profits associated with firms' decisions to destroy the hard evidence. And third, the level of the fine imposed on detected cartels is sufficiently lower than the accrued collusive profits, so that firms have more to lose if they destroy the hard evidence and less to pay if detected by the AA.

We have also showed that the introduction of a LP as a policy instrument reinforce firms' incentives to keep hard evidence. The LP essentially provides to those firms that keep hard evidence an option value, which, *ceteris paribus*, decreases the expected fine. Thus, firms may keep the hard evidence not only to enhance the stability of the cartel but also to exploit the LP. The empirical fact that we are experiencing more cartels self-reporting may, therefore, be a *side-effect* of the introduction of LPs.

Finally, collusive firms may keep hard evidence without reporting to the AA. We have showed that this behavior is more likely to manifest when *i*) the probability in which the cartel collapse in absence of hard evidence increases, *ii*) the collusive profits-fine ratio increases and *iii*) the additional probability of cartel detection decreases. Accordingly the empirical fact that AAs investigations expose hard evidence retained by cartels' members can be justified on three grounds. First, that the cartel is very sensitive to hard evidence, so that firms keep hard evidence to enhance the survival rate of the cartel. Second, that the efficiency of the AA in detecting and prosecuting cartels with hard evidence is sufficiently low, so that the incremental and total expected fine is lower than the expected fine with self-reporting. And third, that the level of the fine imposed on detected cartels is sufficiently lower than the accrued collusive profits, so that firms have more to lose if they destroy the hard evidence and less to pay if detected by the AA. It is worth noting that all results in our paper are obtained without allowing for positive rewards.

Despite the simple setting, our paper sheds some light on firms' decisions to keep or destroy the hard incriminating evidence that is generated by the cartel as well as on firms' decisions to report or not the hard evidence to the AA. By abstracting from behavioral elements or cognitive biases our paper predicts a particular pattern of behavior by colluding firms. However, our understanding of the underlying reasons justifying firms' decisions to keep or destroy the hard evidence may be enriched if we diverge from the realm of rational choice theory by integrating bounded rationality in the analysis. For instance, collusive firms may be time inconsistent or overconfident, although they know they have to destroy the hard evidence when they do it is too late!. Such extension to the literature could offer alternative explanations with regard to the research questions of this paper and indeed may come up with very different patterns of behavior. This research avenue remains open to future exploration.

References

Aubert, C., Rey, P., and Kovacic, W. (2006). The impact of leniency and whistle-blowing programs on cartels. *International Journal of Industrial Organization*, 24(6):1241–1266.

Bryant, P. and Eckard, E. (1991). Price fixing: The probability of getting caught. *The Review of Economics and Statistics*, 73(3):531–36.

Buccirossi, P. and Spagnolo, G. (2006). Leniency policies and illegal transactions. *Journal of Public Economics*, 90(6-7):1281–1297.

Combe, E., Monnier, C., and Legal, R. (2008). Cartels: The probability of getting caught in the european union. *BEER paper n. 12*.

Friedman, J. W. (1971). A non-cooperative equilibrium for supergames. *Review of Economic Studies*, 38(113):1–12.

Fudenberg, D. and Tirole, J. (1991). *Game Theory*. The MIT Press.

Harrington, J. (2008). Optimal corporate leniency programs. *The Journal of Industrial Economics*, 56(2):215–246.

Motchenkova, E. (2004). Effects of leniency programs on cartel stability. *Tilburg University, Tilburg Law and Economic Center*.

Motta, M. and Polo, M. (2003). Leniency programs and cartel prosecution. *International Journal of Industrial Organization*, 21(3):347–379.

Ormosi, P. (2011). How big is a tip of the iceberg? A parsimonious way to estimate cartel detection rate. *SSRN eLibrary*.

Silbye, F. (2010). Asymmetric evidence and optimal leniency programs. In *Topics in competitition policy: cartels, leniency and price discrimination*, pages 41–67. University of Copenhagen.

Spagnolo, G. (2004). Divide et impera: Optimal deterrence mechanisms against cartels and organized crime. Econometric Society 2004 North American Winter Meetings, Econometric Society.

Spagnolo, G. (2008). *Handbook of Antitrust Economics*, chapter Leniency and Whistleblowers in Antitrust, pages 259–304. MIT Press.

Tirole, J. (1988). *The Theory of Industrial Organization*. MIT Press, Cambridge, Massachusetts.

Appendix A - Proof of Proposition 1

Lemma 16 *1.* $p_1 \geq 0$ *iff* $\Delta p \leq \Delta p_1$, *where* $\Delta p_1 \equiv \frac{\pi}{F}\frac{2(\delta+\beta-\beta\delta)}{(\delta+\beta-\beta\delta+1)}$

2. $\Delta p_1 > 1$ *iff* $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$

3. $p_1 + \Delta p \leq 1$ *iff* $\Delta p \geq \Delta p_2$, *where* $\Delta p_2 \equiv \frac{2(\beta+\delta-\beta\delta)}{(1-\beta)(1-\delta)}(\frac{\pi}{F} - 1)$

4. $\Delta p_2 > 1$ *iff* $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$

5. $\Delta p_1 \geq \Delta p_2$ *iff* $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$.

Proof of Lemma 16
(1) To show: $p_1 > 0$ iff $\Delta p < \Delta p_1$, where $\Delta p_1 \equiv \frac{\pi}{F}\frac{2(\delta+\beta-\beta\delta)}{(\delta+\beta-\beta\delta+1)}$.

$$\begin{aligned} p_1 &\geq 0 \Leftrightarrow \\ \frac{\pi}{F} - \frac{(\delta+\beta-\beta\delta+1)\Delta p}{2(\delta+\beta-\beta\delta)} &\geq 0 \Leftrightarrow \\ \Delta p &\leq \frac{\pi}{F}\frac{2(\delta+\beta-\beta\delta)}{(\delta+\beta-\beta\delta+1)} \equiv \Delta p_1 \end{aligned}$$

$\therefore QED.$
(2) To show: $\Delta p_1 > 1$ iff $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$.

$$\begin{aligned} \Delta p_1 &> 1 \Leftrightarrow \\ \frac{\pi}{F}\frac{2(\delta+\beta-\beta\delta)}{(\delta+\beta-\beta\delta+1)} &> 1 \Leftrightarrow \\ \frac{\pi}{F} &> \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} \end{aligned}$$

$\therefore QED.$
(3) To show: $p_1 + \Delta p \leq 1$ iff $\Delta p > \Delta p_2$, where $\Delta p_2 \equiv \frac{2(\beta+\delta-\beta\delta)}{(1-\beta)(1-\delta)}(\frac{\pi}{F} - 1)$.

$$\begin{aligned}
\frac{\pi}{F} - \frac{(\delta + \beta - \beta\delta + 1)\Delta p}{2(\delta + \beta - \beta\delta)} + \Delta p &\leq 1 \Leftrightarrow \\
(\frac{\pi}{F} - 1) &\leq \Delta p(\frac{\delta + \beta - \beta\delta + 1}{2(\delta + \beta - \beta\delta} - 1) \Leftrightarrow \\
\Delta p &\geq \frac{2(\beta + \delta - \beta\delta)}{(1-\beta)(1-\delta)}(\frac{\pi}{F} - 1) \equiv \Delta p_2
\end{aligned}$$

$\therefore QED.$

(4) To show: $\Delta p_2 > 1$ iff $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$.

$$\begin{aligned}
\Delta p_2 &> 1 \Leftrightarrow \\
\frac{2(\beta + \delta - \beta\delta)}{(1-\beta)(1-\delta)}(\frac{\pi}{F} - 1) &> 1 \Leftrightarrow \\
\frac{\pi}{F} - 1 &> \frac{(1-\beta)(1-\delta)}{2(\beta + \delta - \beta\delta)} \Leftrightarrow \\
\frac{\pi}{F} &> \frac{(1-\beta)(1-\delta)}{2(\beta + \delta - \beta\delta)} + 1 \Leftrightarrow \\
\frac{\pi}{F} &> \frac{\delta + \beta - \beta\delta + 1}{2(\delta + \beta - \beta\delta)}
\end{aligned}$$

$\therefore QED.$

(5) To show: $\Delta p_1 > \Delta p_2$ iff $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$.

$$\begin{aligned}
\Delta p_1 &> \Delta p_2 \Leftrightarrow \\
\frac{\pi}{F}\frac{2(\delta + \beta - \beta\delta)}{(\delta + \beta - \beta\delta + 1)} &> \frac{2(\beta + \delta - \beta\delta)}{(1-\beta)(1-\delta)}(\frac{\pi}{F} - 1) \Leftrightarrow \\
\frac{\pi}{F} &< \frac{1 + \delta + \beta - \beta\delta}{2(\delta + \beta - \beta\delta)} \\
\text{true given that } \frac{\pi}{F} &\in [1, \frac{\delta + \beta - \beta\delta + 1}{2(\delta + \beta - \beta\delta)}]
\end{aligned}$$

$\therefore QED.$

If $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ then both $\Delta p_1 \leq 1$ (from Lemma 16(2)) and $\Delta p_2 \leq 1$ (from Lemma 16(4)). Moreover, $\Delta p_1 > \Delta p_2$ (from Lemma 16(5)). Hence, if $\Delta p < \Delta p_2$ then $p_1 > 1 - \Delta p$ so that for all $p \in [0, 1 - \Delta p]$ (KEEP, KEEP) is a NE.

If $\Delta p \in [\Delta p_2, \Delta p_1]$ then $p \in [0, 1 - \Delta p]$ and thus for all $p < p_1$ (KEEP, KEEP) is a NE. Moreover, if $\Delta p > \Delta p_1$ then $p_1 < 0$ so that (KEEP, KEEP) is not a NE. The intuition for the latter result is the following. If the additional probability with which the collusive firms are detected in the presence of hard evidence is sufficiently high then each firm has a unilateral incentive to deviate from the agreement and destroy the hard evidence. By destroying the hard evidence the probability of detection and therefore of paying the fine decreases by $\frac{\Delta p}{2}$. This decrease is higher when Δp is high. If on the other hand $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$ then both $\Delta p_1 > 1$ (from Lemma 16(2)) and $\Delta p_2 > 1$ (from Lemma 16(4)), implying that $p_1 > 0$ and $p_1 > 1 - \Delta p$. In this case (KEEP, KEEP) is a NE of the *Keep-Destroy* subgame for all $p \in [0, 1 - \Delta p]$. Hence, if the ratio of collusive profits to fine is relatively high then no firm has an incentive to deviate from (KEEP, KEEP). Any deviation in that case will result in deprivation of future profits which is very costly given that the level of the fine is relatively low compared to the collusive profits.

