Collusive Price Rigidity under Price-Matching Punishments

Luke Garrod

CCP Working Paper 11-14

Abstract: By analysing an infinitely repeated game where unit costs alternate stochastically between low and high states and where firms follow a price – matching punishment strategy, we demonstrate that the best collusive prices are rigid over time when the two cost levels are sufficiently close. This provides game theoretic support for the results of the kinked demand curve. In contrast to the kinked demand curve it also generates predictions regarding the level and the determinants of the best collusive price, which in turn has implications for the corresponding collusive profits. The relationships between such price rigidity and the expected duration of a high-cost phase, the degree of product differentiation, and the number of firms in the market are also investigated.

Keywords: Tacit collusion, kinked demand curve, price rigidity.

JEL Classification codes: L11, L13, L41

Acknowledgements: I am grateful for comments from Iwan Bos, Steve Davies, Joe Harrington, Morten Hviid, Roman Inderst, Kai-Uwe Kühn, Bruce Lyons, Matt Olczak, Patrick Rey, Paul Seabright, Chris M. Wilson, two anonymous referees, and seminar participants at the International Industrial Organization Conference 2011 and the European Association of Research in Industrial Economics Conference 2011. The support of the Economic and Social Research Council (UK) is gratefully acknowledged. The usual disclaimer applies.

ISSN 1745-9648

Contact Details:

Luke Garrod: School of Business and Economics, Loughborough University, Loughborough, Leicestershire, LE11 3TU, United Kingdom, email: l.garrod@lboro.ac.uk

Collusive Price Rigidity under Price-Matching Punishments*

Luke Garrod†

March 2012

Abstract

By analysing an infinitely repeated game where unit costs alternate stochastically between low and high states and where firms follow a price-matching punishment strategy, we demonstrate that the best collusive prices are rigid over time when the two cost levels are sufficiently close. This provides game theoretic support for the results of the kinked demand curve. In contrast to the kinked demand curve, it also generates predictions regarding the level and the determinants of the best collusive price, which in turn has implications for the corresponding collusive profits. The relationships between such price rigidity and the expected duration of a high-cost phase, the degree of product differentiation, and the number of firms in the market are also investigated.

Keywords: Tacit collusion, kinked demand curve, price rigidity
JEL: L11, L13, L41

1 Introduction

The old theory of the kinked demand curve (Hall and Hitch, 1939; and Sweezy, 1939) was the first attempt to formalise the long-standing belief that tacit collusion and price rigidity are linked. It assumes that there is a prevailing focal price and that rivals will match a firm's price decrease but they will not match a price increase. This rivalry implies that each firm's demand curve has a kink at the focal price, and it follows from the resultant discontinuity in marginal revenue curve that prices remain constant at the focal level for a range of marginal costs. Although the rivalry

*I am grateful for comments from Iwan Bos, Steve Davies, Joe Harrington, Morten Hviid, Roman Inderst, Kai-Uwe Kühn, Bruce Lyons, Matt Olczak, Patrick Rey, Paul Seabright, Chris M. Wilson, two anonymous referees, and seminar participants at the International Industrial Organization Conference 2011 and the European Association of Research in Industrial Economics Conference 2011. The support of the Economic and Social Research Council (UK) is gratefully acknowledged. The usual disclaimer applies.

†School of Business and Economics, Loughborough University, Loughborough, Leicestershire, LE11 3TU, United Kingdom, email: l.garrod@lboro.ac.uk

of the kinked demand curve has an intuitive appeal and some anecdotal support, this theory has been heavily criticised (for example see Tirole, 1988, p.243-245).

Contemporary models of dynamic oligopolistic interaction differ in two respects with the kinked demand curve. First, they are modelled as an explicit dynamic game using the theory of repeated games, where collusive prices are sustainable when the short-term gain from any deviation is outweighed by the long-term loss from a credible retaliation. Second, firms usually more than match lower deviation prices, because the most commonly analysed retaliations are "Nash reversion" (see Friedman, 1971) and "optimal punishment strategies" (see Abreu, 1986, 1988). Using such models, there is a theoretical literature that analyses the effect of temporary changes in market conditions on the best collusive prices that achieve the highest levels of profit possible (for example see Rotemberg and Saloner, 1986; Haltiwanger and Harrington, 1991).[1] A feature of this literature is that, barring the special circumstances when incentives are perfectly aligned, the best collusive prices are not rigid over time. This is at odds with the results of the kinked demand curve and with the belief that tacit collusion and price rigidity are linked.

In this paper, we analyse the rivalry of the kinked demand curve in an infinitely repeated game and show that, in contrast to the previous collusion literature, the best collusive prices can be rigid over time despite small industry-wide changes in unit costs. This provides game theoretic support for the results of the kinked demand curve. We derive this result by extending Lu and Wright (2010) who analyse an infinitely repeated game where, similar to the kinked demand curve, firms match lower deviation prices (provided they are above the one-shot Nash equilibrium price) but do not match higher deviation prices. They show that collusive prices are sustainable under such "price-matching punishments" when products are symmetrically differentiated and when market conditions do not vary over time. We extend their model so that unit costs alternate stochastically between high and low states, and analyse the characteristics of the best collusive subgame perfect Nash equilibrium when prices are rigid over time.

The intuition behind price rigidity in our model is that when unit costs are temporarily high today and they are permanently low in the future, there is an incentive to deviate today from any collusive price above the collusive price that prevails in the future. The reason is that when a firm deviates to that future price today, there is no long-term loss to offset the deviation gain, because the punishment is just to match the deviation price, which still results in the collusive price being set in the future. An implication of this is that the best collusive prices are rigid when the two cost levels are sufficiently close, such that the one-shot Nash equilibrium

[1] Although much of the previous literature focuses on changes in demand, many of the results generalise to changes in costs.

price of the high-cost state is not above the future collusive price. When high costs persist into the future, price matching such a deviation does reduce the profits of future high-cost states, but this long-term loss must outweigh the initial gain for procyclical prices to be sustainable. Given this loss is small and the gain is large when the two cost levels are close, the best collusive prices are rigid over time when the difference between the two cost levels is below some critical threshold. We show that this critical threshold falls as high-cost states are likely to persist for longer into the future, and it equals zero when the high-cost state is expected to last permanently.

In contrast to the kinked demand curve, our model generates predictions regarding the level of the best rigid price and the effects of the other parameters of the model on it. This price uniquely achieves the highest level of profit possible, given that the price does not vary over time. It defines the best collusive price in both cost states when the difference between the two cost levels is below the critical threshold, and it is always between the one-shot Nash equilibrium price of the high-cost state and the monopoly price of the low-cost state for such conditions. The best rigid price is determined by the incentives to collude in low-cost states, and it monotonically increases with the level of high costs at a rate that is less than one-to-one. An implication of these features is that when the best collusive prices are rigid over time, the resultant per-period collusive profits of low-cost states are strictly increasing in the level of high costs, but such profits of high-cost states are strictly decreasing in the level of high costs. In contrast, the corresponding present discounted values of collusive profits are strictly decreasing in the level of high costs, whether the initial period has high or low costs.

Our model also generates predictions regarding the relationship between price rigidity and the number of firms in the market, which has been investigated by several empirical studies (for example see Carlton, 1986, 1989). This relationship ultimately depends upon the degree of product differentiation. Based on an example where demand is derived from the constant elasticity of substitution version of Spence-Dixit-Stiglitz preferences (Spence, 1976; and Dixit and Stiglitz, 1977), we show that the best collusive prices are rigid for the largest difference between the two cost levels when products are differentiated by an intermediate degree. This is because price-matching punishments do not support collusive prices when products are homogeneous, and since firms can set the monopoly prices when they have no close rivals. Finally, we find that the best collusive prices are rigid for a larger difference between the two cost levels in a concentrated market, with few firms, than in a less concentrated market, with a greater number of firms, when the degree of product differentiation is sufficiently low.

The rest of the paper is structured as follows. Section 2 reviews the related literature and provides anecdotal support for a link between price matching and price rigidity. Section 3 outlines the assumptions on demand and costs, and it formally defines the price-matching punishment strategy. In section 4, we first find the conditions for which the best collusive prices are rigid over time. We then analyse the relationship between such price rigidity and the expected duration of a sequence of high-cost states, and after that we investigate the effects of such price rigidity and fluctuating costs on the best collusive profits. Section 5 places more structure on demand to investigate the effects of both the degree of product differentiation and the number of firms in the market on such price rigidity, and section 6 concludes. All proofs are relegated to the appendix.

2 Related Literature and Evidence

In this paper, we propose that the expectation that lower deviation prices will be matched can lead to price rigidity during collusive phases, and there is some anecdotal support for a link between the two. Slade (1987, 1992) analysed a price war between gasoline retailers during 1983 in Vancouver (see also Slade, 1990). She found that there was "a high degree of (lagged) price matching during the war" and that "prices before and after the war were uniform across firms and stable over time" (1992, p.264). In fact, "after the price war came to an end, prices were stable for nearly a year" (1987, p.515). Slade (1989, p.295) also argues that other Canadian markets (including nickel, cigarette, as well as gasoline) had three stylised facts: "First, price is the choice variable and it can be observed by all. Second, price wars are occasional events and are separated by periods of stable prices. Third, during a war there is considerable matching of prices". Similarly, Kalai and Satterthwaite (1994) state that between 1900 and 1958 small firms in the US steel industry believed that the largest producer would match their prices if they undercut it, and observed that "Before World War II certain classes of steel products showed remarkable price rigidity" (p.31).[2]

The anecdotal evidence above suggests that, in at least some situations, price matching is a relevant form of firm behaviour, and this is re-emphasised by Slade's (1987) empirical evidence that finds some support for strategies, similar to price matching, where "small deviations lead to small punishments" over Nash reversion (p.499). This contrasts with the informal reasoning that argues that since collusion is easier to sustain under harsher punishments, then colluding firms would employ the harshest credible punishment. The evidence above also suggests that our price

[2] Levenstein (1997) and Genesove and Mullin (2001) also find that some cartel price wars consisted of mild punishments and price matching, respectively, but due to infrequent price observations it is not possible to determine the extent to which prices vary over time.

rigidity result may be of some empirical relevance for such situations where price matching is prevalent. This differs to previous attempts to model the rivalry of the kinked demand curve in dynamic settings, because they do not find a link between price matching and price rigidity.[3]

Our model also contrasts with the literature that analyses collusion when market conditions vary over time. Rotemberg and Saloner (1986) show that under Nash reversion the deviation gains are greatest in a temporary boom but the long-term losses are constant when future market conditions are independent of current conditions. This implies that any price that is just sustainable in a low-cost boom is easier to sustain in a high-cost bust, so the best collusive prices are procyclical. For similar reasons, the incentive to deviate from a rigid price is greatest in a period of low costs under price-matching punishments, when future fluctuations in costs are independent of or positively correlated with the current level. However, procyclical collusive prices may not be sustainable, because there is a discontinuity in the incentives to collude at the rigid price in high-cost states. This is because price matching reduces only the price set in future high-cost states when a firm deviates in a period of high costs by reducing its price to the low-cost price that prevails in low-cost periods. Therefore, such a deviation from a price above yet very close to the low-cost price can generate a much smaller long-term loss than an otherwise identical deviation from the low-cost price, where price matching reduces the prices set in all future periods. Yet, the deviation gains are effectively the same for such deviations. As a result, a deviation in a period of high costs from a price only slightly above the low-cost price can be profitable, even though a deviation from the low-cost price will be strictly unprofitable.

Finally, this paper is also related to Athey *et al* (2004) who develop an alternative model of collusive price rigidity, where prices are publicly observable but firms experience private shocks to their unit costs in each period. They show that the best collusive prices under Nash reversion may be rigid over time because, although demand is not allocated to the most efficient firm, this inefficiency can be outweighed by the benefit of detecting deviations easily.[4] Their model is similar to Green and Porter (1984) since, due to some information asymmetry, price wars can occur on

[3]Bhaskar (1988) and Kalai and Satterthwaite (1994) show that price rigidity does not occur in a one-shot game when lower prices can be matched immediately before profits are realised. In an infinitely repeated game where a duopoly alternates between committing to price for two periods, Maskin and Tirole (1988) show that price rigidity can occur in a Markov perfect equilibrium when costs fall permanently, because firms attempt to avoid a price war. However, this is because rivals more than match lower prices. In another related infinitely repeated game, Slade (1989) captures the three stylised facts discussed above when an unexpected change in demand is anticipated to be permanent, but stable prices only occur in her model when the new equilibrium is reached.

