Matching Own Prices, Rivals’ Prices, or Both

by

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Abstract: Many retailers promise that they will not be undersold by rivals (price-matching guarantees) and extend their promise to include their own future prices (most-favoured-customer clauses). This is puzzling because the extant literature has shown that each promise independently has the potential to facilitate supracompetitive prices, and so one might think that the two promises are substitutes. In this paper, we consider why a firm might make both promises in the same guarantee, and show that price-matching guarantees and most-favoured-customer clauses complement each other and can lead to higher prices than either one could have facilitated by itself.

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If, within 30 days of purchasing an item from CoolBuys or a CoolBuys Retail Partner, you find the same product advertised at a lower price, we will gladly refund you 100% of the difference. If our own price is reduced within 30 days of your purchase, we would be pleased to refund you the difference as well.1

1 Introduction

Many retailers promise that they will not be undersold by rivals, and extend their promise to include their own future prices if they should decrease. The above quote from CoolBuys is typical. CoolBuys’ pricing policy combines both a low-price guarantee (in this case a price-matching guarantee) and a retroactive most-favored-customer (MFC) clause, in which a promise is made to consumers that if a competitor has a lower advertised price, or if CoolBuys lowers its own advertised price within 30 days of purchase, the difference in prices will be refunded. Arbatskaya et. al. (2004) found that of the 514 low-price guarantees in their sample, obtained from newspaper advertisements in the U.S., 104 of them (more than 20 percent) also contained MFC clauses.

Why firms might want to combine these promises in the same guarantee is puzzling. On the one hand, the literature beginning with Salop (1986), and extended by many others,2 suggests that price-matching guarantees can facilitate monopoly pricing when all firms in the market adopt them.3 But, if monopoly pricing can be supported by price-matching guarantees alone, then it is not clear what additional benefits MFC clauses might offer firms. On the other hand, the literature also suggests that the ability of price-matching guarantees to raise prices may be quite limited, even in the simplest settings.4 In particular, their power to facilitate higher prices may be limited if not all firms offer them or if consumers incur hassle costs when requesting that prices be matched. In the case of symmetric firms, for example, Hviid and Shaffer (1999) show that any amount of hassle costs, no matter how small, eliminate the ability of price-matching guarantees to raise prices above static Bertrand levels. In these cases, it is not clear what additional benefits price-matching guarantees might offer firms that already have MFC clauses, which, as Cooper (1986) and Neilson and Winter (1993)5 suggest, can act as price commitments by the firms adopting them, leading to

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3The by now familiar reasoning is that a firm’s low-price guarantee may eliminate its rivals’ incentives to cut prices, and when all firms have such guarantees, all firms’ incentives to cut prices may be eliminated.
4See, for example, Corts (1995), Hviid and Shaffer (1999), Kaplan (2000), and Logan and Lutter (1989).
5See also Akman and Hviid (2006), Baker (1996), Neilson and Winter (1992), and Schnitzer (1994).
higher equilibrium prices.\textsuperscript{6}

In this paper, we consider whether there may be competition-related motives for combining price-matching guarantees and MFC clauses. In other words, can two facilitating practices jointly lead to higher prices than either one could have facilitated by itself? In the simple two-period, two-firm setting considered here, the answer is yes, even though only one firm makes such promises. What we find is that price-matching guarantees and MFC clauses are complements, each covering for the other’s weakness. The firm’s price-matching guarantees reduce its rival’s incentive to cut prices in each period, whereas its first-period MFC clause reduces its own incentive to cut prices in period two. As a result, in some cases, prices between the Stackelberg prices and the monopoly prices can be supported in each period. With MFC clauses only, prices are less than the Stackelberg prices, whereas with price-matching guarantees only, prices are at the static Bertrand levels.

Although previous literature has explored the use of price-matching guarantees and MFC clauses as possible facilitating practices, there has been no consideration of their joint use. Since firms often do combine these practices in reality, this omission appears to a major gap in the literature. In taking a first step towards closing this gap, the model also has implications for empirical work. Although empirical evidence supports the notion that price-matching guarantees can discourage price-cutting and thereby act as a facilitating practice (see, for example, Hess and Gerstner, 1991; and Arbatskaya et al, 1999; 2004; 2006), Arbatskaya et al (2004) and Manez (2006) show that whether this is so may depend on the details of the guarantees, such as, whether the guarantees cover advertised or selling prices, whether there are hassle costs, and whether the guarantees promise to match or beat rivals’ prices. To this, we can add that it may also depend on whether the guarantees include an MFC clause, as the effect on equilibrium prices may depend crucially on this fact.