Lemma 17 *(KEEP, KEEP) is a NE of the Keep-Destroy game if:*

1. $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ *and either a)* $\Delta p < \Delta p_2$ *and* $p \in [0, 1 - \Delta p]$ *or b)* $\Delta p \in [\Delta p_2, \Delta p_1]$ *and* $p \in [0, p_1]$ *or*

2. $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$

Lemma 18 1. $p_3 \geq 0$ *iff* $\Delta p < \Delta p_3$, *where* $\Delta p_3 = \beta\frac{\pi}{F}$

2. $\Delta p_3 > 1$ *iff* $\frac{\pi}{F} > \frac{1}{\beta}$

3. $p_3 \leq 1 - \Delta p$ *iff* $\Delta p \geq \Delta p_4$, *where* $\Delta p_4 = \frac{\beta}{1-\beta}(\frac{\pi}{F} - 1)$

4. $\Delta p_4 > 1$ *iff* $\frac{\pi}{F} > \frac{1}{\beta}$

5. $\Delta p_2 > \Delta p_4$

6. $\Delta p_3 > \Delta p_2$

7. $\Delta p_1 > \Delta p_3$

8. $\Delta p_1 > \Delta p_3 > \Delta p_2 > \Delta p_4$

9. $p_1 > p_3$

10. $\frac{\delta+\beta-\beta\delta+1}{2(\beta+\delta-\beta\delta)} < \frac{1}{\beta}$

11. $\Delta p_3 > \Delta p_4$ *iff* $\frac{\pi}{F} < \frac{1}{\beta}$.

Proof of Lemma 18

(**1**) To show: $p_3 \geq 0$ iff $\Delta p \leq \Delta p_3$, where $\Delta p_3 \equiv \beta\frac{\pi}{F}$.

$$\begin{aligned} p_3 &\geq 0 \\ \frac{\pi}{F} - \frac{\Delta p}{\beta} &\geq 0 \Leftrightarrow \\ \beta\frac{\pi}{F} &\geq \Delta p \Leftrightarrow \\ \Delta p &\leq \beta\frac{\pi}{F} \equiv \Delta p_3 \end{aligned}$$

$\therefore QED.$

(2) To show: $\Delta p_3 > 1$ iff $\frac{\pi}{F} > \frac{1}{\beta}$.

$$\begin{aligned} \beta\frac{\pi}{F} &> 1 \Leftrightarrow \\ \frac{\pi}{F} &> \frac{1}{\beta} \end{aligned}$$

$\therefore QED.$

(4) To show: $p_3 \leq 1 - \Delta p$ iff $\Delta p \geq \Delta p_4$, where $\Delta p_4 = \frac{\beta}{1-\beta}(\frac{\pi}{F} - 1)$.

$$\begin{aligned} p_3 &\leq 1 - \Delta p \Leftrightarrow \\ \frac{\pi}{F} - \frac{\Delta p}{\beta} &\leq 1 - \Delta p \Leftrightarrow \\ \frac{\pi}{F} - 1 &\leq \Delta p(\frac{1}{\beta} - 1) \Leftrightarrow \\ \Delta p &\geq \frac{\beta}{1-\beta}(\frac{\pi}{F} - 1) \equiv \Delta p_4 \end{aligned}$$

$\therefore QED.$

(**5**) To show: $\Delta p_2 > \Delta p_4$.

$$
\begin{aligned}
\Delta p_2 &> \Delta p_4 \\
\frac{2(\beta+\delta-\beta\delta)}{1-(\beta+\delta-\beta\delta)}(\frac{\pi}{F}-1) &> \frac{\beta}{1-\beta}(\frac{\pi}{F}-1) \\
2(\beta+\delta-\beta\delta)(1-\beta) &> \beta[1-(\beta+\delta-\beta\delta)] \Leftrightarrow \\
(\beta+\delta-\beta\delta)(2-2\beta+\beta) &> \beta \Leftrightarrow \\
(\beta+\delta-\beta\delta)(2-\beta) &> \beta \Leftrightarrow \\
2\beta+2\delta(1-\beta)-\beta^2-\beta\delta(1-\beta) &> \beta \Leftrightarrow \\
\beta(1-\beta)+\delta(2-\beta)(1-\beta) &> 0 \\
& \quad true
\end{aligned}
$$

$\therefore QED.$
(6) To show: $\Delta p_3 > \Delta p_2$ iff $\frac{\pi}{F}[1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$.
By contradiction. Suppose that:

$$
\begin{aligned}
\Delta p_2 &> \Delta p_3 \\
\frac{2(\beta+\delta-\beta\delta)}{1-(\beta+\delta-\beta\delta)}(\frac{\pi}{F}-1) &> \beta\frac{\pi}{F} \\
2(\beta+\delta-\beta\delta)(\frac{\pi}{F}-1) &> \beta[1-(\beta+\delta-\beta\delta)]\frac{\pi}{F} \\
\frac{\pi}{F}(2(\beta+\delta-\beta\delta)-\beta+\beta(\beta+\delta-\beta\delta) &> 2(\beta+\delta-\beta\delta) \\
\frac{\pi}{F} &> \frac{2(\beta+\delta-\beta\delta)}{\beta^2(1-\delta)+\beta(1-\delta)+2\delta}
\end{aligned}
$$

Notice that $\frac{2(\beta+\delta-\beta\delta)}{\beta^2(1-\delta)+\beta(1-\delta)+2\delta} > 1$.

$$
\begin{aligned}
\frac{2(\beta+\delta-\beta\delta)}{\beta^2(1-\delta)+\beta(1-\delta)+2\delta} &> 1 \\
2(\beta+\delta-\beta\delta) &> \beta^2(1-\delta)+\beta(1-\delta)+2\delta \\
2\delta+2\beta(1-\delta) &< \beta^2(1-\delta)+\beta(1-\delta)+2\delta \\
\beta(1-\delta)-\beta^2(1-\delta) &> 0 \\
\beta(1-\beta)(1-\delta) &> 0 \\
\text{always true given that } \beta,\delta &< 1
\end{aligned}
$$

But given that $\frac{\pi}{F}[1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ this leads to a contradiction since:

$$\begin{aligned}
\frac{2(\beta+\delta-\beta\delta)}{\beta^2(1-\delta)+\beta(1-\delta)+2\delta} &> \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} \\
4(\beta+\delta-\beta\delta)(\delta+\beta-\beta\delta) &> (\beta^2(1-\delta)+\beta(1-\delta)+2\delta)(\delta+\beta-\beta\delta+1) \\
& \text{contradiction}
\end{aligned}$$

$\therefore QED.$

(**7**)

To show: $\Delta p_1 > \Delta p_3$.

$$\begin{aligned}
\Delta p_1 &> \Delta p_3 \\
\frac{\pi}{F}\frac{2(\delta+\beta-\beta\delta)}{(\delta+\beta-\beta\delta+1)} &> \beta\frac{\pi}{F} \\
2(\delta+\beta-\beta\delta) &> \beta+\beta(\delta+\beta-\beta\delta) \\
2\beta+2\delta(1-\beta)-\beta-\beta\delta-\beta(1-\delta) &> 0 \\
2\delta(1-\beta) &> 0 \\
\beta(1-\beta)+(1-\beta)[\delta(2-\beta)] &> 0 \\
& \mathit{true}
\end{aligned}$$

$\therefore QED.$

(**8**) To show: $\Delta p_1 > \Delta p_3 > \Delta p_2 > \Delta p_4$ iff $\frac{\pi}{F}[1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$.

From Lemmata 5, 6 and 7.

$\therefore QED.$

(**9**) To show: $p_1 > p_3$.

$$\begin{aligned} p_1 &> p_3 \\ \frac{\pi}{F} - \frac{(\delta+\beta-\beta\delta+1)\Delta p}{2(\delta+\beta-\beta\delta)} &> \frac{\pi}{F} - \frac{\Delta p}{\beta} \\ \frac{\Delta p}{\beta} &> \frac{(\delta+\beta-\beta\delta+1)\Delta p}{2(\delta+\beta-\beta\delta)} \\ (2-\beta)(\delta+\beta-\beta\delta)-\beta &> 0 \\ \beta+2\delta(1-\beta)-\beta^2-\beta\delta(1-\beta) &> 0 \\ \beta(1-\beta)+\delta(2-\beta)(1-\beta) &> 0 \\ \delta &> -\frac{\beta}{2-\beta} < 0 \\ \text{which is always true given that } \delta &\in [0,1] \end{aligned}$$

$\therefore QED.$

(**10**) To show: $\frac{\delta+\beta-\beta\delta+1}{2(\beta+\delta-\beta\delta)} < \frac{1}{\beta}$.

$$\begin{aligned} \frac{\delta+\beta-\beta\delta+1}{2(\beta+\delta-\beta\delta)} &< \frac{1}{\beta} \\ \beta(\delta+\beta-\beta\delta)+\beta &< 2(\beta+\delta-\beta\delta) \\ \beta^2+\beta\delta(1-\beta)+\beta &< 2\beta+2\delta(1-\beta) \\ \beta(1-\beta)+2\delta(1-\beta)-\beta\delta(1-\beta) &> 0 \\ \beta+\delta(2-\beta) &> 0 \\ & \text{true} \end{aligned}$$

$\therefore QED.$

(11) To show: $\Delta p_3 > \Delta p_4$ iff $\frac{\pi}{F} < \frac{1}{\beta}$.

$$\begin{aligned}
\Delta p_3 &> \Delta p_4 \Leftrightarrow \\
\beta\frac{\pi}{F} &> \frac{\beta}{1-\beta}(\frac{\pi}{F}-1) \Leftrightarrow \\
\frac{\pi}{F} &> \frac{1}{1-\beta}(\frac{\pi}{F}-1) \Leftrightarrow \\
(1-\beta)\frac{\pi}{F} &> \frac{\pi}{F}-1 \Leftrightarrow \\
\frac{\pi}{F}(1-\beta-1) &< -1 \Leftrightarrow \\
1 &> \beta\frac{\pi}{F} \Leftrightarrow \\
\frac{\pi}{F} &< \frac{1}{\beta}
\end{aligned}$$

$\therefore QED.$

From Lemmata 17 and 18 we get the following result.