[4]In a similar model, Hanazono and Yang (2007) show that price rigidity can also occur with unobservable demand fluctuations.

the equilibrium path when firms receive a bad signal. In contrast, price wars do not occur on the equilibrium path in our model, because there is symmetric information. Instead, the successfulness of collusion is affected by market conditions in a similar way as Rotemberg and Saloner (1986). Our model adds to our understanding of price rigidity because it is the first to consider the relationship between price rigidity and the degree of product differentiation, and it can be tested empirically since it does not rely on parameters that are likely to be unobservable to an econometrician.

3 The Model

3.1 Basic assumptions

Consider a market where a fixed number of $n \geq 2$ firms each produce a single differentiated product and compete in observable prices over an infinite number of periods. In any period t, firms have constant unit costs, $c_t \geq 0$, face no fixed costs, and have a common discount factor, $\delta \in (0,1)$. They simultaneously choose price in each period and the demand of firm $i = 1, \ldots, n$ in period t is $q_i(p_{it}, \mathbf{p}_{-it}, n)$ where p_{it} is its own price and $\mathbf{p}_{-it}$ is the vector of its rivals' prices. Demand is symmetric, strictly decreasing in p_{it} and $\lim_{p_{it}\to\infty} q_i(p_{it}, \mathbf{p}_{-it}, n) = 0$. Since firms are symmetric, at equal prices $p_{it} = p_t$ for all i, $q_i(p_t, p_t, n) = q(p_t)/n$ where $q(p_t)$ is independent of n. For every price vector $\mathbf{p}_t$=$(p_{it}, \mathbf{p}_{-it})$ where $q_i(p_{it}, \mathbf{p}_{-it}, n) > 0$ for all i, demand is twice continuously differentiable and from Vives (2001, p.148-152) we assume it has the following standard properties:

Assumption 1. $\left|\frac{\partial q_i}{\partial p_{it}}\right| > \sum_{j\neq i} \frac{\partial q_i}{\partial p_{jt}} > 0$

Assumption 2. $\frac{\partial^2 q_i}{\partial p_{it}\partial p_{jt}} \geq 0 \;\forall\; j \neq i$

Assumption 3. $\frac{\partial^2 q_i}{\partial p_{it}^2} + \sum_{j\neq i} \frac{\partial^2 q_i}{\partial p_{it}\partial p_{jt}} < 0.$

These assumptions imply that products are imperfect substitutes, demand exhibits increasing differences in (p_{it}, p_{jt}) and the own effect of a price change dominates the cross effect both in terms of the level and slope of demand.

Firm i's per-period profit in period t is $\pi_{it}(p_{it}, \mathbf{p}_{-it}; c_t, n) = (p_{it} - c_t) q_i(p_{it}, \mathbf{p}_{-it}, n)$, where at equal prices $p_{it} = p_t$ for all i write $\pi_{it}(p_t, p_t; c_t, n) = \pi_t(p_t; c_t, n)$. Assumptions 1 and 2 imply that prices are strategic complements:

$$\frac{\partial^2 \pi_{it}}{\partial p_{it}\partial p_{jt}} > 0 \;\forall\; j \neq i \;\forall\; t. \tag{1}$$

Since unit costs are constant in any period, Assumptions 1 and 3 are sufficient to ensure the best reply mapping is a contraction (see Vives, 2001, p.150):

$$\frac{\partial^2 \pi_{it}}{\partial p_{it}^2} + \sum_{j \neq i} \frac{\partial^2 \pi_{it}}{\partial p_{it} \partial p_{jt}} < 0 \; \forall \; t. \tag{2}$$

This guarantees the existence of a unique one-shot Nash equilibrium price in pure strategies, denoted $p^N(c_t, n)$. It follows from (1) and (2) that each firm's per-period profit is strictly concave in its own price (i.e. $\partial^2 \pi_{it} / \partial p_{it}^2 < 0$), which implies that if rivals charge a price above $p^N(c_t, n)$, then a firm can strictly increase its per-period profit by unilaterally lowering its price towards the one-shot Nash equilibrium price (i.e. $\partial \pi_{it} / \partial p_{it} < 0 \; \forall \; p_{jt} > p^N(c_t, n)$, $j \neq i$). Assumption 1 guarantees that $p^N(c_t, n)$ is strictly increasing in c_t and to ensure that $p^N(c_t, n)$ is strictly decreasing in n, we assume the following sufficient condition:

Assumption 4. $\frac{\partial^2 q_i}{\partial p_{it} \partial n} < 0$.

Finally, to ensure that the monopoly price, $p^m(c_t)$, is unique with $p^m(c_t) > p^N(c_t, n)$ we assume:

Assumption 5. $\frac{d^2 \pi_t}{d p_t^2} = \frac{\partial^2 \pi_{it}}{\partial p_{it}^2} + 2 \sum_{j \neq i} \frac{\partial^2 \pi_{it}}{\partial p_{it} \partial p_{jt}} + \frac{\partial^2 \pi_{it}}{\partial p_{jt}^2} < 0 \; \forall \; t$.

An implication of Assumption 5 is that if all firms set the same price below the monopoly price, then they would strictly increase their per-period profits if all set a higher price (i.e. $d\pi_t / dp_t > 0 \; \forall \; p^N(c_t, n) \leq p_t < p^m(c_t)$). Assumption 1 ensures that $p^m(c_t)$ is strictly increasing in c_t, while the symmetry assumptions on demand and costs guarantee that $p^m(c_t)$ is independent of n.

3.2 Cost fluctuations

In any period, unit costs can be low or high such that $c_t = 0$ or $c_t = c > 0$. To simplify notation, write $p^N(0, n) = p^N(n)$, $p^m(0) = p^m$ and $\pi_{it}(p_{it}, \mathbf{p}_{-it}; 0, n) = \pi_{it}(p_{it}, \mathbf{p}_{-it}; n)$. The current level is common knowledge before firms set prices, and expectations of future levels of c_t for all t follow a Markov process such that:

$$\begin{aligned} \lambda \quad & \equiv \Pr\left(c_t = c \middle| \, c_{t-1} = 0\right) \in (0, 1) \\ \theta \quad & \equiv \Pr\left(c_t = 0 \middle| \, c_{t-1} = c\right) \in (0, 1) \\ \mu \quad & \equiv \Pr\left(c_0 = c\right) \in [0, 1]. \end{aligned}$$

Thus, λ is the transition probability associated with moving from a low-cost state to one of high costs, and θ is the probability that corresponds with a transition from high costs to low costs. The parameter μ describes how the system begins.

This process implies that the probability that costs will be high in the next period is λ if they are currently low, otherwise it is $1-\theta$. Thus, future costs are independent of the current level if $1-\theta-\lambda=0$, and this simple case provides a benchmark for our analysis. In many industries it is natural to expect that future costs will be positively correlated with the current level. Consequently, we also allow for the case where $1-\theta-\lambda>0$, which implies that it is more likely that the current cost level will continue into the next period than change. Following the terminology of Bagwell and Staiger (1997), we refer to the former as zero correlation $(1-\theta-\lambda=0)$ and the latter as positive correlation $(1-\theta-\lambda>0)$.

3.3 Collusive prices and profits

Due to the Markov process that determines future cost levels, collusive profits are the same in any high-cost state regardless of the specific date, other things equal, and likewise for any low-cost state. Thus, the best collusive prices emerge as a pair, and we wish to find the conditions for which these are equal. Analysing the best collusive prices is consistent with the prominent papers in the collusion literature (for example see Rotemberg and Saloner, 1986; Haltiwanger and Harringtion, 1991), and it is also consistent with the kinked demand curve since the most profitable equilibrium is often argued to be the most logical (see Tirole, 1988, p.244).

Let the collusive prices of high- and low-cost states be $p(c)$ and $p(0)$, respectively, and denote $\Omega_H(p(c),p(0))$ as a firm's expected discounted profit in period t and thereafter, if period t is a high-cost state. Similarly, denote $\Omega_L(p(0),p(c))$ as a firm's expected discounted profit in period t and thereafter, if period t is a low-cost state. Suppressing notation slightly, it is possible to write such profits as:

$$\begin{aligned}\Omega_H &= \pi(p(c);c,n)+\delta\theta\Omega_L+\delta(1-\theta)\Omega_H\\ \Omega_L &= \pi(p(0);n)+\delta\lambda\Omega_H+\delta(1-\lambda)\Omega_L.\end{aligned}$$

Solving for Ω_H and Ω_L gives:

$$\begin{aligned}\Omega_H(p(c),p(0)) &= \pi(p(c);c,n)+\tfrac{\delta}{1-\delta}\left[\tfrac{\theta}{\omega}\pi(p(0);n)+(1-\tfrac{\theta}{\omega})\pi(p(c);c,n)\right]\\ \Omega_L(p(0),p(c)) &= \pi(p(0);n)+\tfrac{\delta}{1-\delta}[\tfrac{\lambda}{\omega}\pi(p(c);c,n)+(1-\tfrac{\lambda}{\omega})\pi(p(0);n)],\end{aligned}$$

where $\omega\equiv 1-\delta(1-\theta-\lambda)>0$, $0<\frac{\theta}{\omega}<1$ and $0<\frac{\lambda}{\omega}<1$. The first terms on the right hand-side of the above equations represent the profits from the initial periods, and the second terms represent the discounted profits from all future periods, conditional on expectations of future cost levels.

3.4 Punishment strategy

Drawing on the insights of Lu and Wright (2010), we assume that firm i's price-matching punishment strategy profile for all t is of the form:

$$
\begin{aligned}
&p_{i0} = p_0(c_0) = p(c_0) \\
&p_{it} = p_t(c_t) = \begin{cases} p(c_t) & \text{if } p_{j\tau} = p_\tau(c_\tau) \ \forall\ j\ \forall\ \tau \in \{0, \ldots, t-1\} \\ \max\{p^N(c_t, n), \min\{\mathbf{p}_t^d, p(c_t)\}\} & \text{otherwise} \end{cases}
\end{aligned} \tag{3}
$$

where $\mathbf{p}_t^d$ is a vector of the history of deviation prices at period t (i.e. it includes all prices where $p_{j\tau} \neq p_\tau(c_\tau)\ \forall\ j\ \forall\ \tau \in \{0, \ldots, t-1\}$). This strategy calls for each firm to set the initial collusive prices until a deviation. Following a deviation, the lowest ever deviation price is matched in periods where it is above the one-shot Nash equilibrium price and it is below the initial collusive price of that period. The one-shot Nash equilibrium price is set in any period when the lowest ever deviation price is below the one-shot Nash equilibrium price. Similarly, the initial collusive price is set in any period when the lowest ever deviation price is above this collusive price. This is repeated for future deviations.

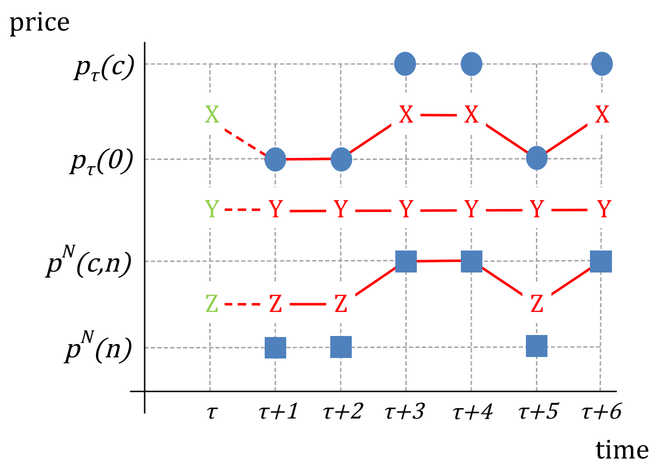

Figure 1: pricing after a one-stage deviation to X, Y and Z in period τ

Figure 1 illustrates the implications for pricing for various one-stage deviations in period τ (i.e. where a firm deviates for one period, then conforms to (3) thereafter). Understanding such deviations are important for our purposes, because we use the one-stage deviation principle (see Fudenberg and Tirole, 1991, p.108-110) to solve for subgame perfect Nash equilibria. This principle states that a strategy profile induces a Nash equilibrium in every subgame, if there is no history that leads to a subgame in which a deviant will choose an action that differs to the one prescribed by the strategy, then conform to the strategy thereafter (assuming the deviant believes others will also conform to the strategy). Thus, to prove subgame perfection, it suffices to show that a one stage-deviation is not profitable in the initial collusive subgames and nor are such deviations in every possible punishment subgame. We say that collusive prices are supportable if the strategy profile in (3) is a subgame perfect Nash equilibrium for all $i = 1, \ldots, n$.