It should be noted that other motivations for price-matching guarantees and MFC clauses are possible, and indeed plausible, in some settings. For example, it has been pointed out that MFC clauses may be offered in input markets as a means to alter the relative bargaining powers of firms when sellers negotiate with buyers sequentially (see Cooper and Fries, 1991; and Neilson and Winter, 1994), and that retailers may adopt price-matching guarantees to signal low prices to consumers or to discriminate in prices between informed and uninformed consumers (see, for example, Moorthy

\textsuperscript{6}The reason is that the concern over having to pay compensation to past consumers makes a firm with an MFC clause more likely to resist lowering its price over time, which effectively allows it to be a price leader. In a two-period model, for example, this is akin to a firm committing in period one to its period-two price, and from standard price-leader, price-follower models, we know that such a precommitment by a first-mover can lead to higher prices.
and Winter, 2006; and Png and Hirschleifer, 1987). Since our focus in this paper is explicitly on their possible role as facilitating practices, we leave these alternative motivations, and how they may be impacted by a firm that unilaterally adopts both practices, for future research.

The rest of the paper is organized as follows. Section two describes the model and gives preliminary results. Section three shows that in equilibrium one firm will unilaterally adopt both practices, and that this will lead to higher prices than either practice could have facilitated by itself. Section four shows that the model is robust to the introduction of hassle costs, and that in equilibrium, the rival firm has no incentive to adopt either practice. Section five offers concluding remarks.

2 Model

Suppose two firms, A and B, sell imperfect substitutes to final consumers and compete by setting prices simultaneously in each of two periods. For simplicity, assume the objective of each firm is to maximize the sum of its (undiscounted) profits over the two periods. We are interested in assessing to what extent higher prices can be supported in this simple setting when a single firm unilaterally adopts one or more facilitating practices. The decision whether or not to adopt a practice is made at the beginning of each period and is assumed to be common knowledge. Later, in Section four, we will show that our results are robust to allowing both firms to adopt facilitating practices.

We consider price-matching guarantees, in which a firm promises not to be undersold by its rivals, and retroactive most-favored customer (MFC) clauses, in which a firm promises to refund to first-period customers the difference in price if its period-two price is less than its period-one price.

The game is as follows. In period one, firm A chooses whether to offer a price-matching guarantee, an MFC clause, or both, to consumers. Firms A and B then compete by setting prices. In period two, firm A chooses whether to extend or to drop its price-matching guarantee if it had one in the first period or, if not, whether to offer a guarantee to consumers for the first time. Firms A and B then compete by setting prices. We use subgame perfection as our solution concept.

To keep things simple, we assume that although the firms sell the same branded product (important in practice for the application of price-matching guarantees), consumers have diverse and symmetric preferences over where to shop. We also assume that their preferences are stable over time, and we let firm i’s demand be given by \( D_i(P_{it}, P_{jt}) \), where \( P_{it} \) is firm i’s price in period t and \( P_{jt} \) is firm j’s price in period t, \( j \neq i \). For all positive values of \( D_i \), we assume that \( D_i \) is

\[ \text{Since the game ends in period two, offering an MFC clause in period two would obviously have no effect.} \]
differentiable, decreasing in $P_{it}$, increasing in $P_{jt}$, and has the property that equal increases in both firms’ prices decrease firm $i$’s demand. All fixed and marginal costs are normalized to zero.

Firm $i$’s profit in period $t$ can thus be written as $\Pi_{it}(P_{it}, P_{jt}) = P_{it} \cdot D_i(P_{it}, P_{jt})$. For all prices $P_{it}$ and $P_{jt}$ such that $D_i > 0$, we make the following standard assumptions on second derivatives.

**Assumption 1:** Firm $i$’s marginal profit in period $t$ is increasing in $P_{jt}$:

$$\frac{\partial \Pi_{it}}{\partial P_{jt}} > 0.$$  

**Assumption 2:** Equal increases in $P_{it}$ and $P_{jt}$ are less profitable for firm $i$ the higher is $P_{it}$:

$$\frac{\partial^2 \Pi_{it}}{\partial P_{it}^2} + \frac{\partial \Pi_{it}}{\partial P_{it} \partial P_{jt}} < 0.$$  

**Assumption 3:** Equal increases in $P_{it}$ and $P_{jt}$ are less profitable for firm $i$ the lower is $P_{jt}$:

$$\frac{\partial^2 \Pi_{it}}{\partial P_{jt}^2} + \frac{\partial \Pi_{it}}{\partial P_{it} \partial P_{jt}} > 0.$$  

Assumptions 1 and 2 imply that firm $i$’s profit is concave in $P_{it}$, and that prices are strategic complements (see Bulow et al, 1985). Together they ensure the existence of a unique Bertrand equilibrium in prices (see Friedman, 1983). Assumptions 2 and 3 imply that own price effects dominate cross-price effects for firm $i$, and are satisfied, as is Assumption 1, if demands are linear.\(^8\)

There are no structural linkages across periods, so that in the absence of a price-matching guarantee or MFC clause by firm A, each firm charges its differentiated-products Bertrand price in each period. Formally, firm $i$’s one-period Bertrand problem in period $t$ can be written as

$$\max_{P_{it}} P_{it} \cdot D_i(P_{it}, P_{jt})$$

with associated first order condition

$$P_{it} \cdot \frac{\partial D_i(P_{it}, P_{jt})}{\partial P_{it}} + D_i(P_{it}, P_{jt}) = 0.$$  

Let the Bertrand-best reply of firm $i$ in period $t$, denoted $BR_{it}(P_{jt})$, be the price $P_{it}$ that solves (2) given $P_{jt}$. Assumptions 1-3 ensure that firm $i$’s best-reply is single-valued, continuous, differentiable, and upward sloping, with $\frac{\partial BR_{it}}{\partial P_{jt}} \in [0, 1]$. These assumptions also ensure that the one-period Bertrand problem, given in (1) for firm $i$, has a unique symmetric solution, which we denote by the Bertrand price pair $PB \equiv (P_B, P_B)$. The one-period equilibrium is illustrated in Figure 1 below.