Lemma 19 *The Keep-Destroy subgame has two NE if:*

1. *For* $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$
   (a) $\Delta p < \Delta p_2$ *and* $p \in [0, 1-\Delta p]$
   (b) $\Delta p \in [\Delta p_2, \Delta p_1]$ *and* $p \in [0, p_1]$
2. *For* $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$, $\forall \Delta p$ *and* $\forall p$.

Corollary 9 *For* $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ *the unique NE is (*Destroy, Destroy*) if:*

1. $\Delta p \in [\Delta p_2, \Delta p_1]$ *and* $p \in [p_1, 1-\Delta p]$ *or*
2. $\Delta p > \Delta p_1$.

From Lemma 19 we distinguish three cases:

1. $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ and $\Delta p < \Delta p_2$
2. $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$, $p < p_1$ and $\Delta p \in [\Delta p_2, \Delta p_1]$

3. $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$

We will analyze each of the three cases below.

Case 1 $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ *and* $\Delta p < \Delta p_2$

Given that $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$, then from Lemma 18(8) $\Delta p_3 > \Delta p_2 > \Delta p_4$. Given that $\Delta p_3 > \Delta p_2$, from Lemma 18(1) $p_3 \geq 0$. Moreover, if $\Delta p < \Delta p_4$ then $p_3 > 1 - \Delta p$ so that for all p (Keep, Keep) Pareto dominates (Destroy, Destroy). If on the other hand $\Delta p \in [\Delta p_4, \Delta p_2]$ then from 18(3) $p_3 \leq 1 - \Delta p$ and thus for $p < p_3$ (Keep, Keep) Pareto dominates (Destroy, Destroy), while for $p \geq p_3$ (Destroy, Destroy) Pareto dominates (Keep, Keep). These results are summarized in the following lemma.

Lemma 20 *For* $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ *and* $\Delta p < \Delta p_2$ *the Pareto dominant NE of the Keep-Destroy subgame is:*

1. *(*Keep, Keep*) if:*
   - *(a)* $\Delta p < \Delta p_4 (< \Delta p_2)$ *and* $\forall p \in [0, 1 - \Delta p]$
   - *(b)* $\Delta p \in [\Delta p_4, \Delta p_2]$ *and* $p < p_3$
2. *(*Destroy, Destroy*) if* $\Delta p \in [\Delta p_4, \Delta p_2]$ *and* $p \geq p_3$*.*

Case 2 $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$*,* $p \leq p_1$ *and* $\Delta p \in [\Delta p_2, \Delta p_1]$

From Lemma 18(8) we know that $\Delta p_1 > \Delta p_3 > \Delta p_2 > \Delta p_4$. Hence, $\Delta p_3 \in [\Delta p_2, \Delta p_1]$. Given that $\Delta p_4 < \Delta p_2$ for all $\Delta p \in [\Delta p_2, \Delta p_1]$, $p_3 \leq 1 - \Delta p$. Then, for $\Delta p \in [\Delta p_2, \Delta p_3]$, $p_3 \in [0, p_1]$, while for $\Delta p \in (\Delta p_3, \Delta p_1]$, $p_3 < 0$. Hence, for $\Delta p \in [\Delta p_2, \Delta p_3]$ if $p < p_3$ (Keep, Keep) Pareto dominates (Destroy, Destroy), while for $p \in [p_3, p_1]$ (Destroy, Destroy) Pareto dominates (Keep, Keep). On the other hand, if $\Delta p \in (\Delta p_3, \Delta p_1]$ then for all $p \leq p_1$ (Destroy, Destroy) Pareto dominates (Keep, Keep).

From the above analysis and from Corollary 9 we get the following result.

Lemma 21 *For* $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$*,* $p < p_1$ *and* $\Delta p \in [\Delta p_2, \Delta p_1]$ *the Pareto dominant NE of the Keep-Destroy subgame is:*

1. *(*Keep, Keep*) if* $\Delta p \in [\Delta p_2, \Delta p_3]$ *and* $p < p_3$
2. *(*Destroy, Destroy*) if:*

(a) $\Delta p \in [\Delta p_2, \Delta p_3]$ *and* $p \in [p_3, p_1]$

(b) $\Delta p \in [\Delta p_2, \Delta p_1]$ *and* $p > p_1$

(c) $\Delta p \in (\Delta p_3, \Delta p_1]$ *and* $p \leq p_1$

(d) $\Delta p > \Delta p_1$

Conjoining Lemmata 20 and 21 we obtain the following lemma.

Lemma 22 *For* $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ *the Pareto dominant NE of the Keep-Destroy subgame is:*

1. *(*Keep, Keep*) if:*

   (a) $\Delta p < \Delta p_4$ *and* $\forall p \in [0, 1-\Delta p]$

   (b) $\Delta p \in [\Delta p_2, \Delta p_3]$ *and* $p < p_3$

2. *(*Destroy, Destroy*) if:*

   (a) $\Delta p \in [\Delta p_4, \Delta p_3]$ *and* $p \geq p_3$

   (b) $\Delta p > \Delta p_3$ *and* $\forall p \in [0, 1-\Delta p]$

Case 3 $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$.

In that case we know from Lemma 19 that there are two NE for all $p \in [0, 1-\Delta p]$ and for all $\Delta p \in [0, 1)$. From Lemmata 18(2), 18(4), 18(9) and 18(10) for $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{\beta}]$, $\Delta p_4 < \Delta p_3 < 1$. Hence, if $\Delta p < \Delta p_4$, then $p_3 > 1 - \Delta p$ and thus (Keep, Keep) Pareto dominates (Destroy, Destroy). If $\Delta p \in [\Delta p_4, \Delta p_3]$ then for $p < p_3$ (Keep, Keep) Pareto dominates (Destroy, Destroy), while for $p \geq p_3$ (Destroy, Destroy) Pareto dominates (Keep, Keep). Moreover, for all $\Delta p > \Delta p_3$, $p_3 < 0$ so that (Destroy, Destroy) Pareto dominates (Keep, Keep). If on the other hand $\frac{\pi}{F} > \frac{1}{\beta}$ then both we know from Lemma 18(2)and 18(4) that both $\Delta p_3 \geq 1$ and $\Delta p_4 > 1$, implying that $p_3 \geq 0$ and $p_3 > 1 - \Delta p$. In that case for all p and for all Δp (Keep, Keep) Pareto dominates (Destroy, Destroy).

We can summarize the above results into the following lemma.

Lemma 23 *For* $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$ *the Pareto dominant NE of the Keep-Destroy subgame is:*

1. *(*Keep, Keep*)*

(a) *For* $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{\beta}]$

i. $\Delta p < \Delta p_4$ *and* $\forall p \in [0, 1-\Delta p]$

ii. $\Delta p \in [\Delta p_4, \Delta p_3]$ *and* $p < p_3$

(b) *For* $\frac{\pi}{F} > \frac{1}{\beta}$

2. *(*DESTROY, DESTROY*)*

(a) *For* $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{\beta}]$

i. $\Delta p \in [\Delta p_4, \Delta p_3]$ *and* $p \geq p_3$

ii. $\Delta p > \Delta p_3$ *and* $\forall p \in [0, 1-\Delta p]$

From Lemmata 22 and 23 we reach the following proposition.

Proposition 1 *For given policy and other parameters (p, Δp, F, β, π) the Pareto dominant SPE of the baseline model without LP is:*

1. *(*KEEP, KEEP*) iff:*

(a) $\frac{\pi}{F} \in [1, \frac{1}{\beta}]$ *and*

i. $\Delta p \in [0, \Delta p_4]$ *and* $p \in [0, 1-\Delta p]$ *or*

ii. $\Delta p \in [\Delta p_4, \Delta p_3]$ *and* $p \in [0, p_3)$ *or*

(b) $\frac{\pi}{F} > \frac{1}{\beta}$, $\Delta p \in [0, 1)$ *and* $p \in [0, 1-\Delta p]$

2. *(*DESTROY, DESTROY*) iff* $\frac{\pi}{F} \in [1, \frac{1}{\beta}]$ *and*

(a) $\Delta p \in [\Delta p_4, \Delta p_3]$ *and* $p \in [p_3, 1-\Delta p]$ *or*

(b) $\Delta p \in (\Delta p_3, 1)$ *and* $p \in [0, 1-\Delta p]$

where $p_3 \equiv \frac{\pi}{F} - \frac{\Delta p}{\beta}$, $\Delta p_3 \equiv \beta\frac{\pi}{F}$ *and* $\Delta p_4 \equiv \frac{\beta}{1-\beta}(\frac{\pi}{F} - 1)$.

Appendix B - Proofs for Section 4

Proof of Lemma 6

Lemma 24 1. *For* $\delta\frac{\pi}{F} > 1$ *the revelation subgame has two NE.*

2. *For* $\delta\frac{\pi}{F} \leq 1$ *and* $p + \Delta p \leq \delta\frac{\pi}{F}$ *the revelation subgame has also two NE.*

Corollary 10 *For $\delta\frac{\pi}{F} \in [\frac{1}{2}, 1]$ and $p + \Delta p > \delta\frac{\pi}{F}$ the revelation subgame has one NE (*Keep and Report*, *Keep and Report*).*

As discussed before in the case where we have two NE we apply the Pareto dominance criterion. It is easy to show that $V_{KR} > V_{KNR}$ iff $p + \Delta p > \frac{1}{2}$.

Lemma 25 *(*Keep and Report*, *Keep and Report*) Pareto dominates (*Keep and Not Report*, *Keep and Not Report*) in the revelation subgame if $p + \Delta p > \frac{1}{2}$.*

Therefore, from Lemmata 24 and 25 we obtain the following result:

Lemma 26

1. *For $\delta\frac{\pi}{F} \in [\frac{1}{2}, 1]$*
   - *(a) (*Keep and Report*, *Keep and Report*) Pareto dominates (*Keep and Not Report*, *Keep and Not Report*) iff $\frac{1}{2} < p + \Delta p \leq \delta\frac{\pi}{F}$*
   - *(b) (*Keep and Not Report*, *Keep and Not Report*) Pareto dominates (*Keep and Report*, *Keep and Report*) iff $p + \Delta p \leq \frac{1}{2}$*
2. *For $\delta\frac{\pi}{F} > 1$*
   - *(a) (*Keep and Report*, *Keep and Report*) Pareto dominates (*Keep and Not Report*, *Keep and Not Report*) iff $p + \Delta p > \frac{1}{2}$*
   - *(b) (*Keep and Not Report*, *Keep and Not Report*) Pareto dominates (*Keep and Report*, *Keep and Report*) iff $p + \Delta p \leq \frac{1}{2}$*

Conflation of Corollary 10 and Lemma 26 above boils down to the following lemma.