The collusive prices are initially procyclical in Figure 1, because firms set $p_\tau(c)$ and $p_\tau(0)$ in high- and low-cost states, respectively. If a firm deviated to Y in period τ, then Y is matched thereafter. If it deviated to Z, however, then Z is matched in future low-cost states, otherwise $p^N(c, n)$ is set. Departing slightly from the kinked demand curve but consistent with Lu and Wright (2010), firms do not match prices below $p^N(c_t, n)$ in period t because doing so seems unreasonable. This assumption is not crucial in determining the range of rigid prices for which collusion is sustainable or the parameter space where the best collusive prices are rigid. This is because a deviation to Z is always less profitable than a deviation to Y in a period of low costs when the best collusive prices are rigid, and a deviation to Z would never occur in a period of high costs, even for prices that are not supportable. This assumption ensures that (3) defines a Nash equilibrium in punishment subgames for histories where the lowest ever deviation price is below $p^N(c_t, n)$ in period t, and that it is possible to check that (3) induces a Nash equilibrium in punishment subgames that start with a period of low costs for histories where the lowest deviation price is between $p^N(n)$ and $p^N(c, n)$.

The strategy profile (3) also has a similar feature for one-stage deviations to prices above the lowest initial collusive price, because if a firm deviated to X, then X is matched only in periods when the initial collusive price is above X, otherwise $p_\tau(0)$ is set. Figure 1 illustrates the case for procyclical prices, but it equally applies to the case of countercyclical prices (where $p_\tau(c)$ is set in low-cost states and $p_\tau(0)$ in high-cost states). This resembles the rivalry of the kinked demand curve, where firms do not match price increases. This is because for deviations where a firm raises its price from $p_\tau(0)$ to a price above $p_\tau(0)$, the deviation is never matched in future low-cost states and it is also not matched in future high-cost states when there is price rigidity. A slight difference is that X is not matched in future low-cost states, if prices are procyclical and a firm lowered its price from $p_\tau(c)$ to X in period τ. However, the rationale for the strategy is the same: each firm expects to lose sales, if it set X in periods when its rivals are expected to set $p_\tau(0)$.[5]

[5] An alternative strategy is one where downward deviations from $p_t(c)$ to X are matched in *all* future periods, other things equal. Since this alternative and (3) are equivalent for rigid prices, the range of rigid prices sustainable and the characteristics of the best rigid price are the same. We

4 Price Rigidity under Price-Matching Punishments

4.1 A Theory of Price Rigidity

We wish to find the conditions for which the best collusive prices are rigid. Prices are procyclical when the level of high costs is so large that $p^N(c,n) > p^m$, so we initially consider the case where $c \in (0, \overline{c}]$ such that $p^N(\overline{c}, n) = p^m$. Similarly, price rigidity can only occur when (3) defines a Nash equilibrium in subgames where firms should set p, such that $p^N(n) \leq p \leq p^N(c,n)$, in low-cost states and $p^N(c,n)$ in high-cost states. Otherwise, there would be some punishment subgames where firms will not conform to (3) for histories when the lowest ever deviation price is between $p^N(n)$ and $p^N(c,n)$. Clearly a firm will not deviate from $p^N(c,n)$ in a period of high costs, so consider firm i's incentive to deviate from any such p in a period of low costs. Firm i's present discounted value of deviation profits if it sets the same or a lower price $p_i \in [p^N(n), p]$ in the initial period and then conforms to (3) thereafter is:

$$\Omega^z_{iL}(p_i, p, p^N(c,n)) \equiv \pi_i(p_i, p; n) + \tfrac{\delta}{1-\delta}[\tfrac{\lambda}{\omega}\pi(p^N(c,n); c, n) + (1 - \tfrac{\lambda}{\omega})\pi(p_i; n)]. \quad (4)$$

The first term on the right-hand side of (4) is the profit from the initial period. The second term is the expected discounted profits from future periods, given p_i will be matched in future low-cost states but $p^N(c,n)$ is set in future high-cost states. It follows from this that firm i will not deviate from p by setting p_i if $\Omega_L(p, p^N(c,n)) \geq \Omega^z_{iL}(p_i, p, p^N(c,n))$ for all $p_i \in [p^N(n), p]$.

Lemma 1 *For every $n \geq 2$, $\delta \in (0,1)$ and $1 - \theta - \lambda \geq 0$, there exists a unique $\widehat{c} \in (0, \overline{c})$ such that the one-shot Nash equilibrium price of the high-cost state, $p^N(c,n)$, and p, where $p^N(n) < p \leq p^N(c,n)$, are supportable in high- and low-cost states, respectively, if and only if $c \in (0, \widehat{c}]$.*

When the difference between the two cost levels is sufficiently small, $p^N(c,n)$ is close enough to $p^N(n)$ such that a firm will not deviate from $p^N(c,n)$ in a collusive subgame that starts with a period of low costs. This is because the deviation gain from setting p between $p^N(n)$ and $p^N(c,n)$ is outweighed by the long-term loss from matching p in future periods of low costs. In punishment subgames that start with a period of low costs, the condition for a firm to want to deviate from p is the same as the condition to deviate from $p^N(c,n)$, except that the price is lower. Since

focus on (3) because there is an asymmetry in this alternative strategy, since a firm is unable to increase the low-cost price by raising its price from $p_t(0)$ to X in a period of low costs, but it is able to by lowering its price from $p_t(c)$ to X in a period of high costs. An implication of this is that the parameter space where the best collusive prices are rigid under (3) is a strict subset of that under this alternative strategy, so it is robust to both strategies.

the standard properties of the underlying competition game imply that it is less profitable to deviate from a price close to $p^N(n)$ than a higher price, a firm will not deviate in any such punishment subgame, if it will not deviate from $p^N(c,n)$ in the collusive subgame. Consequently, the punishment is credible and harsh enough to support $p^N(c,n)$ in both cost states when the difference between the two cost levels is sufficiently small.

In the next subsection, we limit our attention to equilibria with the same price $p^c > p^N(c,n)$ in both cost states. This allows us to characterise the best rigid price that achieves the highest level of profit possible, given that the price does not vary over time. In the subsection after, we find the conditions for which firms can do no better than set the best rigid price in both cost states.

4.1.1 Best rigid price

Under the conditions of Lemma 1, a rigid price p^c is only supportable when firms will not deviate from any rigid price p, such that $p^N(c,n) \leq p \leq p^c$, in both high- and low-cost periods. Otherwise there is at least one collusive/punishment subgame where a firm will not conform to (3). Depending upon whether the cost state is high or low in the initial period, firm i's present discounted values of deviation profits if it sets the same or lower price $p \in [p^N(c,n), p^c]$ in the initial period and then conforms to (3) thereafter are:

$$\Omega^y_{iH}(p,p^c) \equiv \pi_i(p,p^c;c,n) + \tfrac{\delta}{1-\delta}\left[\tfrac{\theta}{\omega}\pi(p;n) + (1-\tfrac{\theta}{\omega})\pi(p;c,n)\right] \tag{5}$$
$$\Omega^y_{iL}(p,p^c) \equiv \pi_i(p,p^c;n) + \tfrac{\delta}{1-\delta}[\tfrac{\lambda}{\omega}\pi(p;c,n) + (1-\tfrac{\lambda}{\omega})\pi(p;n)], \tag{6}$$

respectively. The first terms on the right-hand side of (5) and (6) are the profits from the initial periods. This profit is lower in (5) than (6), because per-period profits are strictly decreasing in c_t. The second terms are the expected discounted profits from future periods, given p will be matched forever. When there is positive correlation, the second term is lower in (5) than (6), but they are equal under zero correlation.

The above implies that if $\Omega_H(p^c,p^c) \geq \Omega^y_{iH}(p,p^c)$ for all $p \in [p^N(c,n), p^c]$, then firm i will not deviate from p^c by setting any such p in high-cost states. Likewise, it will not deviate from p^c by setting any such p in low-cost states if $\Omega_L(p^c,p^c) \geq \Omega^y_{iL}(p,p^c)$ for all $p \in [p^N(c,n), p^c]$. We can write the slack in these constraints as:

$$\begin{aligned}
\xi^y_{iH}(p,p^c) \equiv \quad & -[\pi_i(p,p^c;c,n) - \pi(p^c;c,n)] \\
& +\tfrac{\delta}{1-\delta}\left(\tfrac{\theta}{\omega}[\pi(p^c;n) - \pi(p;n)] + (1-\tfrac{\theta}{\omega})[\pi(p^c;c,n) - \pi(p;c,n)]\right) \\
\xi^y_{iL}(p,p^c) \equiv \quad & -[\pi_i(p,p^c;n) - \pi(p^c;n)] \\
& +\tfrac{\delta}{1-\delta}\left(\tfrac{\lambda}{\omega}[\pi(p^c;c,n) - \pi(p;c,n)] + (1-\tfrac{\lambda}{\omega})[\pi(p^c;n) - \pi(p;n)]\right),
\end{aligned}$$

respectively. Consider the difference between the two. After some rearranging and cancellation, we find:

$$\xi^y_{iH}(p,p^c) - \xi^y_{iL}(p,p^c) = c[q_i(p,p^c;n) - \tfrac{1}{n}q(p^c)] + \tfrac{\delta}{\omega}(1-\theta-\lambda)\tfrac{c}{n}[q(p) - q(p^c)].$$

The first term on the right-hand side of the above is the difference between the deviation gains, and the second term is the difference between the long-term losses due to the price-matching punishment. It follows from the assumptions on demand and $1-\theta-\lambda \geq 0$ that $\xi^y_{iH}(p,p^c) > \xi^y_{iL}(p,p^c)$ for any $p \in [p^N(c,n), p^c)$, so firm i's incentive to deviate is greatest in a period of low costs under zero and positive correlation. This is because the deviation gain is largest and the long-term loss is smallest when a firm deviates in a period of low costs. The latter effect, unlike the former, is distinct from a similar model where firms follow optimal punishments.[6] The reason for it in our model is that a matched price causes a smaller loss in per-period profits when unit costs are lower, so the long-term loss is smaller when more low-cost states are expected in the future, which is the case in a period of low costs under positive correlation. Thus, if $\xi^y_{iL}(p,p^c) \geq 0$ for all $p \in [p^N(c,n), p^c]$, then firm i will not deviate from p^c by setting any such p in low- and high-cost states.

Lemma 2 *For every $n \geq 2$, $\delta \in (0,1)$ and $1-\theta-\lambda \geq 0$, there exists a unique best rigid price, $p^y_L(c,n,\delta,\lambda,\theta)$, that is supportable in both cost states if and only if $c \in (0,\widehat{c}]$, where $p^N(c,n) < p^y_L(c,n,\delta,\lambda,\theta) < p^m(c)$. Any rigid price p^c such that $p^N(c,n) \leq p^c \leq p^y_L(c,n,\delta,\lambda,\theta)$ is also supportable.*

When the difference between the two cost levels is sufficiently small, there exists some rigid price p^c that is above yet close enough to $p^N(c,n)$, such that a firm will not deviate from p^c in a collusive subgame that starts with a period of low costs. This is because the deviation gain from setting any p between $p^N(c,n)$ and p^c in a low-cost state is outweighed by the long-term loss from matching p in all future periods.[7] In punishment subgames that start with a period of low costs, the condition for a firm to want to deviate from p is the same as the condition to deviate from p^c, except the price is lower. Since the standard properties of the underlying competition game imply that it is less profitable to deviate from a price close to $p^N(c,n)$ than a higher price, a firm will not deviate in any such punishment subgame, if it will not deviate from p^c in the collusive subgame. Furthermore, this and the analysis above implies

[6] This is because under optimal punishments the long-term loss is larger when the future collusive profits are greater, since profits are zero in the punishment phase. Consequently, the long-term loss would be largest when a firm deviates in a period of low costs under positive correlation.

[7] Furthermore, it is never profitable to deviate from a rigid price above $p^N(c,n)$ to a price below $p^N(c,n)$ in a period of low-costs for all $c \in (0,\widehat{c}]$.

that a firm will also not deviate from p^c or from any rigid price between $p^N(c, n)$ and p^c in subgames that start with a period of high costs. Consequently, the punishment is credible and harsh enough to support p^c in both cost states, when p^c is sufficiently close to $p^N(c, n)$ and when the difference between the two cost levels is sufficiently small.