Lemma 27 *The Pareto dominant NE of the Revelation subgame is:*

1. *(*Keep and Not Report*, *Keep and Not Report*) iff $p + \Delta p \leq \frac{1}{2}$;*
2. *(*Keep and Report*, *Keep and Report*) iff $p + \Delta p > \frac{1}{2}$.*

Proof of Lemma 24

Lemma 28 *1.* $p_4 < \delta\frac{\pi}{F}$

2. $p_4 < 0$ *iff* $\frac{\pi}{F} > \frac{1}{2\beta}$

3. $p_4 < \frac{1}{2} - \Delta p$ *iff* $\Delta p < \Delta p_5$*, where* $\Delta p_5 = \frac{\beta}{1-\beta}\frac{\pi}{F} - \frac{\beta}{2(1-\beta)}$

4. $\Delta p_5 > 1$ *iff* $\frac{\pi}{F} > \frac{2-\beta}{2\beta}$

Proof of Lemma 28

(1) To show: $p_4 < \delta\frac{\pi}{F}$.

$$\begin{aligned} p_4 &< \delta\frac{\pi}{F} \\ \frac{F - 2\beta\Pi_M}{2(1-\beta)F} &< \delta\frac{\pi}{F} \\ \beta &> \frac{1 - 2\delta\frac{\pi}{F}}{2(1-\delta)\frac{\pi}{F}} \equiv \beta_1 \end{aligned}$$

Notice that:

$$\begin{aligned} \beta_1 &> 0 \\ \frac{1 - 2\delta\frac{\pi}{F}}{2(1-\delta)\frac{\pi}{F}} &> 0 \\ \frac{\pi}{F} &< \frac{1}{2\delta} \\ \text{contradiction since } \frac{\pi}{F} &\geq 1 \end{aligned}$$

$\therefore$ Given that $\beta_1 < 0$ it is always true that $p_4 < \delta\frac{\pi}{F}$.

(2) To show: $p_4 < 0$ iff $\frac{\pi}{F} > \frac{1}{2\beta}$.

$$\begin{aligned} p_4 &< 0 \iff \\ \frac{F - 2\beta\Pi_M}{2(1-\beta)F} &< 0 \iff \\ \frac{\pi}{F} &> \frac{1}{2\beta} \end{aligned}$$

$\therefore QED.$
(3) To show: $p_4 < \frac{1}{2} - \Delta p$ iff $\Delta p < \Delta p_5$, where $\Delta p_5 = \frac{\beta}{1-\beta}\frac{\pi}{F} - \frac{\beta}{2(1-\beta)}$.

$$\begin{aligned} p_4 &< \frac{1}{2} - \Delta p \iff \\ \Delta p &< \frac{\beta}{1-\beta}\frac{\pi}{F} - \frac{\beta}{2(1-\beta)} \equiv \Delta p_5 \end{aligned}$$

$\therefore QED.$
(4) To show: $\Delta p_5 > 1$ iff $\frac{\pi}{F} > \frac{2-\beta}{2\beta}$.

$$\begin{aligned} \Delta p_5 &> 1 \iff \\ \frac{\beta}{1-\beta}\frac{\pi}{F} - \frac{\beta}{2(1-\beta)} &> 1 \iff \\ \frac{\pi}{F} &> \frac{2-\beta}{2\beta} \end{aligned}$$

$\therefore QED.$
Consider first Lemma 9(1), that is, $\delta\frac{\pi}{F} < 1 - \Delta p$ so that the *Keep-Destroy* subgame has two NE for $p \in [\frac{1}{2} - \Delta p, \delta\frac{\pi}{F}]$. Then, given Lemmata 10 and 28 above we can now proceed to the analysis to find the NE at the *Keep-Destroy* subgame. To begin with suppose that $\beta < \frac{1}{2}$. Then, $1 < \frac{1}{2\beta} < \frac{2-\beta}{2\beta}$. From Lemma 28(2) if $\frac{\pi}{F} > \frac{1}{2\beta}$, $p_4 < 0$ implying that (Keep and Report, Keep and Report) Pareto dominates (Destroy, Destroy) for all $p \in [\max\{0, \frac{1}{2} - \Delta p\}, \delta\frac{\pi}{F}]$. If however $\frac{\pi}{F} < \frac{1}{2\beta}$ then from Lemma 28(3) $p_4 \in [0, \frac{1}{2} - \Delta p]$ if $\Delta p < \Delta p_5$. Therefore, if $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$ and $\Delta p < \Delta p_5$ then (Keep and Report, Keep and Report) Pareto dominates (Destroy, Destroy) for all $p \in [\frac{1}{2} - \Delta p, \delta\frac{\pi}{F}]$. If on the other hand $\Delta p > \Delta p_5$ then for all $p \in [\max\{0, \frac{1}{2} - \Delta p\}, p_4]$ (Destroy, Destroy) Pareto dominates (Keep and Report, Keep and Report), whereas for all $p \in (p_4, \delta\frac{\pi}{F}]$ (Keep and Report, Keep and Report) Pareto dominates (Destroy, Destroy).

Suppose now that $\beta \in [\frac{1}{2}, \frac{2}{3}]$. Then $\frac{2-\beta}{2\beta} > 1 > \frac{1}{2\beta}$. From Lemma 28(2) if $\frac{\pi}{F} \in [1, \frac{2-\beta}{2\beta}]$ then $p_4 < 0$ so that (Keep and Report, Keep and Report) Pareto dominates (Destroy, Destroy) for all $p \in [\frac{1}{2} - \Delta p, \delta\frac{\pi}{F}]$. Moreover, if $\frac{\pi}{F} > \frac{2-\beta}{2\beta}$ then again $p_4 < 0$ so that (Keep and Report, Keep and Report) Pareto dominates (Destroy, Destroy) for all $p \in [\max\{0, \frac{1}{2} - \Delta p\}, \delta\frac{\pi}{F}]$.

Finally, suppose that $\beta > \frac{2}{3}$. Then $1 > \frac{2-\beta}{2\beta} > \frac{1}{2\beta}$. From Lemma 28(2) for all $\frac{\pi}{F} \geq 1$, $p_4 < 0$ implying that (Keep and Report, Keep and Report) Pareto dominates (Destroy, Destroy) for all $p \in [\max\{0, \frac{1}{2} - \Delta p\}, \delta \frac{\pi}{F}]$.

Consider now Lemma 9(2), that is $\delta \frac{\pi}{F} > 1$ so that for all $p \in [\frac{1}{2} - \Delta p, 1 - \Delta p]$ there are two NE. In that case we have to check whether p_4 is greater or lower than $1 - \Delta p$.

Lemma 29 *1. $p_4 < 1 - \Delta p$ iff $\Delta p < \Delta p_6$, where $\Delta p_6 = \frac{1-2\beta}{2(1-\beta)} + \frac{\beta}{1-\beta}\frac{\pi}{F}$*

2. $\Delta p_6 > 1$ iff $\frac{\pi}{F} > \frac{1}{\beta}$

3. $\Delta p_6 > \Delta p_5$

Proof of Lemma 29
(1) To show: $p_4 < 1 - \Delta p$ iff $\Delta p < \Delta p_6$ where $\Delta p_6 = \frac{1-2\beta}{2(1-\beta)} + \frac{\beta}{1-\beta}\frac{\pi}{F}$.

$$\begin{aligned} p_4 &< 1 - \Delta p \Leftrightarrow \\ \frac{F - 2\beta\Pi_M}{2(1-\beta)F} &< 1 - \Delta p \Leftrightarrow \\ \Delta p &< \frac{1-2\beta}{2(1-\beta)} + \frac{\beta}{1-\beta}\frac{\pi}{F} \equiv \Delta p_6 \end{aligned}$$

$\therefore QED.$
(**2**) To show: $\Delta p_6 > 1$ iff $\frac{\pi}{F} > \frac{1}{\beta}$.

$$\begin{aligned} \Delta p_6 &> 1 \Leftrightarrow \\ \frac{1-2\beta}{2(1-\beta)} + \frac{\beta}{1-\beta}\frac{\pi}{F} &> 1 \Leftrightarrow \\ \frac{\pi}{F} &> \frac{1}{\beta} \end{aligned}$$

$\therefore \Delta p_6 > 1$ if $\frac{\pi}{F} > \frac{1}{\beta}$.

(**3**) To show: $\Delta p_6 > \Delta p_5$.

$$\begin{aligned}
\Delta p_6 &> \Delta p_5 \\
\frac{1-2\beta}{2(1-\beta)} + \frac{\beta}{1-\beta}\frac{\pi}{F} &> \frac{\beta}{1-\beta}\frac{\pi}{F} - \frac{\beta}{2(1-\beta)} \\
\frac{1-2\beta}{2(1-\beta)} &> -\frac{\beta}{2(1-\beta)} \\
& \frac{1-2\beta+\beta}{2(1-\beta)} \\
\frac{1}{2} &> 0
\end{aligned}$$

$\therefore$ *QED.*

Taking into consideration Lemma 29 above we can find the Pareto dominant NE of the *Keep-Destroy* subgame for different combinations of parameters' values. To begin with suppose that $\beta < \frac{1}{2}$. Then, $1 < \frac{1}{2\beta} < \frac{2-\beta}{2\beta} < \frac{1}{\beta}$. In that case, if $\frac{\pi}{F} > \frac{1}{2\beta}$ then $p_4 < 0$ implying that (Keep and Report, Keep and Report) Pareto dominates (Destroy, Destroy) for all $p \in [\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$. If on the other hand, $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$ then we can distinguish the following three cases:

1. From Lemma 28(3) if $\Delta p < \Delta p_5$ then $p_4 < \frac{1}{2} - \Delta p$ so that for all $p \in [\frac{1}{2} - \Delta p, 1 - \Delta p]$ (Keep and Report, Keep and Report) Pareto dominates (Destroy, Destroy).

2. From Lemmata 28(3), 29(1) and 29(3) if $\Delta p \in [\Delta p_5, \Delta p_6]$ then $p_4 > \frac{1}{2} - \Delta p$ so that for $p \in [0, p_4]$ (Destroy, Destroy) Pareto dominates (Keep and Report, Keep and Report), while for $p \in (p_4, 1 - \Delta p]$ (Keep and Report, Keep and Report) Pareto dominates (Destroy, Destroy).

3. From Lemma 29(1) if $\Delta p > \Delta p_6$ then $p_4 > 1-\Delta p$ so that for all $p \in [0, 1-\Delta p]$ (Destroy, Destroy) Pareto dominates (Keep and Report, Keep and Report).

Suppose now that $\beta \in [\frac{1}{2}, \frac{2}{3}]$. Then, $\frac{1}{2\beta} < 1 < \frac{2-\beta}{2\beta} < \frac{1}{\beta}$. From Lemma 28(2) for all $\frac{\pi}{F} \geq 1$ then $p_4 < 0$ so that (Keep and Report, Keep and Report) Pareto dominates (Destroy, Destroy) for all $p \in [\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$.

Finally, if $\beta > \frac{2}{3}$ then $\frac{1}{2\beta} < \frac{2-\beta}{2\beta} < 1 < \frac{1}{\beta}$. From 28(2) then for all $\frac{\pi}{F} \geq 1$, $p_4 < 0$ implying that (Keep and Report, Keep and Report) Pareto dominates (Destroy, Destroy) for all $p \in (\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$.

We can now summarize the above results in the lemma below.

Lemma 30 *The Pareto dominant NE of the Keep-Destroy subgame for $p + \Delta p > \frac{1}{2}$ and $\delta \frac{\pi}{F} \leq 1$ is:*

1. *(*Keep and Report, Keep and Report*) iff:*

   (a) $\beta < \frac{1}{2}$, $\frac{\pi}{F} > \frac{1}{2\beta}$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$;

   (b) $\beta < \frac{1}{2}$, $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p < \Delta p_5$ *and* $p \in [\frac{1}{2} - \Delta p, 1 - \Delta p]$;

   (c) $\beta < \frac{1}{2}$, $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p > \Delta p_5$ *and* $p \in [p_4, 1 - \Delta p]$;

   (d) $\beta > \frac{1}{2}$, $\frac{\pi}{F} \geq 1$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$.

2. *(*Destroy, Destroy*) iff* $\beta < \frac{1}{2}$, $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p > \Delta p_5$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, p_4]$.

Lemma 31 *The Pareto dominant NE of the Keep-Destroy subgame for $p > \frac{1}{2} - \Delta p$ and $\delta \frac{\pi}{F} > 1$ is:*

1. *(*Keep and Report, Keep and Report*) iff:*

   (a) $\beta < \frac{1}{2}$, $\frac{\pi}{F} > \frac{1}{2\beta}$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$;

   (b) $\beta < \frac{1}{2}$, $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p < \Delta p_5$ *and* $p \in (\frac{1}{2} - \Delta p, 1 - \Delta p]$;

   (c) $\beta < \frac{1}{2}$, $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_6]$ *and* $p \in (p_4, 1 - \Delta p]$;

   (d) $\beta > \frac{1}{2}$, $\frac{\pi}{F} \geq 1$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$.

2. *(*Destroy, Destroy*) iff:*

   (a) $\beta < \frac{1}{2}$ *and* $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$ *and* $\Delta p \in [\Delta p_5, \Delta p_6]$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, p_4]$;

   (b) $\beta < \frac{1}{2}$ *and* $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$ $\Delta p > \Delta p_6$ *and* $p \in [0, 1 - \Delta p]$.

From Lemmata 30 and 31 we get the following Lemma.

Lemma 32 *The Pareto dominant SPE of the model with LP for $p + \Delta p > \frac{1}{2}$ is:*

1. *(*Keep and Report, Keep and Report*) if:*

   (a) $\beta < \frac{1}{2}$ *and*

      i. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p < \Delta p_5$ *and* $p \in [\frac{1}{2} - \Delta p, 1 - \Delta p]$

      ii. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_6]$ *and* $p \in (p_4, 1 - \Delta p]$

iii. $\frac{\pi}{F} > \frac{1}{2\beta}$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$

(b) $\beta > \frac{1}{2}$, $\frac{\pi}{F} \geq 1$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$.

2. (Destroy, Destroy) *if* $\beta < \frac{1}{2}$, $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$ *and*

(a) $\Delta p \in [\Delta p_5, \Delta p_6]$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, p_4]$

(b) $\Delta p > \Delta p_6$ *and* $p \in [0, 1 - \Delta p]$.

Proof of Lemma 15

Lemma 33 1. $p_5 \geq 0$ *iff* $\Delta p \leq \Delta p_7$, *where* $\Delta p_7 = \frac{\pi}{F}\frac{2(\delta+\beta-\beta\delta)}{\delta+\beta-\beta\delta+1}$

2. $\Delta p_7 \leq 1$ *iff* $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$

3. $\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} > \frac{1}{2\delta}$

4. $p_5 + \Delta p < \frac{1}{2}$ *iff* $\Delta p > \Delta p_8$, *where* $\Delta p_8 = \frac{(2\frac{\pi}{F}-1)(\beta+\delta-\beta\delta)}{1-(\beta+\delta-\beta\delta)}$

5. $\Delta p_8 > 1$

6. $\Delta p_8 > \Delta p_7$

Proof of Lemma 33

(1) To show: $p_5 > 0$ iff $\Delta p < \Delta p_7$, where $\Delta p_7 = \frac{\pi}{F}\frac{2(\delta+\beta-\beta\delta)}{\delta+\beta-\beta\delta+1}$.

$$\begin{aligned} p_5 &> 0 \Leftrightarrow \\ \frac{\pi}{F} - \frac{(\delta+\beta-\beta\delta+1)\Delta p}{2(\delta+\beta-\beta\delta)} &> 0 \Leftrightarrow \\ \Delta p &< \frac{\pi}{F}\frac{2(\delta+\beta-\beta\delta)}{\delta+\beta-\beta\delta+1} \equiv \Delta p_7 \end{aligned}$$

$\therefore QED.$

(2) To show: $\Delta p_7 < 1$ iff $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$.

$$\begin{aligned} \Delta p_7 &< 1 \Leftrightarrow \\ \frac{\pi}{F}\frac{2(\delta+\beta-\beta\delta)}{\delta+\beta-\beta\delta+1} &< 1 \Leftrightarrow \\ \frac{\pi}{F} &< \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} \end{aligned}$$

$\therefore QED.$

(3) To show: $\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} > \frac{1}{2\delta}$.

$$\begin{aligned} \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} &> \frac{1}{2\delta} \\ \beta &< \frac{\delta^2}{(1-\delta)^2} \end{aligned}$$

Given that $\delta \in [\frac{1}{2}, 1]$, $\lim_{\delta\to\frac{1}{2}} \frac{\delta^2}{(1-\delta)^2} = 1$ and $\lim_{\delta\to 1} \frac{\delta^2}{(1-\delta)^2} = \infty$. Thus, given that $\beta \in [0, 1]$, $\beta < \frac{\delta^2}{(1-\delta)^2}$. Another way to see this is the following. Observe that $\frac{\delta+\beta-\beta\delta+1}{\delta+\beta-\beta\delta} > 1$ and that $\underset{\delta\in[\frac{1}{2},1]}{max} \frac{1}{2\delta} = 1$.

$\therefore QED.$

(4) To show: $p_5 + \Delta p < \frac{1}{2}$ iff $\Delta p > \Delta p_8$, where $\Delta p_8 = \frac{(2\frac{\pi}{F}-1)(\beta+\delta-\beta\delta)}{\Delta p_8 1-(\beta+\delta-\beta\delta)}$.

$$\begin{aligned} p_5 + \Delta p &< \frac{1}{2} \Leftrightarrow \\ \frac{\pi}{F} - \frac{(\delta+\beta-\beta\delta+1)\Delta p}{2(\delta+\beta-\beta\delta)} + \Delta p &< \frac{1}{2} \Leftrightarrow \\ \Delta p &> \frac{(2\frac{\pi}{F}-1)(\beta+\delta-\beta\delta)}{1-(\beta+\delta-\beta\delta)} \equiv \Delta p_8 \end{aligned}$$

$\therefore QED.$

(5) To show: $\Delta p_8 > 1$.

$$\begin{aligned} \frac{(2\frac{\pi}{F}-1)(\beta+\delta-\beta\delta)}{1-(\beta+\delta-\beta\delta)} &> 1 \\ (2\frac{\pi}{F}-1)(\beta+\delta-\beta\delta) &> 1-(\beta+\delta-\beta\delta) \\ \frac{\pi}{F} &> \frac{1-(\beta+\delta-\beta\delta)+(\beta+\delta-\beta\delta)}{2(\beta+\delta-\beta\delta)} \\ \frac{\pi}{F} &> \frac{1}{2(\beta+\delta-\beta\delta)} \end{aligned}$$

$$\begin{aligned}
\frac{1}{2(\beta+\delta-\beta\delta)} &< 1 \\
1 &< 2(\beta+\delta-\beta\delta) \\
\beta+\delta-\beta\delta &> \frac{1}{2} \\
\text{true given that } \delta &\geq \frac{1}{2}
\end{aligned}$$

$\therefore \Delta p_8 > 1$.

(6) To show: $\Delta p_7 > \Delta p_8$.

$$\begin{aligned}
\Delta p_7 &> \Delta p_8 \\
\frac{\pi}{F}\frac{2(\delta+\beta-\beta\delta)}{\delta+\beta-\beta\delta+1} &> \frac{(2\frac{\pi}{F}-1)(\beta+\delta-\beta\delta)}{1-(\beta+\delta-\beta\delta)} \\
\frac{\pi}{F} &< \frac{1+\delta+\beta-\beta\delta}{4(\delta+\beta-\beta\delta)}
\end{aligned}$$

Notice however that given that we are in the area where $\frac{\pi}{F} > \frac{1}{2\delta}$ for this to hold it must be the case that:

$$\begin{aligned}
\frac{1+\delta+\beta-\beta\delta}{4(\delta+\beta-\beta\delta)} &> \frac{1}{2\delta} \\
\delta(1-\beta)+(1-\delta)(2-\beta) &< 0
\end{aligned}$$

This, however, implies that $\frac{\pi}{F} < \frac{1+\delta+\beta-\beta\delta}{4(\delta+\beta-\beta\delta)} < \frac{1}{2\delta}$ which is a contradiction.
$\therefore$ *QED*.
Notice that Lemma 33(5) imply that $p_5 > \frac{1}{2} - \Delta p$.