The best rigid price has the unique property that a small deviation from it in a period of low costs that is matched in all future periods balances the first-order increase in the deviation profit with the first-order decrease in future profits (i.e. the argument maximising (6) is p^c). This implies that the (unconstrained) optimal 'deviation' price from the best rigid price in a period of low costs is equal to the best rigid price. At any rigid price above this level, there is an incentive to deviate in a period of low costs (i.e. for any $p^c > p^y_L(c, n, \delta, \lambda, \theta)$, then $\xi^y_{iL}(p, p^c) < 0$ for some $p < p^c$). Since it is less profitable to deviate in a period of high costs than one of low costs, the (constrained) optimal 'deviation' price from the best rigid price in a period of high costs also equals the best rigid price.[8] The best rigid price is equivalent to the best collusive price analysed by Lu and Wright (2010) as $c \to 0$, and it is strictly increasing in the level of high costs. The reason is that a given rigid price is easier to support in a period of low costs when the high-cost level is closer to $\widehat{c}$ than when it is close to zero, because the long-term loss from a small deviation increases with c. In contrast to Lu and Wright (2010), the monopoly price of the low-cost state may be supportable. This is because a small deviation from p^m in a period of low costs can balance the first-order increase in the deviation profit with the first-order decrease in future profits of high-cost states.[9]

4.1.2 Best collusive prices and price rigidity

The best collusive prices are rigid if a firm will deviate from any procyclical or countercyclical prices that would be more profitable than setting the best rigid price in both cost states. To see that such countercyclical prices are not supportable, suppose that the initial collusive prices are $p(0)$ and $p(c)$ for low- and high-cost states, respectively, where $p(0)$ is above $p(c)$. A necessary (but not sufficient) condition for such prices to be more profitable than setting the best rigid price in both cost states is that $p(0)$ must be strictly greater than the best rigid price. Consider firm i's incentive to deviate in a period of low costs. Firm i's present discounted value of

[8] A firm would want to deviate from the best rigid price to a higher price in a period of high costs, if firms matched such a deviation price in all future periods. However, this would not be a credible strategy even if such deviations were matched, because a firm would want to deviate from such a price in punishment subgames that start with a period of low costs when the price should be matched.

[9] There is no first-order decrease in the profits of future low-cost states, because such profits are flat at p^m. It is this feature that determines that p^m is not supportable by price-matching punishments, when all future periods are expected to have low costs.

deviation profits if it sets the same or a lower price $p \in [p(c), p(0)]$ in the initial period, then conforms to (3) thereafter is:

$$\Omega^x_{iL}(p, p(0), p(c)) \equiv \pi_i(p, p(0); n) + \tfrac{\delta}{1-\delta}[\tfrac{\lambda}{\omega}\pi(p(c); c, n) + (1 - \tfrac{\lambda}{\omega})\pi(p; n)].$$

Thus, a firm will not deviate from $p(0)$ by setting p if $\Omega_L(p(0), p(c)) \geq \Omega^x_{iL}(p, p(0), p(c))$ for all $p \in [p(c), p(0)]$, where the slack in this constraint is:

$$\xi^x_{iL}(p, p(0)) \equiv -[\pi_i(p, p(0); n) - \pi(p(0); n)] + \tfrac{\delta}{1-\delta}(1 - \tfrac{\lambda}{\omega})[\pi(p(0); n) - \pi(p; n))].$$

Notice that $\xi^x_{iL}(p, p(0))$ does not depend on $p(c)$, because the punishment results in firms still setting $p(c)$ in high-cost states, and as a consequence it is the same as $\xi^y_{iL}(p, p(0))$, except that there is no long-term loss in profits of future high-cost states. This implies that since it is profitable for a firm to deviate from a rigid price above the best rigid price in a period of low costs, then an otherwise identical deviation is even more profitable when prices are countercyclical (i.e. for any $p(0) > p^y_L(c, n, \delta, \lambda, \theta)$, $\xi^x_{iL}(p, p(0)) < \xi^y_{iL}(p, p(0)) < 0$ for some $p < p(0)$). Therefore, the best collusive prices cannot be countercyclical.

Now consider whether the best collusive prices can be procyclical, where $p(c)$ is above $p(0)$. First consider how this affects the best collusive price of the low-cost state, denoted $p^*(0) \in [p^N(c, n), p^m]$. Notice that firm i's present discounted value of deviation profits is equivalent to (6), if it deviates from $p(0)$ to some $p \in [p^N(c, n), p(0)]$ in a period of low costs. Thus, such a deviation is not profitable if $\Omega_L(p(0), p(c)) \geq \Omega^y_{iL}(p, p(0))$ for all $p \in [p^N(c, n), p(0)]$. Since $\Omega_L(p(0), p(c))$ increases with $p(c)$ but $\Omega^y_{iL}(p, p(0))$ is independent of $p(c)$, then there is still no incentive to deviate from the best rigid price when prices are procyclical. However, a price above the best rigid price is not supportable in low-cost states, because the punishment for such a price is not credible. This is because for any such price there are some punishment subgames where firms should match a price above the best rigid price in all future periods, but each firm has an incentive to deviate from it in such punishment subgames that start with a period of low costs. Consequently, the best rigid price is the highest price that is supportable in low-cost states when prices are rigid or procyclical. However, it is more profitable to set the monopoly price when the best rigid price is above it, so $p^*(0)$ is the lower of the best rigid price and the monopoly price of the low-cost state for all $c \in (0, \widehat{c}]$.

Finally, to find whether procyclical prices are supportable, consider firm i's incentive to deviate from $p(c)$ above $p^*(0)$ in a period of high costs, while holding the collusive price of low-cost states fixed at $p^*(0)$. Firm i's present discounted value of deviation profits if it sets the same or a lower price $p \in [p^*(0), p(c)]$ in the initial period, then conforms to (3) thereafter is:

$$\Omega^x_{iH}(p, p(c), p^*(0)) \equiv \pi_i(p, p(c); c, n) + \tfrac{\delta}{1-\delta}\left[\tfrac{\theta}{\omega}\pi(p^*(0); n) + (1 - \tfrac{\theta}{\omega})\pi(p; c, n)\right]. \tag{7}$$

The first term on the right-hand side of (7) is the profit from the initial period. The second term represents the expected discounted profits from future periods, given p will be matched in future high-cost states but $p^*(0)$ is set in future low-cost states. It follows from this that firm i will not deviate from $p(c)$ by setting p if $\Omega_H(p(c), p^*(0)) \geq \Omega^x_{iH}(p, p(c), p^*(0))$ for all $p \in [p^*(0), p(c)]$. The slack in this constraint is:

$$\xi^x_{iH}(p, p(c)) \equiv -[\pi_i(p, p(c); c, n) - \pi(p(c); c, n)] + \tfrac{\delta}{1-\delta}(1 - \tfrac{\theta}{\omega})[\pi(p(c); c, n) - \pi(p; c, n))],$$

which does not depend on $p^*(0)$ because the punishment results in firms still setting $p^*(0)$ in low-cost states.

To see that the best collusive prices can be rigid under price-matching punishments, suppose firm i deviates from $p(c)$ to $p = p^*(0)$. Given the punishment is limited to future high-cost states when such a deviation is matched, there is no long-term loss for such a deviation when all future periods are expected to have low costs (i.e. $\theta = 1$ and $\lambda = 0$). Thus, each firm will have an incentive to deviate from any $p(c)$ above $p^*(0)$, and the best collusive prices are rigid at $p^*(0)$ for all $c \in (0, \widehat{c}]$. Proposition 1 shows that the best collusive prices can still be rigid when high costs persist into the future.

Proposition 1 *For every $n \geq 2$, $\delta \in (0, 1)$ and $1 - \theta - \lambda \geq 0$, there exists a unique $c^* \in (0, \widehat{c})$ such that the best rigid price, $p^y_L(c, n, \delta, \lambda, \theta)$, is the best collusive price in both cost states if and only if $c \in (0, c^*]$, where $p^N(c, n) < p^y_L(c, n, \delta, \lambda, \theta) < p^m$.*

When the difference between the two cost levels is below the critical threshold, a firm will want to deviate from a price above yet very close to the best rigid price in high-cost states. To see this point, consider a deviation from such a $p(c)$ to a price equal to or just above $p^*(0)$ in a period of high costs.[10] Notice that $\xi^x_{iH}(p, p(c))$ is the same as $\xi^y_{iL}(p, p(c))$ as $c \to 0$, except that there is no long-term loss in profits of future low-cost states. This implies that since it is profitable to deviate from a rigid price above the best rigid price in a period of low costs, then an otherwise identical deviation is even more profitable in a period of high costs as $c \to 0$ when prices are procyclical (i.e. for any $p(c) > p^y_L(c, n, \delta, \lambda, \theta)$, $\xi^x_{iH}(p, p(c)) < \xi^y_{iL}(p, p(c)) < 0$ for some $p < p(c)$ as $c \to 0$). As the level of high costs increases towards c^*, the

[10] It is never profitable to deviate from any procyclical $p(c)$ by setting a price below $p^*(0)$ in a period of high costs for all $c \in (0, \widehat{c}]$.

profitability of such a deviation falls.[11] However, a price above yet very close to the best rigid price is not supportable until the level of high costs exceeds c^*.

Prices above yet very close to the best rigid price may not be supportable in high-cost states, even though the best rigid price is always supportable. This is because there is a discontinuity in the incentives to collude at $p^*(0)$, which arises due to the fact that a deviation from $p(c)$ to a price that is equal to or just above $p^*(0)$ only lowers prices in future high-cost states. Consequently, such a deviation from a price above yet very close to the rigid price in a period of high costs generates a much smaller long-term loss than an otherwise identical deviation from the rigid price, where price matching reduces the prices of all future periods. Yet, the deviation gains are effectively the same for such deviations. As a result, for some positive values of c, it can be the case that a deviation from a price above yet very close to the best rigid price is strictly profitable in a period of high costs, even though a deviation from the best rigid price is strictly unprofitable.

When the difference between the two cost levels is so large that the best collusive price of the low-cost state is p^m, the best collusive prices are procyclical. This is because p^m is the best collusive price of low-cost states, if the first-order increase in the deviation profit from a small deviation from p^m in a period of low costs is outweighed by the first-order decrease in profits of future high-cost states (there is no first-order decrease in profits of future low-cost states, since such profits are flat at p^m). In comparison to this, a small deviation from a price above yet very close to p^m in a period of high costs leads to a smaller first-order increase in the deviation profits and a (weakly) larger first-order decrease in profits of future high-cost states. This implies that the best collusive prices will be procyclical, because a firm will not deviate from a price above yet very close to p^m in a period of high costs, if a firm will not deviate from p^m in a period of low costs.

4.2 Price rigidity and the expected duration of a high-cost phase

The best collusive prices are rigid when the difference between the two costs levels is below the critical threshold. Proposition 2 now shows that this critical threshold depends upon the extent to which a high-cost state is likely to persist into the future. To see this point, define a high-cost phase as a sequence of high-cost states that begins in a period where costs change from the low- to the high-cost state and ends the period before they change back. The expected duration of a high-cost phase is $\Sigma_{t=1}^{\infty} t\theta(1-\theta)^{t-1} = 1/\theta$, which implies that the lower the probability that costs

[11] This is because the deviation gain strictly decreases and long-term loss strictly increases as the level of high costs rises, due to the fact that high-cost states are less profitable than before; and it is despite of the fact that the deviation occurs from a slightly higher price, since the best rigid price strictly increases with the level of high costs.

will change from the high- to the low-cost state in the following period, the longer a high-cost phase is likely to last. Similarly, we can define a low-cost phase with an expected duration of $1/\lambda$.

Proposition 2 *For every $n \geq 2$, $\delta \in (0,1)$ and $1-\theta-\ \lambda \geq 0$, the critical difference between the two cost levels, c^*, is strictly decreasing in the expected duration of a high-cost phase.*

As the expected duration of a high-cost phase increases, other things equal, it is easier to support a price above yet very close to the best rigid price in high-cost states. This comes about from two opposing effects. First, a direct effect reduces the profitability of a small deviation from such a price. Second, an indirect effect raises the profitability of such a deviation, because the deviation occurs from a slightly higher price than before, since the best rigid price strictly increases with the expected duration of a high-cost phase. Both effects are caused by the fact that the punishment strategy leads to larger long-term losses when future periods are likely to consist of more high-cost states. The direct effect dominates the indirect effect, which implies that, for a given difference between the two cost levels, procyclical prices are easier to support as the expected duration of a high-cost phase increases, so the critical threshold falls. When there is zero correlation (so that the expected duration of a low-cost phase decreases at the same rate as the expected duration of a high-cost phase increases) both the direct and indirect effects are larger than under positive correlation, but the direct effect still dominates.