Corollary 11 *$p_5 > \frac{1}{2} - \Delta p$.*

From Lemma 33(1) $p_5 > 0\ \forall \Delta p$ iff $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$, while $p_5 > 0$ for all $\Delta p \leq \Delta p_7$ iff $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$. Therefore, for $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ and $\Delta p < \Delta p_7$ and for all $p \in [0, \frac{1}{2} - \Delta p]$ (Keep and Not Report, Keep and Not Report) is a NE of the *Keep-Destroy* subgame. If however $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ then from Lemmata 33(1) and 33(2) and $\Delta p > \Delta p_7$ then $p_5 < 0$. In the latter case (Keep Not Report, Keep Not Report) is not a NE. On the other hand, from Lemmata 33(1), 33(2), 33(4) and 33(5) if $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$ then both $\Delta p_8 > 1$ and $\Delta p_7 > 1$, so that for all $p \in [0, \frac{1}{2} - \Delta p]$ (Keep and Not Report, Keep and Not Report) form a NE of the *Keep-Destroy* subgame.

Lemma 34 *The Pareto dominant NE of the* Keep-Destroy *subgame for $p < \frac{1}{2} - \Delta p$ is:*

1. $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ *and* $\Delta p \leq \Delta p_7$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

2. $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

Corollary 12 *If* $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ *and* $\Delta p > \Delta p_7$ *the unique NE of the Keep-Destroy subgame for* $p \leq \frac{1}{2} - \Delta p$ *is (*Destroy, Destroy*).*

Lemma 35 1. $p_3 \geq 0$ *iff* $\Delta p \leq \Delta p_3$*, where* $\Delta p_3 \equiv \beta\frac{\pi}{F}$.

2. $\Delta p_3 > 1$ *iff* $\frac{\pi}{F} > \frac{1}{\beta}$

3. $p_3 \leq \frac{1}{2} - \Delta p$ *iff* $\Delta p \geq \Delta p_5$*, where* $\Delta p_5 \equiv \frac{\beta}{1-\beta}(\frac{\pi}{F} - \frac{1}{2})$

4. $\Delta p_5 > 1$ *iff* $\frac{\pi}{F} > \frac{2-\beta}{2\beta}$ *(from Lemma 28(4))*

5. $\Delta p_5 > \Delta p_3$ *iff* $\frac{\pi}{F} > \frac{1}{2\beta}$

6. $\frac{1}{\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$

7. $\frac{2-\beta}{2\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$

8. $\frac{2-\beta}{2\beta} < \frac{1}{\beta}$

9. $\frac{1}{2\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$ *if* $\beta < 0.414$ *or if* $\beta \in [0.414, 0.5]$ *and* $\delta > \frac{\beta^2}{(1-\beta)^2}$

Proof of Lemma 35
(1) To show: $p_3 > 0$ if $\Delta p < \Delta p_3$, where $\Delta p_3 = \beta\frac{\pi}{F}$.

$$\begin{aligned} p_3 &> 0 \Leftrightarrow \\ \frac{\pi}{F} - \frac{\Delta p}{\beta} &> 0 \Leftrightarrow \\ \Delta p &< \beta\frac{\pi}{F} \equiv \Delta p_3 \end{aligned}$$

$\therefore$ *QED.*

(2) To show: $\frac{1}{\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$.
Let $x \equiv \delta + \beta - \beta\delta$. Notice that $x < 1$. Then,

$$\begin{aligned} \frac{1}{\beta} &> \frac{x+1}{2x} \\ 2x &> \beta x + x \\ x(2-\beta) &> \beta \end{aligned}$$

$$\begin{aligned} \frac{1}{\beta} &> \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} \\ (2\delta+\beta(1-\delta))(1-\beta) &> 0 \end{aligned}$$

Given that $x > \beta$ it is the case also that $x(2-\beta) > \beta$.
$\therefore QED$.

(3) To show: $p_3 < \frac{1}{2} - \Delta p$ if $\Delta p > \Delta p_5$, where $\Delta p_5 = \frac{\beta}{1-\beta}(\frac{\pi}{F} - \frac{1}{2})$.

$$\begin{aligned} p_3 &< \frac{1}{2} - \Delta p \Leftrightarrow \\ \Delta p &> \frac{\beta}{1-\beta}(\frac{\pi}{F} - \frac{1}{2}) \equiv \Delta p_5 \end{aligned}$$

$\therefore QED$.

(4) To show: $\Delta p_5 > 1$ if $\frac{\pi}{F} > \frac{2-\beta}{2\beta}$.

$$\begin{aligned} \Delta p_5 &> 1 \Leftrightarrow \\ \frac{\beta}{1-\beta}(\frac{\pi}{F} - \frac{1}{2}) &> 1 \Leftrightarrow \\ \frac{\pi}{F} &> \frac{2-\beta}{2\beta} \end{aligned}$$

$\therefore QED$.
(5) To show: $\Delta p_3 > 1$ if $\frac{\pi}{F} > \frac{1}{\beta}$.

$$
\begin{aligned}
\Delta p_3 &> 1 \Leftrightarrow \\
\beta \frac{\pi}{F} &> 1 \Leftrightarrow \\
\frac{\pi}{F} &> \frac{1}{\beta}
\end{aligned}
$$

$\therefore QED.$

(6) To show: $\frac{2-\beta}{2\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$.

By contradiction. Suppose that $\frac{2-\beta}{2\beta} < \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$. Then:

$$
\begin{aligned}
\frac{2-\beta}{2\beta} &< \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} \\
\delta &< \frac{\beta(2\beta-1)}{2\beta^2-4\beta+2}
\end{aligned}
$$

Now, observe that $\frac{\beta(2\beta-1)}{2\beta^2-4\beta+2} < \frac{1}{2}$.

$$
\begin{aligned}
\frac{\beta(2\beta-1)}{2\beta^2-4\beta+2} &< \frac{1}{2} \\
\beta^2+\beta &< 2 \\
&\text{always true}
\end{aligned}
$$

This implies that $\delta < \frac{\beta(2\beta-1)}{2\beta^2-4\beta+2} < \frac{1}{2}$ which is a contradiction, given assumption 3.

$\therefore QED.$

(7) To show: $\frac{2-\beta}{2\beta} < \frac{1}{\beta}$.

$$
\begin{aligned}
\frac{2-\beta}{2\beta} &< \frac{1}{\beta} \\
\beta &> 0
\end{aligned}
$$

$\therefore QED.$

(8) To show: $\Delta p_5 > \Delta p_3$ iff $\frac{\pi}{F} > \frac{1}{2\beta}$.

$$\begin{aligned} \Delta p_5 &> \Delta p_3 \Leftrightarrow \\ \frac{\beta}{1-\beta}(\frac{\pi}{F} - \frac{1}{2}) &> \beta\frac{\pi}{F} \Leftrightarrow \\ \frac{\pi}{F} &> \frac{1}{2\beta} \end{aligned}$$

$\therefore QED.$

(9) To show: $\frac{1}{2\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$ if $\beta < 0,414$ or if $\beta \in [0.414, 0,5]$ and $\delta > \frac{\beta^2}{(1-\beta)^2}$. Let $x \equiv \delta + \beta - \beta\delta$. Then:

$$\begin{aligned} \frac{1}{2\beta} &> \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} \\ x &> \beta x + \beta \\ x &> \frac{\beta}{1-\beta} \end{aligned}$$

Now notice that $\frac{\beta}{1-\beta} > 1$ if $\beta > \frac{1}{2}$. However, this leads to a contradiction since $x < 1$. Therefore. if $\beta > \frac{1}{2}$ then $\frac{1}{2\beta} < \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$.

Now suppose that $\beta < \frac{1}{2}$. In this case,

$$\begin{aligned} \frac{1}{2\beta} &> \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} \\ \delta &> \frac{\beta^2}{(1-\beta)^2} \end{aligned}$$

Notice that $\frac{\beta^2}{(1-\beta)^2} < \frac{1}{2}$ if $\beta < 0,414$. Therefore, for $\beta < 0,414$ $\frac{1}{2\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$. Moreover, for $\beta \in [0.414, 0,5]$ and $\delta > \frac{\beta^2}{(1-\beta)^2}$. Conversely, if $\beta \in [0.414, 0,5]$ and $\delta < \frac{\beta^2}{(1-\beta)^2}$ then $\frac{1}{2\beta} < \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$.

$\therefore QED.$

Case 1 $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ *so that* $\Delta p_7 < 1$

1. From Lemmata 35(8) and 35(9) if $\beta < 0.414$ or $\beta \in [0.414, 0.5]$ and $\delta > \frac{\beta^2}{(1-\beta)^2}$ then $\frac{1}{\beta} > \frac{2-\beta}{2\beta} > \frac{1}{2\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} > 1$. Therefore, given that $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$

and taking into account Lemmata 35(1), 35(2), 35(3), 35(4) and 35(5) we may distinguish three subcases:

(a) if $\Delta p < \Delta p_5$ then $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$. Hence (KEEP AND NOT REPORT, KEEP AND NOT REPORT) Pareto dominates (DESTROY, DESTROY) for all $p \in [0, \frac{1}{2} - \Delta p]$.

(b) if $\Delta p \in [\Delta p_5, \Delta p_3]$ then $p_3 \geq 0$ and $p_3 \leq \frac{1}{2} - \Delta p$. Hence for $p \in [0, p_3)$ (KEEP AND NOT REPORT, KEEP AND NOT REPORT) Pareto dominates (DESTROY, DESTROY), while for $p \in [p_3, \frac{1}{2} - \Delta p]$ (DESTROY, DESTROY) Pareto dominates (KEEP AND NOT REPORT, KEEP AND NOT REPORT).

(c) if $\Delta p \in [\Delta p_3, \frac{1}{2}]$ then $p_3 < 0$ and $p_3 \leq \frac{1}{2} - \Delta p$. Hence (DESTROY, DESTROY) Pareto dominates (KEEP AND NOT REPORT, KEEP AND NOT REPORT) for all $p \in [0, \frac{1}{2} - \Delta p]$. Notice that we restrict $\Delta p < \frac{1}{2}$ so that the set $[0, \frac{1}{2} - \Delta p]$ is non-empty.

2. From Lemmata 35(7), 35(8) and 35(9) if $\beta \in [0.414, 0.5]$ and $\delta < \frac{\beta^2}{(1-\beta)^2}$ then $\frac{1}{\beta} > \frac{2-\beta}{2\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} > \frac{1}{2\beta} > 1$.