We have already seen that the best collusive prices are rigid when a high-cost phase is expected to last only one period and the following low-cost phase lasts forever, provided the one-shot Nash equilibrium price of the high-cost state is not above the best rigid price (i.e. $c^* \to \widehat{c}$ as $\theta \to 1$ and $\lambda \to 0$). On the other hand, when a high-cost phase is expected to last forever, the best collusive prices are procyclical, regardless of the expected duration of a low-cost phase (i.e. $c^* \to 0$ as $\theta \to 0$ for all $0 < \lambda < 1$). This is because for such conditions it is more profitable to deviate from a rigid price in a period of low costs than to deviate from a price above yet very close to the rigid price in a period of high costs (i.e. as $\theta \to 0$, $\xi^x_{iH}(p, p^c) > \xi^y_{iL}(p, p^c)$ for all $p < p^c$). Therefore, provided a firm will not deviate from the rigid price in low-cost states, it will not deviate from a price above yet very close to the rigid price in high-cost states.

4.3 Price rigidity and profits over the fluctuations

The preceding analysis showed that the best rigid price strictly increases with the level of high costs. Proposition 3 shows that this implies that there are also general properties for the resultant collusive profits when the best collusive prices are rigid.

Proposition 3 *For any $c \in (0, c^*)$, per-period profits when costs are high (low) are strictly decreasing (increasing) in the level of high costs, c, when firms set the best rigid price. The present discounted values of collusive profits are strictly decreasing in c when firms set the best rigid price in both states, whether the initial period has high or low costs.*

Clearly, per-period profits are greater in a low-cost state than in a high-cost state, when firms set the best rigid price in both states. As the level of high costs rises towards c^*, the difference in such profits becomes larger for two reasons. First, per-period profits in low-cost states are larger than before, since the best rigid price rises with the level of high cost but it remains below p^m. Second, per-period profits in high-cost states are smaller than before, because the best rigid price rises with the level of high costs at a rate that is less than one-to-one. In contrast, the present discounted values of collusive profits are equal when the high-cost level is equal to the low-cost level, but such profits fall as the high-cost level rises towards c^*, regardless of whether the initial period has low or high costs.

5 An Example

We complement the above analysis by assuming that demand is derived from the constant elasticity of substitution version of Spence-Dixit-Stiglitz preferences (Spence, 1976; and Dixit and Stiglitz, 1977). We do this for three reasons. First, we want to show that the best collusive prices are rigid for reasonably large differences in the two cost levels. Second, we want to investigate the effect of the degree of product differentiation on such price rigidity. To the author's knowledge, there is no other model of collusive price rigidity that considers this, since both Athey *et al* (2004) and Hanazono and Yang (2007) analyse homogeneous products. Third, we want to investigate the effect of the number of firms in the market on such price rigidity, and this ultimately depends upon the degree of product differentiation. We use Spence-Dixit-Stiglitz preferences, because it falls into the class of our general model and it generates results with simpler intuition than alternatives, since it isolates the competitive effects of product differentiation as there is no market expansion effect.[12]

[12] Spence-Dixit-Stiglitz preferences is one example of differentiated demand analysed by Kühn and Rimler (2007) for collusion models under Nash reversion and optimal punishment strategies. It has not been analysed for collusion under price-matching punishments before. Similar results as those presented here can be derived using the standard Bertrand competition model with linear demands.

A representative consumer's utility function is $U(\mathbf{x}) = \frac{n^{1-\kappa}}{1-\kappa}\left(\frac{1}{n}\Sigma x_i^{1-\phi\kappa}\right)^{\frac{1-\kappa}{1-\phi\kappa}} + m$, where $\mathbf{x}$ is the vector of consumption of the n products, m is expenditure on other goods, $\phi \in (0,1)$ measures the degree of product differentiation, where products are less differentiated the closer ϕ is to zero, and $\kappa \in (0,1)$ is a parameter. It follows from this utility function that the direct demand function for firm i is:

$$q_i(p_i, \mathbf{p}_{-i}, \phi, n) = \frac{1}{n} p_i^{-\frac{1}{\kappa}} \left[\frac{n}{\Sigma_j \left(p_i/p_j\right)^{\frac{1-\phi\kappa}{\phi\kappa}}}\right]^{\frac{1-\phi}{1-\phi\kappa}}.$$

This implies that total demand at equal prices is independent of both the degree of product differentiation and the number of firms, i.e. $q(p) = p^{-1/\kappa}$. It is straightforward to show that the monopoly price is $p^m(c_t) = c_t/(1-\kappa)$ and that the one-shot Nash equilibrium price is $p^N(c_t, n, \phi) = c_t/\left[1 - \kappa/\left(1 + \frac{1-\phi}{\phi}\frac{n-1}{n}\right)\right]$. To ensure that the monopoly price is above the one-shot Nash equilibrium price for both cost states, we assume that the level of low costs is $\underline{c} \in (0, c)$ and we normalise the high-cost level relative to the low-cost level later.

For this example, the best rigid price is:

$$p_L^y(c, n, \delta, \lambda, \theta, \phi) = \frac{\underline{c}}{1-\kappa/\left(1+(1-\delta)\frac{1-\phi}{\phi}\frac{n-1}{n}\right)} + \frac{\delta\lambda(c-\underline{c})}{\omega\left(1-\kappa+(1-\delta)\frac{1-\phi}{\phi}\frac{n-1}{n}\right)}, \qquad (8)$$

which applies for $\underline{c} < c \leq \widehat{c}$.[13] The first term on the right hand-side of (8) is equivalent to the best collusive price analysed in Lu and Wright (2010) and the second term captures the effect of varying costs. This price equals $\underline{c}$ when products are homogeneous, and it is everywhere strictly increasing in the degree of product differentiation, ϕ. It is above the one-shot Nash equilibrium price of the low-cost state for all $0 < \phi \leq 1$, and it is above the monopoly price of the low-cost state when products are not substitutable. It is everywhere strictly decreasing in the number of firms, n, but it is always above the one-shot Nash equilibrium price of the low-cost state, even when there is a large number of firms in the market.[14]

The price in (8) defines the best collusive price in both cost states when the difference between the two cost states is below the critical threshold. It follows from Proposition 1 that $c^* = \frac{\underline{c}}{1-K} \in (\underline{c}, \widehat{c})$ where:

$$K \equiv \frac{\kappa\frac{\delta\theta}{\omega}(1-\delta)\frac{1-\phi}{\phi}\frac{n-1}{n}}{\left((1-\kappa)(1-\frac{\delta\theta}{\omega})+(1-\delta)\frac{1-\phi}{\phi}\frac{n-1}{n}\right)\left(1-\frac{\delta\lambda}{\omega}+(1-\delta)\frac{1-\phi}{\phi}\frac{n-1}{n}\right)} \in (0,1).$$

[13] Following Lemma 1, $\widehat{c} = \left(1 - \kappa/\left[1 + \frac{1-\phi}{\phi}\frac{n-1}{n}\right]\right)\frac{\underline{c}}{1-\kappa(1-\frac{\delta\lambda}{\omega})/\left[(1-\delta)\left(1+\frac{1-\phi}{\phi}\frac{n-1}{n}\right)+\delta(1-\frac{\lambda}{\omega})\right]}$.

[14] This is because the underlying competition game is one of true monopolistic competition, where the price is above marginal cost even as $n \to \infty$.

To illustrate the properties of c^*, Figure 2 plots $\Delta c^* \equiv \frac{c^*-\underline{c}}{\underline{c}} = \frac{K}{1-K}$ as a function of ϕ for three levels of n. This has two interpretations. First, Δc^* is the critical proportional difference between the two cost states, where the best collusive prices are rigid for any proportional difference that does not exceed this level. Second, it measures proportional difference between the monopoly price when the cost state is c^* and the monopoly price of the low-cost state (i.e. $[p^m(c^*) - p^m(\underline{c})]/p^m(\underline{c}) = \Delta c^*$). Parameter values are chosen such that the monopoly price of the low-cost state is equal to unity, future costs are independent of the current level, and each cost state is equally likely in any future period.

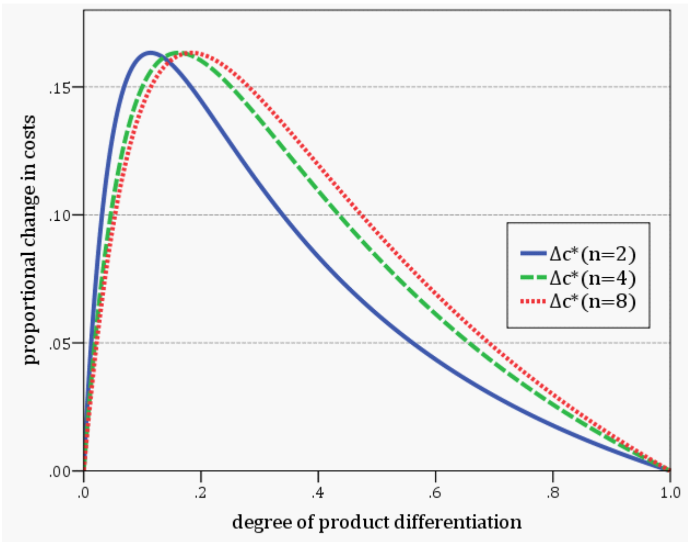

Figure 2: ($\underline{c} = 0.5$, $\lambda = 1 - \theta = 0.5$, $\kappa = 0.5$, $\delta = 0.9$)

The Figure shows that there is a non-monotonic relationship between Δc^* and the degree of product differentiation. For certain intermediate degrees of differentiation, the best collusive prices are rigid when the monopoly price of the high-cost state is 16% higher than the monopoly price of the low-cost state, and such price rigidity can occur for even larger differences between the two cost levels when the expected duration of a high-cost phase is shorter.[15] The best collusive prices are not rigid when products are homogeneous or when each product has no close substitutes. This is because the punishment strategy does not support collusive prices when the products are homogeneous, since an infinitesimally small deviation from the collusive price captures the whole market and the price-matching strategy leads to virtually no long-term loss. Consequently, firms set the one-shot Nash equilibrium price in each cost state. In contrast, firms can set the monopoly price in each cost state

[15] For example, if $\lambda = 0.01$ and $\theta = 0.99$, the shape of Δc^* is similar to that of Figure 2, except that the best collusive prices can be rigid when the monopoly price of the high-cost state is 40% higher than the monopoly price of the low-cost state.

when they are local monopolies, with no close substitutes.

Finally, Figure 2 shows that Δc^* is larger for concentrated markets, with few firms, than for less concentrated markets, with a greater number of firms, when the degree of product differentiation is sufficiently low; the opposite relationship may exist otherwise. This is not inconsistent with empirical research that shows that prices are less responsive to changes in market conditions in some cases when the markets are more concentrated (see Dixon, 1983; Carlton, 1986; Bedrossian and Moschos, 1988; Geroski, 1992; Weiss, 1995) but the opposite relationship exists in others (see Domberger, 1979; and Kardasz and Stollery, 1988). The reason behind this result in our model is that it can either be more or less profitable to deviate from a price above yet very close to the best rigid price in a period of high costs, as the number of firms increases. This is because there are two opposing effects. First, a direct effect raises the profitability of a small deviation from such a price. Second, an indirect effect reduces the profitability of such a deviation, because the deviation occurs from a slightly lower price than before, since the best rigid price strictly decreases with the number of firms in the market. Both effects are caused by the fact that the deviation gains are larger and the punishment strategy leads to smaller long-term losses when there are a greater number of firms in the market (from Assumption 4 and symmetric demand, respectively). In our general framework, it is not possible to sign the overall effect. In our example, however, the indirect effect dominates the direct effect when the degree of product differentiation is sufficiently low. This implies that, for a given difference between the two cost levels, procyclical prices are easier to support as the number of firms in the market increases, so the critical threshold falls.

6 Concluding remarks

In this paper, we analysed an infinitely repeated game where unit costs alternate stochastically between low and high states and where firms employ a price-matching punishment strategy. This provided game theoretic support for the results of the kinked demand curve, because we showed that the best collusive prices can be rigid over time when the difference between the two costs levels is below some critical threshold. Moreover, we showed that this critical threshold is closer to zero as high-cost states are likely to persist for longer into the future, and it equals zero when the high-cost state is expected to last permanently. When the best collusive prices are rigid over time, the best rigid price is always between the one-shot Nash equilibrium price of the high-cost state and the monopoly price of the low-cost state, and it monotonically increases with the level of high costs at a rate that is less than one-to-one. As a result, an increase in the level of high costs raises the resultant per-

period profits of a low-cost state, but it reduces such per-period profits of a high-cost state. Nevertheless, the corresponding present discounted values of collusive profits are decreasing in the level of high costs, whether the initial period has high or low costs. Finally, when demand is derived from the constant elasticity of substitution version of Spence-Dixit-Stiglitz preferences, we found that the best collusive prices are rigid for the largest difference between the two cost levels when products are differentiated by an intermediate degree; and that the best collusive prices are rigid for a larger difference between the two cost levels in a concentrated market than in a less concentrated market, when the degree of product differentiation is sufficiently low.

Throughout the paper, we have considered only two cost states, but periods of price rigidity are not restricted to this special case. For example, when a medium-cost state is added and there is zero correlation, the best rigid price is unaffected by the introduction of the third state, if the expected level of future costs is unchanged compared to the two-state model. Moreover, holding the expected level of future costs constant also ensures that future high-cost states are less likely in this three-state model than the two-state model. As a result, there is a greater incentive to deviate from a procyclical price in a period of high costs in this three-state model than the two-state model, when such a deviation only leads to a long-term loss in profits of future high-cost states. Thus, when the difference between the low- and the high-cost states is such that the best collusive prices are rigid in the two-state model, the best collusive prices in this three-state model will either be rigid for every cost state or partially rigid (where the best collusive prices are rigid in medium- and high-cost states, at a price above the best collusive price of low-cost states). Applying this logic to more than three cost states suggests that it is even more difficult to support procyclical prices in the highest-cost state than in the two-state model, so periods of price rigidity can occur for any number of states.

Finally, an important avenue for future research is to investigate whether there exists any circumstances where firms will choose to support collusive prices through a weaker punishment, such as price matching, rather than harsher punishment strategies, such as Nash reversion or optimal punishment strategies. Such a theoretical justification for price matching may provide a better indication of the industry characteristics where price rigidity is likely to prevail. It would also resolve the tension between the informal reasoning behind the belief that firms will employ the harshest credible punishment with the evidence that, at least in some situations, tacitly colluding firms (and even some cartels) do not employ such punishments.

References

[1] Abreu, D (1986) "Extremal Equilibria of Oligopolistic Supergames," *Journal of Economic Theory*, 39, 191-225

[2] Abreu, D (1988) "On the Theory of Infinitely Repeated Games with Discounting," *Econometrica*, 56, 383-396

[3] Athey, S, Bagwell, K and Sanchirico, C (2004) "Collusion and Price Rigidity," *Review of Economic Studies*, 71(2), 317-349

[4] Bagwell, K and Staiger, R (1997) "Collusion over the Business Cycle," *RAND Journal of Economics*, 28(1), 82-106

[5] Bedrossian, A and Moschos, D (1988) "Industrial Structure, Concentration and the Speed of Price Adjustments," *Journal of Industrial Economics*, 36, 459-475

[6] Bhaskar, V (1988) "The Kinked Demand Curve: A Game Theoretic Approach," *International Journal of Industrial Organization*, 6, 373-384

[7] Carlton, D (1986) "The Rigidity of Prices," *American Economic Review*, 76(4), 637-658

[8] Carlton, D (1989) "The Theory and the Facts of How Markets Clear: Is Industrial Organization Valuable for Understanding Macroeconomics?" in Schmalensee, R and Willig, R (eds) *Handbook of Industrial Organization*, Oxford: North-Holland

[9] Dixit, A and Stiglitz, J (1977) "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67, 63-78

[10] Dixon, R (1983) "Industry Structure and the Speed of Price Adjustments," *Journal of Industrial Economics*, 31, 25-37

[11] Domberger, S (1979) "Price Adjustment and Market Structure," *Economic Journal*, 89, 96-108

[12] Friedman, J (1971) "A Non-Cooperative Equilibrium for Supergames," *Review of Economic Studies*, 28, 1-12

[13] Fudenberg, D and Tirole, J (1991) *Game Theory*, Cambridge: MIT Press

[14] Genesove, D and Mullin, M. (2001) "Rules, Communication, and Collusion: Narrative Evidence from the Sugar Institute Case," *American Economic Review*, 91, 379-398

[15] Geroski, P (1992) "Price Dynamics in UK Manufacturing: A Microeconomic View," *Economica*, 59, 403-19

[16] Green, E and Porter, R (1984) "Non-cooperative Collusion under Imperfect Price Information," *Econometrica*, 52, 87-100

[17] Hall, R and Hitch, C (1939) "Price Theory and Business Behaviour," *Oxford Economic Papers*, 2, 12-45

[18] Haltiwanger, J and Harrington, J (1991) "The Impact of Cyclical Demand Movements on Collusive Behavior," *RAND Journal of Economics*, 22(1), 89-106

[19] Hanazono, M and Yang, H (2007) "Collusion, Fluctuating Demand, and Price Rigidity," *International Economic Review*, 48(2) 483-515

[20] Kalai, E and Satterthwaite, M (1994) "The Kinked Demand Curve, Facilitating Practices, and Oligopolistic Coordination," in Gilles, R and Ruys, P (eds) *Imperfections and Behavior in Economic Organizations*, Kluwer Academic Publishers

[21] Kardasz, S and Stollery, K (1988) "Market Structure and Price Adjustment in Canadian Manufacturing Industries," *Journal of Economics and Business*, 40, 335-342

[22] Kühn K-U and Rimler, M (2007) "The Comparative Statics of Collusion Models," mimeo, University of Michigan

[23] Levenstein, M. (1997) "Price Wars and the Stability of Collusion: A Study of the Pre-World War I Bromine Industry," *Journal of Industrial Economics*, 45, 117-136

[24] Lu, Y and Wright, J (2010) "Tacit Collusion with Price-Matching Punishments," *International Journal of Industrial Organization*, 28, 298-306

[25] Maskin, E and Tirole, J (1988) "A Theory of Dynamic Oligopoly II: Price Competition, Kinked Demand Curves and Edgeworth Cycles," *Econometrica*, 56(3), 571-599

[26] Rotemberg, J and Saloner, G (1986) "A Supergame-Theoretic Model of Price Wars during Booms," *American Economic Review*, 76, 390-407

[27] Slade, M (1987) "Interfirm Rivalry in a Repeated Game: An Empirical Test of Tacit Collusion," *Journal of Industrial Economics*, 35, 499-516

[28] Slade, M (1989) "Price Wars in Price Setting Supergames," *Economica*, 223, 295-310

[29] Slade, M (1990) "Strategic Pricing Models and Interpretation of Price War Data," *European Economic Review*, 34, 524-537

[30] Slade, M (1992) "Vancouver's Gasoline Price Wars: An Empirical Exercise in Uncovering Supergame Strategies," *Review of Economic Studies*, 59, 257-276

[31] Spence, M (1976) "Product Differentiation and Welfare," *American Economic Review*, 66, 407-414

[32] Sweezy, P (1939) "Demand Under Conditions of Oligopoly," *Journal of Political Economy*, 47, 568-573

[33] Tirole, J (1988) *Theory of Industrial Organization*, Cambridge: MIT Press

[34] Vives, X (2001) *Oligopoly Pricing: Old Ideas and New Tools*, Cambridge: MIT Press

[35] Weiss, C (1995) "Determinants of Price Flexibility in Oligopolistic Markets: Evidence from Austrian Manufacturing," *Journal of Economics & Business*, 47, 423-439

A Proofs

Proof of Lemma 1. Suppose the collusive price is $p^N(c,n)$ in both cost states. To prove subgame perfection, it suffices to check that there is no history that leads to a subgame in which a one-stage deviation is profitable. For every history, the lowest ever deviation price below $p^N(c,n)$ at some period τ is $\min\{\mathbf{p}_\tau^d, p^N(c,n)\}$. If $\min\{\mathbf{p}_\tau^d\} \leq p^N(n)$, then (3) trivially defines a Nash equilibrium in the subsequent punishment subgames, whether they start with a period of high or low costs. Otherwise, the subsequent punishment subgames are identical to a history in which firms had set $p^N(c,n)$ in high-cost states and $\min\{\mathbf{p}_\tau^d, p^N(c,n)\} \in (p^N(n), p^N(c,n)]$ in low-cost states. Clearly, (3) defines a Nash equilibrium in the subgames that start with a period of high costs. Thus, we must find the conditions for which a firm will not deviate from $p^N(c,n)$ or from any price between $p^N(n)$ and $p^N(c,n)$ in subgames that start with a period of low costs.

Suppose we consider some collusive price $p \in (p^N(n), p^N(c,n)]$ that is set in low-cost states, where $p^N(c,n)$ is set in high-cost states. Consider firm i setting the same or a lower price $p_i \in [p^N(n), p]$ in a period of low costs. From (4) define:

$$\Delta\Omega_L^z(p) \equiv \left[\tfrac{\partial \pi_i(p_i,p;n)}{\partial p_i} + \tfrac{\delta}{1-\delta}(1-\tfrac{\lambda}{\omega})\tfrac{d\pi(p_i;n)}{dp_i}\right]_{p_i=p}.$$

Firm i will not deviate from p if $\Delta\Omega_L^z(p) \geq 0$, otherwise $\Omega_L(p, p^N(c,n)) < \Omega_{iL}^z(p_i, p, p^N(c,n))$ for some $p_i < p$. We wish to show that if $\Delta\Omega_L^z(p^N(c,n)) \geq 0$, then $\Delta\Omega_L^z(p) \geq 0$ $\forall$ $p \in (p^N(n), p^N(c,n))$. Differentiating $\Delta\Omega_L^z(p)$ with respect to p yields:

$$\frac{d(\Delta\Omega_L^z(p))}{dp} = \left[\frac{\partial^2\pi_i(p_i,p;n)}{\partial p_i^2} + \sum_{j\neq i}\frac{\partial^2\pi_i(p_i,p;n)}{\partial p_i \partial p_j} + \frac{\delta}{1-\delta}(1-\frac{\lambda}{\omega})\frac{d^2\pi(p_i;n)}{dp_i^2}\right]_{p_i=p_j=p}.$$

It follows from (2) and Assumption 5 that $\frac{d(\Delta\Omega_L^z(p))}{dp} < 0$. Hence, if $\Delta\Omega_L^z(p^N(c,n)) \geq 0$, then $\Delta\Omega_L^z(p) > 0$ $\forall$ $p \in (p^N(n), p^N(c,n))$. Thus, a firm will not deviate from a price between $p^N(n)$ and $p^N(c,n)$, if it will not deviate from $p^N(c,n)$.

There exists a unique $\widehat{c} \in (0,\overline{c})$ such that $\Delta\Omega_L^z(p^N(\widehat{c},n)) = 0$ because $\Delta\Omega_L^z(p^N(n)) >$

0, $\Delta\Omega_L^z(p^N(\bar{c},n)) = \Delta\Omega_L^z(p^m) < 0$ and:

$$\frac{d(\Delta\Omega_L^z(p^N))}{dc} = \left[\frac{d(\Delta\Omega_L^z(p))}{dp}\frac{dp^N}{dc}\right]_{p=p^N(c,n)} < 0,$$

since $\frac{d(\Delta\Omega_L^z(p))}{dp} < 0$ and $\frac{dp^N}{dc} > 0$. Thus, $\Delta\Omega_L^z(p^N(c,n)) \geq 0$ if and only if $c \in (0,\widehat{c}]$. The above analysis implies that $p^N(c,n)$ and p, such that $p^N(n) < p \leq p^N(c,n)$, are supportable in high- and low-cost states, respectively, if and only if $c \in (0,\widehat{c}]$. ■

Proof of Lemma 2. Suppose the collusive price is p^c in both cost states. To prove subgame perfection, it suffices to check that there is no history that leads to a subgame in which a one-stage deviation is profitable. For every history, the lowest ever deviation price below p^c at some period τ is $\min\{\mathbf{p}_\tau^d, p^c\}$. If $\min\{\mathbf{p}_\tau^d, p^c\} \leq p^N(c,n)$, then (3) defines a Nash equilibrium in the subsequent punishment subgames, whether they start with a period of high or low costs, if and only if $c \in (0,\widehat{c}]$ (from Lemma 1). Otherwise, the subsequent punishment subgames are identical to a history in which firms had set $\min\{\mathbf{p}_\tau^d, p^c\} \in (p^N(c,n), p^c]$ in both high- and low-cost states. Thus, we must find the conditions for which a firm will not deviate from p^c or from any rigid price between $p^N(c,n)$ and p^c in subgames that start with a period of high or low costs.