   (a) If $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$ then from Lemma 35(5) $\Delta p_3 > \Delta p_5$. Taking into consideration Lemmata 35(1), 35(2), 35(3) and 35(4) we may distinguish the following three cases:

      i. if $\Delta p < \Delta p_5$ then $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$. Hence (KEEP AND NOT REPORT, KEEP AND NOT REPORT) Pareto dominates (DESTROY, DESTROY) for all $p \in [0, \frac{1}{2} - \Delta p]$.

      ii. if $\Delta p \in [\Delta p_5, \Delta p_3]$ then $p_3 \geq 0$ and $p_3 \leq \frac{1}{2} - \Delta p$. Hence for $p \in [0, p_3)$ (KEEP AND NOT REPORT, KEEP AND NOT REPORT) Pareto dominates (DESTROY, DESTROY), while for $p \in [p_3, \frac{1}{2} - \Delta p]$ (DESTROY, DESTROY) Pareto dominates (KEEP AND NOT REPORT, KEEP AND NOT REPORT).

      iii. if $\Delta p \in [\Delta p_3, \frac{1}{2}]$ then $p_3 < 0$ and $p_3 \leq \frac{1}{2} - \Delta p$. Hence (DESTROY, DESTROY) Pareto dominates (KEEP AND NOT REPORT, KEEP AND NOT REPORT) for all $p \in [0, \frac{1}{2} - \Delta p]$.

   (b) If $\frac{\pi}{F} \in [\frac{1}{2\beta}, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ then from Lemma 35(5) $\Delta p_5 > \Delta p_3 > \frac{1}{2}$. Taking into consideration Lemmata 35(1), 35(2), 35(3) and 35(4) if $\Delta p < \frac{1}{2}$ then $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$. Hence, (KEEP AND NOT REPORT, KEEP AND NOT REPORT) Pareto dominates (DESTROY, DESTROY).

3. From Lemmata 35(7), 35(8) if $\beta \in [\frac{1}{2}, \frac{2}{3}]$ then $\frac{1}{\beta} > \frac{2-\beta}{2\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} > 1 > \frac{1}{2\beta}$. By taking into consideration Lemmata 35(2), 35(4) and 35(5) if $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ then $\Delta p_5 > \Delta p_3 > \frac{1}{2}$. By Lemmata 35(1) and 35(3) if $\Delta p < \frac{1}{2}$ then $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$. Hence for $\Delta p < \frac{1}{2}$ (KEEP AND NOT REPORT, KEEP AND NOT REPORT) Pareto dominates (DESTROY, DESTROY) for all $p \in [0, \frac{1}{2} - \Delta p]$.

4. From Lemmata 35(7), 35(8) if $\beta > \frac{2}{3}$ then $\frac{1}{\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} > 1 > \frac{2-\beta}{2\beta} > \frac{1}{2\beta}$. By taking into account Lemmata 35(2), 35(4) and 35(5) $1 > \Delta p_5 > \Delta p_3 > \frac{1}{2}$. Hence, from Lemmata 35(1) and 35(3) if $\Delta p < \frac{1}{2}$ then $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$. Therefore, for $\Delta p < \frac{1}{2}$ (KEEP AND NOT REPORT, KEEP AND NOT REPORT) Pareto dominates (DESTROY, DESTROY) for all $p \in [0, \frac{1}{2} - \Delta p]$.

Case 2 $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$ *so that* $\Delta p_7 > 1$

Consider the second case now where $\Delta p_7 > 1$ so that for all Δp and $p \leq \frac{1}{2} - \Delta p$ we have two NE.

1. From Lemmata 35(7), 35(8) and 35(9) if $\beta < 0.414$ or $\beta \in [0.414, 0.5]$ and $\delta > \frac{\beta^2}{(1-\beta)^2}$ then $\frac{1}{\beta} > \frac{2-\beta}{2\beta} > \frac{1}{2\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} > 1$. Therefore, given that $\frac{\pi}{F} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}$ and taking into account Lemmata 35(1), 35(2), 35(3), 35(4) and 35(5) we may then distinguish four cases:

   (a) if $\frac{\pi}{F} > \frac{1}{\beta}$ then both $\Delta p_5 > \Delta p_3 > 1$ implying that $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$. Therefore for $\Delta p < \frac{1}{2}$ (KEEP AND NOT REPORT, KEEP AND NOT REPORT) Pareto dominates (DESTROY, DESTROY) for all $p \in [0, \frac{1}{2} - \Delta p]$.

   (b) if $\frac{\pi}{F} \in [\frac{2-\beta}{2\beta}, \frac{1}{\beta}]$ then $\Delta p_5 > 1 > \Delta p_3 > \frac{1}{2}$. Therefore, for $\Delta p < \frac{1}{2}$ then $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$ and hence (KEEP AND NOT REPORT, KEEP AND NOT REPORT) Pareto dominates (DESTROY, DESTROY) for all $p \in [0, \frac{1}{2} - \Delta p]$.

   (c) if $\frac{\pi}{F} \in [\frac{1}{2\beta}, \frac{2-\beta}{2\beta}]$ then $1 > \Delta p_5 > \Delta p_3 > \frac{1}{2}$. Therefore, for $\Delta p < \frac{1}{2}$ then $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$ and hence (KEEP AND NOT REPORT, KEEP AND NOT REPORT) Pareto dominates (DESTROY, DESTROY) for all $p \in [0, \frac{1}{2} - \Delta p]$.

   (d) $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{2\beta}]$ then $\frac{1}{2} > \Delta p_3 > \Delta p_5$. Therefore:

      i. if $\Delta p < \Delta p_5$, then $p_3 > 0$ and $p_3 > \frac{1}{2} - \Delta p$ and hence (KEEP AND NOT REPORT, KEEP AND NOT REPORT) Pareto dominates (DESTROY, DESTROY).

ii. if $\Delta p \in [\Delta p_5, \Delta p_3]$ then $p_3 \geq 0$ and $p_3 \leq \frac{1}{2} - \Delta p$ and hence for $p < p_3$ (Keep and not Report, Keep and not Report) Pareto dominates (Destroy, Destroy), while for $p \in [p_3, \frac{1}{2} - \Delta p]$ (Destroy, Destroy) Pareto dominates (Keep and not Report, Keep and not Report).

iii. if $\Delta p > (\Delta p_3, \frac{1}{2}]$ then $p_3 < 0$ and hence (Destroy, Destroy) Pareto dominates (Keep and not Report, Keep and not Report) for all $p \in [0, \frac{1}{2} - \Delta p]$.

2. From Lemmata 35(7), 35(8) and 35(9) if $\beta \in [0,414, 0,5]$ and $\delta < \frac{\beta^2}{(1-\beta)^2}$ then $\frac{1}{\beta} > \frac{2-\beta}{2\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} > \frac{1}{2\beta} > 1$. By taking into account Lemmata 35(1), 35(2), 35(3), 35(4) and 35(5) we may then distinguish the following three cases:

   (a) if $\frac{\pi}{F} > \frac{1}{\beta}$ then both $\Delta p_5 > \Delta p_3 > \frac{1}{2}$ implying that for all $\Delta p < \frac{1}{2}$ $p_3 > 0$ and $p_3 > \frac{1}{2} - \Delta p$. Therefore (Keep and not Report, Keep and not Report) Pareto dominates (Destroy, Destroy) for all $p \in [0, \frac{1}{2} - \Delta p]$.

   (b) if $\frac{\pi}{F} \in [\frac{2-\beta}{2\beta}, \frac{1}{\beta}]$ then $\Delta p_5 > 1 > \Delta p_3 > \frac{1}{2}$. Therefore, for $\Delta p < \frac{1}{2}$ then $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$ and hence (Keep and not Report, Keep and not Report) Pareto dominates (Destroy, Destroy) for all $p \in [0, \frac{1}{2} - \Delta p]$.

   (c) if $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{2-\beta}{2\beta}]$ then $1 > \Delta p_5 > \Delta p_3 > \frac{1}{2}$. Therefore, for $\Delta p < \frac{1}{2}$ then $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$ and hence (Keep and not Report, Keep and not Report) Pareto dominates (Destroy, Destroy) for all $p \in [0, \frac{1}{2} - \Delta p]$.

3. From Lemmata 35(7) and 35(8) if $\beta \in [\frac{1}{2}, \frac{2}{3}]$ then $\frac{1}{\beta} > \frac{2-\beta}{2\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} > 1 > \frac{1}{2\beta}$. By taking into consideration Lemmata 35(1), 35(2), 35(3), 35(4) and 35(5) we may then distinguish the following three cases:

   (a) if $\frac{\pi}{F} > \frac{1}{\beta}$ then both $\Delta p_5 > \Delta p_3 > 1$ implying that for all $\Delta p < \frac{1}{2}$, $p_3 > 0$ and $p_3 > \frac{1}{2} - \Delta p$. Therefore, for all $\Delta p < \frac{1}{2}$ (Keep and not Report, Keep and not Report) Pareto dominates (Destroy, Destroy) for all $p \in [0, \frac{1}{2} - \Delta p]$.

   (b) if $\frac{\pi}{F} \in [\frac{2-\beta}{2\beta}, \frac{1}{\beta}]$ then $\Delta p_5 > 1 > \Delta p_3 > \frac{1}{2}$. Therefore, for $\Delta p < \frac{1}{2}$ then $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$ and hence (Keep and not Report, Keep and not Report) Pareto dominates (Destroy, Destroy) for all $p \in [0, \frac{1}{2} - \Delta p]$.

(c) if $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{2-\beta}{2\beta}]$ then $1 > \Delta p_5 > \Delta p_3 > \frac{1}{2}$. Therefore, for $\Delta p < \frac{1}{2}$ then $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$ and hence (Keep and not Report, Keep and not Report) Pareto dominates (Destroy, Destroy) for all $p \in [0, \frac{1}{2} - \Delta p]$.

4. From Lemmata Lemmata 35(7) and 35(8) if $\beta > \frac{2}{3}$ then $\frac{1}{\beta} > \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)} > 1 > \frac{2-\beta}{2\beta} > \frac{1}{2\beta}$. By taking into consideration Lemmata 35(1), 35(2), 35(3), 35(4) and 35(5) we may then distinguish the following two cases:

   (a) $\frac{\pi}{F} > \frac{1}{\beta}$ then both $\Delta p_5 > \Delta p_3 > \frac{1}{2}$ implying that for all $\Delta p < \frac{1}{2}$, $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$. Therefore (Keep and not Report, Keep and not Report) Pareto dominates (Destroy, Destroy) for all $p \in [0, \frac{1}{2} - \Delta p]$.

   (b) $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ then $\Delta p_5 > 1 > \Delta p_3 > \frac{1}{2}$. Therefore, for $\Delta p < \frac{1}{2}$ then $p_3 \geq 0$ and $p_3 > \frac{1}{2} - \Delta p$ and hence (Keep and not Report, Keep and not Report) Pareto dominates (Destroy, Destroy) for all $p \in [0, \frac{1}{2} - \Delta p]$.