Suppose we consider some collusive price $p \in (p^N(c,n), p^c]$ that is set in both cost states, where $c \in (0,\widehat{c}]$. First, consider firm i setting a lower price $p_i \in [p^N(n), p^N(c,n)]$ in a period of low costs, so its present discounted value of deviation profits are given by (4). It is more profitable to deviate from p to $p^N(c,n)$ than to any price below $p^N(c,n)$, because $\Delta\Omega_L^z(p^N(c,n)) \geq 0 \ \forall \ c \in (0,\widehat{c}]$ and prices are strategic complements. Thus, we must consider deviations where firm i sets the same or a lower price $p_i \in [p^N(c,n), p]$. From (5) and (6), respectively, define:

$$\Delta\Omega_H^y(p) \equiv \left[\frac{\partial\pi_i(p_i,p;c,n)}{\partial p_i} + \frac{\delta}{1-\delta}\left(\frac{\theta}{\omega}\frac{d\pi(p_i;n)}{dp_i} + [1-\frac{\theta}{\omega}]\frac{d\pi(p_i;c,n)}{dp_i}\right)\right]_{p_i=p}$$
$$\Delta\Omega_L^y(p) \equiv \left[\frac{\partial\pi_i(p_i,p;n)}{\partial p_i} + \frac{\delta}{1-\delta}\left(\frac{\lambda}{\omega}\frac{d\pi(p_i;c,n)}{dp_i} + [1-\frac{\lambda}{\omega}]\frac{d\pi(p_i;n)}{dp_i}\right)\right]_{p_i=p}.$$

Firm i will not deviate from p in a period of low costs if $\Delta\Omega_L^y(p) \geq 0$, otherwise $\xi_{iL}^y(p_i,p) < 0$ for some $p_i < p$. We wish to show that if $\Delta\Omega_L^y(p^c) \geq 0$, then $\Delta\Omega_L^y(p) \geq 0$ and $\Delta\Omega_H^y(p) \geq 0 \ \forall \ p \in (p^N(c,n), p^c]$. First, differentiating $\Delta\Omega_L^y(p)$ with respect to p yields:

$$\frac{d(\Delta\Omega_L^y(p))}{dp} = \left[\frac{\partial^2\pi_i(p_i,p;n)}{\partial p_i^2} + \sum_{j\neq i}\frac{\partial^2\pi_i(p_i,p;n)}{\partial p_i\partial p_j} + \frac{\delta}{1-\delta}\left(\frac{\lambda}{\omega}\frac{d^2\pi(p_i;c,n)}{dp_i^2} + [1-\frac{\lambda}{\omega}]\frac{d^2\pi(p_i;n)}{dp_i^2}\right)\right]_{p_i=p_j=p}.$$

It follows from (2) and Assumption 5 that $\frac{d(\Delta\Omega_L^y(p))}{dp} < 0$. Hence, if $\Delta\Omega_L^y(p^c) \geq 0$, then $\Delta\Omega_L^y(p) > 0 \ \forall \ p \in (p^N(c,n), p^c)$. Thus, if firm i will not deviate from p^c in a period of low costs, then it will not deviate from a lower rigid price between $p^N(c,n)$ and p^c. Next, consider:

$$\Delta\Omega_H^y(p) - \Delta\Omega_L^y(p) = -\left[c\frac{\partial q_i(p_i,p;n)}{\partial p_i} + \frac{\delta}{\omega}(1-\theta-\lambda)\frac{c}{n}\frac{dq(p_i)}{dp_i}\right]_{p_i=p}.$$

Assumption 1 and $1-\theta-\lambda \geq 0$ imply that the above is positive. So, if $\Delta\Omega_L^y(p^c) \geq 0$, then $\Delta\Omega_H^y(p) > \Delta\Omega_L^y(p) \geq 0 \; \forall \; p \in (p^N(c,n), p^c]$. Thus, firm i will also not deviate from p^c or any rigid price between $p^N(c,n)$ and p^c in a period of high costs.

Given $\frac{d(\Delta\Omega_L^y(p))}{dp} < 0$, there exists a unique best rigid price, $p_L^y(c,n,\delta,\lambda,\theta)$, which is the level of p that solves $\Delta\Omega_L^y(p) = 0$. It satisfies $p^N(c,n) < p_L^y(c,n,\delta,\lambda,\theta) < p^m(c)$ since $\Delta\Omega_L^y(p^N(c,n)) > \Delta\Omega_L^z(p^N(c,n)) \geq 0 \; \forall \; c \in (0,\widehat{c}]$ and $\Delta\Omega_L^y(p^m(c)) < 0$. The above analysis implies that any rigid price p^c such that $p^N(c,n) < p^c \leq p_L^y(c,n,\delta,\lambda,\theta)$ is supportable if and only if $c \in (0,\widehat{c}]$. ■

Proof of Proposition 1. Suppose the collusive prices are $p(0)$ and $p(c) > p(0)$ in low- and high-cost states, respectively, where without loss of generality let $p(0) > p^N(c,n)$. To prove subgame perfection, it suffices to check that there is no history that leads to a subgame in which a one-stage deviation is profitable. For every history, the lowest ever deviation price below $p(c_\tau)$ at some period τ is $\min\{\mathbf{p}_\tau^d, p(c_\tau)\}$. If $\min\{\mathbf{p}_\tau^d\} \leq p^N(c,n) < p(0)$, then (3) defines a Nash equilibrium in the subsequent punishment subgames, whether they start with a period of high or low costs, if and only if $c \in (0,\widehat{c}]$ (from Lemma 1). If $p^N(c,n) < \min\{\mathbf{p}_\tau^d\} \leq p(0)$, then (3) defines a Nash equilibrium in the subsequent punishment subgames, whether they start with a period of high or low costs, if and only if $p(0) \leq p_L^y(c,n,\delta,\lambda,\theta)$ (from Lemma 2). Otherwise, the subsequent punishment subgames are identical to a history in which firms had set $\min\{\mathbf{p}_\tau^d, p(c)\} \in (p(0), p(c)]$ in high-cost states and $p(0)$ in low-cost states. Thus, to find when procyclical prices are supportable, we must find the conditions for which a firm will not deviate from $p(c)$ or from any price between $p(0)$ and $p(c)$ in subgames that start with a period of high costs. Moreover, we have to check that a firm will not deviate from $p(0) \leq p_L^y(c,n,\delta,\lambda,\theta)$ when $p(c) > p(0)$ in subgames that start with a period of low costs.

Suppose we consider some collusive prices such that $p(0) > p^N(c,n)$ is set in low-cost states and $p \in (p(0), p(c)]$ is set in high-cost states, where $c \in (0,\widehat{c}]$. Furthermore, recall that there are punishment subgames in which a firm will not conform to (3) for any $p(0)$ above $p_L^y(c,n,\delta,\lambda,\theta)$, so it must be the case that $p(0) \leq p_L^y(c,n,\delta,\lambda,\theta)$. Consider firm i deviating from such a $p(0)$ in a period of low costs. It is not profitable to set a higher price, because this decreases profits in the initial period and it decreases profits of future high-cost states. Moreover, it is not profitable to deviate to $p_i \in [p^N(c,n), p(0)]$, because given $\Omega_L(p(0), p(0)) \geq$

$\Omega^y_{iL}(p_i, p(0))\ \forall\ p_i \in [p^N(c,n), p(0)]$ if $p(0) \leq p^y_L(c,n,\delta,\lambda,\theta)$ (from Lemma 2), then $\Omega_L(p(0),p) > \Omega^y_{iL}(p_i,p(0))$ for any such p_i. Finally, it is not profitable to set $p_i \in [p^N(n), p^N(c,n))$ because it is more profitable to deviate from $p(0)$ to $p^N(c,n)$ than to any price below $p^N(c,n)$, since $\Delta\Omega^z_L(p^N(c,n)) \geq 0\ \forall\ c \in (0,\widehat{c}]$ and prices are strategic complements. This implies that a firm will not deviate from any $p(0)$, such that $p^N(c,n) < p(0) \leq p^y_L(c,n,\delta,\lambda,\theta)$, when $p > p(0)$. Thus, the best collusive price of the low-cost state is $p^*(0) = \min\{p^y_L(c,n,\delta,\lambda,\theta), p^m\}\ \forall\ c \in (0,\widehat{c}]$.

Now suppose the collusive prices are $p \in (p^*(0), p(c)]$ in high-cost states and $p^*(0)$ in low-cost states, where $c \in (0,\widehat{c}]$. First, consider firm i setting a lower price $p_i \in [p^N(c,n), p^*(0)]$, so its present discounted value of deviation profits are given by (5). It is more profitable to deviate from p to $p^*(0)$ than to a price below $p^*(0)$ in a period of high costs, because $\Delta\Omega^y_H(p^*(0)) > \Delta\Omega^y_L(p^*(0)) \geq 0\ \forall\ c \in (0,\widehat{c}]$ and prices are strategic complements. Thus, we must consider deviations where firm i sets the same or a lower price $p_i \in [p^*(0), p]$. From (7), define:

$$\Delta\Omega^x_H(p) \equiv \left[\frac{\partial\pi_i(p_i,p;c,n)}{\partial p_i} + \frac{\delta}{1-\delta}\left(1-\frac{\theta}{\omega}\right)\frac{d\pi(p_i;c,n)}{dp_i}\right]_{p_i=p}.$$

Firm i will not deviate from p if $\Delta\Omega^x_H(p) \geq 0$, otherwise $\xi^x_{iH}(p_i,p) < 0$ for some $p_i < p$. We wish to show that if $\Delta\Omega^x_H(p(c)) \geq 0$, then $\Delta\Omega^x_H(p) \geq 0\ \forall\ p \in (p^*(0), p(c))$. Differentiating $\Delta\Omega^x_H(p)$ with respect to p yields:

$$\frac{d(\Delta\Omega^x_H(p))}{dp} = \left[\frac{\partial^2\pi_i(p_i,p;c,n)}{\partial p_i^2} + \sum_{j\neq i}\frac{\partial^2\pi_i(p_i,p;c,n)}{\partial p_i\partial p_j} + \frac{\delta}{1-\delta}\left(1-\frac{\theta}{\omega}\right)\frac{d^2\pi(p_i;c,n)}{dp_i^2}\right]_{p_i=p_j=p}.$$

It follows from (2) and Assumption 5 that $\frac{d(\Delta\Omega^x_H(p))}{dp} < 0$. Hence, if $\Delta\Omega^x_H(p(c)) \geq 0$, then $\Delta\Omega^x_H(p) > 0\ \forall\ p \in (p^*(0), p(c))$. Thus, a firm will not deviate from a price between $p^*(0)$ and $p(c)$, if it will not deviate from $p(c)$. This implies that the best collusive prices are procyclical when $\Delta\Omega^x_H(p^*(0)) > 0$, otherwise they are rigid over time at $p^*(0)$. Notice that $\Delta\Omega^x_H(p^m) > 0$ if $\Delta\Omega^y_L(p^m) \geq 0$, so a firm will not deviate from a price above yet very close to p^m in a period of high costs when $p^*(0) = p^m$. Thus, the best collusive prices can only be rigid at $p^y_L(c,n,\delta,\lambda,\theta) < p^m$, and this occurs if and only if $\Delta\Omega^x_H(p^y_L(c,n,\delta,\lambda,\theta)) \leq 0$.