We can now summarize the results of the above analysis to the following lemma.

Lemma 36 *The SPE of the game is (*Keep and Not Report, Keep and Not Report*) if :*

1. $\beta < 0.414$

   (a) $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{2\beta}]$ *and* $\Delta p_7 > 1$ *OR* $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$ *and* $\Delta p_7 < 1$

      i. $\Delta p < \Delta p_5$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

      ii. $\Delta p \in [\Delta p_5, \Delta p_3]$ *and* $p \in [0, p_3)$

   (b) $\frac{\pi}{F} \in [\frac{1}{2\beta}, \frac{1}{\beta}]$, $\Delta p < \frac{1}{2}(< \Delta p_3 < 1 < \Delta p_7)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

   (c) $\frac{\pi}{F} > \frac{1}{\beta}$, $\Delta p < \frac{1}{2} < 1 < \Delta p_7$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

2. $\beta \in [0.414, 0.5]$

   (a) $\delta < \frac{\beta^2}{(1-\beta)^2}$

      i. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p < \Delta p_5(< \frac{1}{2} < \Delta p_7 < 1)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

      ii. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_3](< \frac{1}{2} < \Delta p_7 < 1)$ *and* $p \in [0, p_3)$

      iii. $\frac{\pi}{F} \in [\frac{1}{2\beta}, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$, $\Delta p < \frac{1}{2}(< \Delta p_3 < \Delta p_7 < 1)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

iv. $\frac{\pi}{F} \in [\frac{1}{2\beta}, \frac{1}{\beta}]$, $\Delta p < \frac{1}{2} (< \Delta p_3 < 1 < \Delta p_7)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

v. $\frac{\pi}{F} > \frac{1}{\beta}$, $\Delta p < \frac{1}{2}$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

(b) $\delta > \frac{\beta^2}{(1-\beta)^2}$

i. $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$, $\Delta p < \Delta p_5 (< \Delta p_7 < 1)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

ii. $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$, $\Delta p \in [\Delta p_5, \Delta p_3] (< \frac{1}{2} < \Delta p_7 < 1)$ *and* $p \in [0, p_3)$

iii. $\frac{\pi}{F} \in [\frac{1}{2\beta}, \frac{1}{\beta}]$, $\Delta p < \frac{1}{2} (< \Delta p_3 < 1 < \Delta p_7)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

iv. $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{2\beta}]$, $\Delta p < \Delta p_5 (< 1 < \Delta p_7)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

v. $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_3] (< \frac{1}{2} < 1 < \Delta p_7)$ *and* $p \in [0, p_3)$

vi. $\frac{\pi}{F} > \frac{1}{\beta}$, $\Delta p < \frac{1}{2} (< 1 < \Delta p_7)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

3. $\beta > 0.5$

(a) $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$, $\Delta p < \frac{1}{2} (< \Delta p_3 < \Delta p_7 < 1)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

(b) $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{\beta}]$, $\Delta p < \frac{1}{2} (< \Delta p_3 < 1 < \Delta p_7)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

(c) $\frac{\pi}{F} > \frac{1}{\beta}$, $\forall \Delta p < \frac{1}{2} (< 1 < \Delta p_7)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

Lemma 36(2(a)iii) and 36(2(a)iv) imply the following result.

Corollary 13 *(*Keep and not Report, Keep and Not Report*) is the SPE if* $\beta \in [0.414, 0.5]$, $\delta < \frac{\beta^2}{(1-\beta)^2}$, $\frac{\pi}{F} \in [\frac{1}{2\beta}, \frac{1}{\beta}]$, $\Delta p < \frac{1}{2}$ *and* $p \in [0, \frac{1}{2} - \Delta p]$.

From Lemma 36(2(b)i) and 36(2(b)iv) we can get the corollary below.

Corollary 14 *(*Keep and not Report, Keep and Not Report*) is the SPE if* $\beta \in [0.414, 0.5]$, $\delta > \frac{\beta^2}{(1-\beta)^2}$, $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p < \Delta p_5$ *and* $p \in [0, \frac{1}{2} - \Delta p]$.

Lemma 36(2(b)ii) and 36(2(b)v) imply the following.

Corollary 15 *(*Keep and not Report, Keep and Not Report*) is the SPE if* $\beta \in [0.414, 0.5]$, $\delta > \frac{\beta^2}{(1-\beta)^2}$, $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_3]$ *and* $p \in [0, p_3)$.

Lemma 36(3a) and 36(3b) imply the following.

Corollary 16 *(*Keep and not Report, Keep and Not Report*) is the SPE if* $\frac{\pi}{F} \in [1, \frac{1}{\beta}]$, $\Delta p < \frac{1}{2}$ *and* $p \in [0, \frac{1}{2} - \Delta p]$.

From Lemma 36 and Corollaries 13, 14, 15 and 16 we get the following result, which identifies the parameters' values for which (KEEP AND NOT REPORT, KEEP AND NOT REPORT) is the Pareto dominant SPE for $p \leq \frac{1}{2} - \Delta p$.

Similarly, by summarizing the results of our previous analysis we find the regions for the parameters' values for which (DESTROY, DESTROY) form a Pareto dominant SPE for $p \leq \frac{1}{2} - \Delta p$.

Lemma 37 *For $p \in [0, \frac{1}{2} - \Delta p]$ (DESTROY, DESTROY) is the Pareto dominant SPE if:*

1. $\beta < 0.414$
   (a) $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$, $\Delta p \in [\Delta p_5, \Delta p_3](< \Delta p_7 < 1)$ *and* $p \in [p_3, \frac{1}{2} - \Delta p]$
   (b) $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$, $\Delta p \in [\Delta p_3, \frac{1}{2}](< \Delta p_7 < 1)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$
   (c) $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_3, \frac{1}{2}](< 1 < \Delta p_7)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$
   (d) $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{2\beta}]$, $[\Delta p_5, \Delta p_3](< \frac{1}{2} < 1 < \Delta p_7)$ *and* $p \in [p_3, \frac{1}{2} - \Delta p]$
2. $\beta \in [0.414, 0.5]$
   (a) $\delta < \frac{\beta^2}{(1-\beta)^2}$
      i. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_3](< \Delta p_7 < 1)$ *and* $p \in [p_3, \frac{1}{2} - \Delta p]$
      ii. $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_3, \frac{1}{2}](< 1 < \Delta p_7)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$
   (b) $\delta > \frac{\beta^2}{(1-\beta)^2}$
      i. $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$, $\Delta p \in [\Delta p_5, \Delta p_3](< \Delta p_7 < 1)$ *and* $p \in [p_3, \frac{1}{2} - \Delta p]$
      ii. $\frac{\pi}{F} \in [1, \frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}]$, $\Delta p \in [\Delta p_3, \frac{1}{2}](< \Delta p_7 < 1)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$
      iii. $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_3, \frac{1}{2}](< 1 < \Delta p_7)$ *and* $p \in [0, \frac{1}{2} - \Delta p]$
      iv. $\frac{\pi}{F} \in [\frac{\delta+\beta-\beta\delta+1}{2(\delta+\beta-\beta\delta)}, \frac{1}{2\beta}]$, $[\Delta p_5, \Delta p_3](< \frac{1}{2} < 1 < \Delta p_7)$ *and* $p \in [p_3, \frac{1}{2} - \Delta p]$

Taking into consideration Corollary 12 we get the following result, which identifies the parameters' values for which (DESTROY, DESTROY) is the Pareto dominant SPE for $p \in [0, \frac{1}{2} - \Delta p]$.

From Lemmata 36, 37 and 32 we obtain the following proposition.

Proposition 2 *For given policy and other parameters (p, Δp, F, β, π) the Pareto dominant SPE of the extended game with LP is:*

1. (Destroy, Destroy) *iff* $\beta < \frac{1}{2}, \frac{\pi}{F} \in [1, \frac{1}{2\beta}]$ *and*

   (a) $\Delta p \in [\Delta p_5, \Delta p_3]$ *and* $p \in [p_3, \frac{1}{2} - \Delta p]$ *or*

   (b) $[\Delta p_3, \frac{1}{2})$ *and* $p \in [0, \frac{1}{2} - \Delta p]$ *or*

   (c) $\Delta p \in [\Delta p_5, \Delta p_6]$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, p_4]$ *or*

   (d) $\Delta p > \Delta p_6$ *and* $p \in [0, 1 - \Delta p]$

2. (Keep and Not Report, Keep and Not Report) *iff:*

   (a) $\beta < \frac{1}{2}$ *and*

      i. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p < \Delta p_5$ *and* $p \in [0, \frac{1}{2} - \Delta p]$ *or*

      ii. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_3]$ *and* $p \in [0, p_3)$ *or*

      iii. $\frac{\pi}{F} > \frac{1}{2\beta}$, $\Delta p < \frac{1}{2}$ *and* $p \in [0, \frac{1}{2} - \Delta p]$ *or*

   (b) $\beta > \frac{1}{2}, \frac{\pi}{F} \geq 1$, $\Delta p < \frac{1}{2}$ *and* $p \in [0, \frac{1}{2} - \Delta p]$

3. (Keep and Report, Keep and Report) *iff:*

   (a) $\beta < \frac{1}{2}$ *and*

      i. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p < \Delta p_5$ *and* $p \in [\frac{1}{2} - \Delta p, 1 - \Delta p]$ *or*

      ii. $\frac{\pi}{F} \in [1, \frac{1}{2\beta}]$, $\Delta p \in [\Delta p_5, \Delta p_6]$ *and* $p \in (p_4, 1 - \Delta p]$ *or*

      iii. $\frac{\pi}{F} > \frac{1}{2\beta}$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$ *or*

   (b) $\beta > \frac{1}{2}$, $\frac{\pi}{F} \geq 1$ *and* $p \in [\max\{0, \frac{1}{2} - \Delta p\}, 1 - \Delta p]$

where $p_3 = \frac{\pi}{F} - \frac{\Delta p}{\beta}$, $p_4 = \frac{F - 2\beta\Pi_M}{2(1-\beta)F}$, $\Delta p_3 = \beta\frac{\pi}{F}$, $\Delta p_5 = \frac{\beta}{1-\beta}\frac{\pi}{F} - \frac{\beta}{2(1-\beta)}$ *and* $\Delta p_6 = \frac{1-2\beta}{2(1-\beta)} + \frac{\beta}{1-\beta}\frac{\pi}{F}$.