Finally, we must show that there exists a unique $c^* \in (0,\widehat{c})$ such that $\Delta\Omega^x_H(p^y_L(c^*,n,\delta,\lambda,\theta)) = 0$, where $\Delta\Omega^x_H(p^y_L(c,n,\delta,\lambda,\theta)) \leq 0\ \forall\ c \in (0,c^*]$. First notice that $c^* > 0$, because $\lim_{c\to 0}\Delta\Omega^x_H(p^c) < \lim_{c\to 0}\Delta\Omega^y_L(p^c)\ \forall\, p^c \in [p^N(c,n), p^m]$, so $\lim_{c\to 0}\Delta\Omega^x_H(p^y_L(c,n,\delta,\lambda,\theta)) < 0$ since $\lim_{c\to 0}\Delta\Omega^y_L(p^y_L(c,n,\delta,\lambda,\theta)) = 0$. Next, consider:

$$\begin{aligned}\frac{d(\Delta\Omega^x_H(p^y_L))}{dc} &= -\left[\frac{\partial q_i(p_i,p;n)}{\partial p_i} + \frac{\delta}{(1-\delta)}\frac{1}{n}\left[1-\frac{\theta}{\omega}\right]\frac{dq(p_i)}{dp_i} - \frac{d(\Delta\Omega^x_H(p))}{dp}\frac{dp^y_L}{dc}\right]_{p_i=p=p^y_L(c,n,\delta,\lambda,\theta)}\\ &= -\left[\frac{\partial q_i(p_i,p;n)}{\partial p_i} + \frac{\delta}{(1-\delta)}\frac{1}{n}\left[1-\frac{\theta}{\omega}-\alpha\frac{\lambda}{\omega}\right]\frac{dq(p_i)}{dp_i}\right]_{p_i=p=p^y_L(c,n,\delta,\lambda,\theta)}\end{aligned}$$

where $\alpha \equiv \frac{d(\Delta\Omega_H^x(p))/dp}{d(\Delta\Omega_L^y(p))/dp} > 0$ and:

$$\frac{dp_L^y}{dc} = \frac{1}{\frac{d(\Delta\Omega_L^y(p))}{dp}} \left. \frac{\delta}{(1-\delta)} \frac{1}{n} \frac{\lambda}{\omega} \frac{dq(p_i)}{dp_i} \right|_{p_i=p=p_L^y(c,n,\delta,\lambda,\theta)} > 0.$$

If $[1 - \frac{\theta}{\omega} - \alpha\frac{\lambda}{\omega}] > 0$, then $\frac{d(\Delta\Omega_H^x(p_L^y))}{dc} > 0$, where $\alpha \leq 1$ is sufficient for this to be true. Subtracting $\frac{d(\Delta\Omega_L^y(p))}{dp}$ from $\frac{d(\Delta\Omega_H^x(p))}{dp}$ yields:

$$-\left[c\left(\frac{\partial^2 q_i(p_i,p;n)}{\partial p_i^2} + \sum_{j\neq i} \frac{\partial^2 q_i(p_i,p;n)}{\partial p_i \partial p_j}\right) + \frac{\delta}{1-\delta}(1-\frac{\lambda}{\omega})\frac{d^2\pi(p_i;n)}{dp_i^2} + \frac{\delta}{\omega}(1-\theta-\lambda)\frac{d^2\pi(p_i;c,n)}{dp_i^2}\right]_{p_i=p_j=p}, \tag{9}$$

which is positive from Assumptions 3 and 5, and $1-\theta-\lambda \geq 0$, so $\alpha < 1$. Hence, $\frac{d(\Delta\Omega_H^x(p_L^y))}{dc} > 0$, which implies that c^* is unique and that $\Delta\Omega_H^x(p_L^y(c,n,\delta,\lambda,\theta)) < 0$ $\forall\ c \in (0,c^*)$. Finally, to see that $c^* < \widehat{c}$, consider:

$$\Delta\Omega_H^x(p) - \Delta\Omega_L^y(p) = \left[\frac{\partial\pi_i(p_i,p;c,n)}{\partial p_i} + \frac{\delta}{\omega}(1-\theta-\lambda)\frac{d\pi(p_i;c,n)}{dp_i} - \Delta\Omega_L^z(p)\right]_{p_i=p}. \tag{10}$$

When evaluated at $p_L^y(c,n,\delta,\lambda,\theta)$, (10) is non-positive for all $c \in (0,c^*]$. Differentiating (10) with respect to p yields (9), which is positive. This implies that (10) is negative for all $p \in [p^N(c,n), p_L^y(c,n,\delta,\lambda,\theta))$. Thus, for (10) to be negative when evaluated at $p^N(c,n)$, it follows that $\Delta\Omega_L^z(p^N(c,n)) > 0$ since the first term on the right-hand side of (10) is zero at $p^N(c,n)$ and the second is non-negative at $p^N(c,n)$ $\forall\ 1-\theta-\lambda \geq 0$. Given $\Delta\Omega_L^z(p^N(\widehat{c},n)) = 0$, then $c^* < \widehat{c}$ since $\frac{d(\Delta\Omega_L^z(p^N))}{dc} < 0$ (see the Proof of Lemma 1). ■

Proof of Proposition 2. Totally differentiating $\Delta\Omega_H^x(p_L^y(c,n,\delta,\lambda,\theta)) = 0$ yields:

$$\begin{aligned} \frac{dc^*}{d\lambda} &= -\frac{1}{\frac{d(\Delta\Omega_H^x(p_L^y))}{dc}} \frac{d(\Delta\Omega_H^x(p_L^y))}{d\lambda} \\ \frac{dc^*}{d\theta} &= -\frac{1}{\frac{d(\Delta\Omega_H^x(p_L^y))}{dc}} \frac{d(\Delta\Omega_H^x(p_L^y))}{d\theta} \end{aligned}$$

where $\frac{d(\Delta\Omega_H^x(p_L^y))}{dc} > 0$ (from the Proof of Proposition 1).

The total derivative of $\Delta\Omega_H^x(p_L^y(c,n,\delta,\lambda,\theta))$ with respect to λ $\forall\ 1-\theta-\lambda = 0$ is:

$$\begin{aligned} \frac{d(\Delta\Omega_H^x(p_L^y))}{d\lambda} &= \left[\frac{\delta}{1-\delta}\frac{d\pi(p_i;c,n)}{dp_i} + \frac{d(\Delta\Omega_H^x(p))}{dp}\frac{dp_L^y}{d\lambda}\right]_{p_i=p=p_L^y(c,n,\delta,\lambda,\theta)} \\ &= \frac{\delta}{1-\delta}\left[\frac{d\pi(p_i;n)}{dp_i} - (1-\alpha)\frac{c}{n}\frac{dq(p_i)}{dp_i}\right]_{p_i=p=p_L^y(c,n,\delta,\lambda,\theta)} \end{aligned}$$

where $\alpha \equiv \frac{d(\Delta\Omega_H^x(p))/dp}{d(\Delta\Omega_L^y(p))/dp} \in (0,1)$ (from the Proof of Proposition 1) and:

$$\frac{dp_L^y}{d\lambda} = \frac{1}{\frac{d(\Delta\Omega_L^y(p))}{dp}} \left. \frac{\delta}{(1-\delta)} \frac{c}{n} \frac{dq(p_i)}{dp_i} \right|_{p_i=p=p_L^y(c,n,\delta,\lambda,\theta)} > 0.$$

The total derivative of $\Delta\Omega_H^x(p_L^y(c,n,\delta,\lambda,\theta))$ with respect to θ $\forall$ $1-\theta-\lambda>0$ is:

$$\begin{aligned}\frac{d(\Delta\Omega_H^x(p_L^y))}{d\theta} &= -\left[\frac{\delta(1-\delta(1-\lambda))}{(1-\delta)\omega^2}\frac{d\pi(p_i;c,n)}{dp_i} - \frac{d(\Delta\Omega_H^x(p))}{dp}\frac{dp_L^y}{d\theta}\right]_{p_i=p=p_L^y(c,n,\delta,\lambda,\theta)} \\ &= -\frac{\delta}{(1-\delta)\omega^2}\left[(1-\delta[1-\lambda(1-\alpha)])\frac{d\pi(p_i;c,n)}{dp_i} + \alpha\delta\lambda\frac{d\pi(p_i;n)}{dp_i}\right]_{p_i=p=p_L^y(c,n,\delta,\lambda,\theta)}\end{aligned}$$

where:

$$\frac{dp_L^y}{d\theta} = -\frac{1}{\frac{d(\Delta\Omega_L^y(p))}{dp}}\left.\frac{\delta^2\lambda}{(1-\delta)\omega^2}\frac{c}{n}\frac{dq(p_i)}{dp_i}\right|_{p_i=p=p_L^y(c,n,\delta,\lambda,\theta)} < 0.$$

It follows from Assumptions 1 and 5, and $0<\alpha<1$ that $\frac{d(\Delta\Omega_H^x(p_L^y))}{d\lambda}>0$ and $\frac{d(\Delta\Omega_H^x(p_L^y))}{d\theta}<0$, so $\frac{dc^*}{d\lambda}<0$ $\forall$ $1-\theta-\lambda=0$ and $\frac{dc^*}{d\theta}>0$ $\forall$ $1-\theta-\lambda>0$. ■

Proof of Proposition 3. It follows from (2) and Assumptions 1 and 5 that $\frac{dp_L^y}{dc}\in(0,\frac{\lambda}{\omega})$ (see the Proof of Proposition 1). This guarantees that per-period profits are strictly increasing in c in low-cost states but they are strictly decreasing in c in high-cost states, when such profits are evaluated at $p_L^y(c,n,\delta,\lambda,\theta)$.

Next, evaluate $p(0)$ and $p(c)$ in $\Omega_L(p(0),p(c))$ at $p_L^y(c,n,\delta,\lambda,\theta)$ and totally differentiate with respect to c. This yields:

$$\begin{aligned}\frac{d\Omega_L(p_L^y,p_L^y)}{dc} &= -\left[\frac{\delta}{1-\delta}\frac{\lambda}{\omega}\frac{q(p)}{n} - \sum_{j\neq i}\frac{\partial\pi_i(p,p;n)}{\partial p_j}\frac{dp_L^y}{dc}\right]_{p_j=p=p_L^y(c,n,\delta,\lambda,\theta)} \\ &= -\frac{\delta}{1-\delta}\frac{\lambda}{\omega}\left[\frac{q(p)}{n} + \frac{(p-c)}{n}\frac{dq(p)}{dp}\frac{1}{-\frac{d(\Delta\Omega_L^y(p))}{dp}}\Sigma_{j\neq i}\frac{\partial q_i(p,p;n)}{\partial p_j}\right]_{p_j=p=p_L^y(c,n,\delta,\lambda,\theta)}\end{aligned}$$

The above is negative since $\frac{d\pi(p;c,n)}{dp}=\frac{q(p)}{n}+\frac{(p-c)}{n}\frac{dq(p)}{dp}>0$ when p is evaluated at $p_L^Y(c,n,\delta,\lambda,\theta)\in(p^N(c,n),p^m)$, and $\left|\frac{d(\Delta\Omega_L^y(p))}{dp}\right|>\left|\frac{dq(p)}{dp}\right|>\Sigma_{j\neq i}\frac{\partial q_i(p,p;n)}{\partial p_j}$. This implies that each firm's present discounted value of collusive profits is strictly decreasing in c in low-cost states.

Finally, evaluate $p(c)$ and $p(0)$ in $\Omega_H(p(c),p(0))$ at $p_L^y(c,n,\delta,\lambda,\theta)$ and totally differentiate with respect to c. This yields:

$$\begin{aligned}\frac{d\Omega_H(p_L^y,p_L^y)}{dc} &= \left[-\left(1+\frac{\delta}{1-\delta}(1-\frac{\theta}{\omega})\right)\frac{q(p)}{n} + \frac{d\Omega_H(p)}{dp}\frac{dp_L^y}{dc}\right]_{p=p_L^y(c,n,\delta,\lambda,\theta)} \\ &= -\left[\frac{q(p)}{n}\left(1-\frac{dp_L^y}{dc}\right) + \frac{q(p)}{n}\frac{\delta}{1-\delta}\left(1-\frac{\theta}{\omega}-\frac{dp_L^y}{dc}\right)\right. \\ &\quad \left. -\frac{1}{n}\frac{dq(p)}{dp}\frac{dp_L^y}{dc}\left((p-c)+\frac{\delta}{1-\delta}(p-(1-\frac{\theta}{\omega})c)\right)\right]_{p=p_L^y(c,n,\delta,\lambda,\theta)}.\end{aligned}$$

It follows from Assumption 1 and $0<\frac{dp_L^y}{dc}<\frac{\lambda}{\omega}\leq(1-\frac{\theta}{\omega})<1$ $\forall$ $1-\theta-\lambda\geq 0$ that the above is negative. This implies that each firm's present discounted value of collusive profits is strictly decreasing in c in high-cost states. ■