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\(^8\)When demand is linear, $\frac{\partial D_i}{\partial P_{jt}} = \frac{\partial D_i}{\partial P_{it}}$, which is positive given that firm $i$’s demand is increasing in $P_{jt}$. It follows from this that Assumptions 1 and 3 hold (since $\frac{\partial^2 D_i}{\partial P_{jt}^2} = 0$). Moreover, Assumption 2 holds since $\frac{\partial D_i}{\partial P_{it}^2} + \frac{\partial D_i}{\partial P_{it} \partial P_{jt}} = 2\frac{\partial D_i}{\partial P_{jt}}$, which is negative given that equal increases in both firms’ prices decrease firm $i$’s demand.
Figure 1: Bertrand Equilibrium

Figure 1 identifies two additional price pairs, \( \mathbf{P}^M \equiv (P^M, P^M) \) and \( \mathbf{P}^S \equiv (P^S_A, P^S_B) \), that will also play prominent roles in the next section. The superscript \( \text{M} \) on the former price pair stands for monopoly, and we define it to be the price pair that maximizes firm \( i \)'s profit along the line \( P_i = P_j \), i.e., \( P^M \equiv \arg \max P \Pi_i(P, P) \). We assume that firm \( i \)'s profit is concave\(^9\) along this line, so that \( P^M \) can be obtained as the price \( P_i = P_j \) that solves the first-order condition

\[
\begin{align*}
  P_i \cdot \frac{\partial D_i(P_i, P_j)}{\partial P_i} + P_i \cdot \frac{\partial D_i(P_i, P_j)}{\partial P_j} + D_i(P_i, P_j) = 0. \tag{3}
\end{align*}
\]

The superscript \( \text{S} \) on the price pair \( \mathbf{P}^S \) stands for Stackelberg, and we define its first element, \( P^S_A \), to be the price that maximizes firm \( i \)'s profit along firm \( j \)'s best reply, i.e., \( P^S_A \equiv \arg \max P \Pi_i(P_i, BR_j(P_i)) \). We assume that firm \( i \)'s profit is concave along firm \( j \)'s best reply, so that \( P^S_A \) can be obtained as the price \( P_i \) that solves the first-order condition\(^10\)

\[
\begin{align*}
  P_i \cdot \frac{\partial D_i(P_i, BR_j)}{\partial P_i} + P_i \cdot \frac{\partial D_i(P_i, BR_j)}{\partial P_j} + \frac{\partial BR_j}{\partial P_i} + D_i(P_i, BR_j) = 0. \tag{4}
\end{align*}
\]

\(^9\)This implies \( \frac{\partial^2 \Pi_i}{\partial P_i^2} + \frac{\partial^2 \Pi_i}{\partial P_j^2} + 2 \frac{\partial \Pi_i}{\partial P_i} \frac{\partial \Pi_i}{\partial P_j} < 0 \), which is satisfied, for example, when demand is linear.

\(^10\)Let \( \psi_{ij} \) denote the left-hand side of (4). Then our assumption that \( \Pi_i(P_i, BR_j(P_i)) \) is concave in \( P_i \) implies that \( \frac{\partial \psi_{ij}}{\partial P_i} + \frac{\partial \psi_{ij}}{\partial P_j} \frac{\partial BR_j}{\partial P_i} < 0 \), which can be shown to be satisfied, for example, when demand is linear.
We define \( P^S_B \equiv BR_B(P^S_A) \) to be the price that maximizes firm B’s profit given that firm A charges \( P^S_A \). Note that Assumption 3 implies that \( P^M > P^S \), as we have illustrated in Figure 1.

2.1 Preliminary Results

Price-matching guarantees

It is useful to begin by considering what would happen if A offers a price-matching guarantee in period two without having offered an MFC clause in period one. In this case, one might think that second-period prices, which are by assumption independent of first-period prices, will be lower than if A did not offer a price-matching guarantee because if period two prices were such that \( P_{B2} < P_{A2} \), then firm A would be obligated to sell to consumers at firm B’s lower price. However, as pointed out by Salop (1986) and many others, A’s price-matching guarantee will alter B’s incentives, and may instead have the perverse effect of causing firm B not to charge a lower price in the first place.

To see how B’s incentives are altered by A’s guarantee, note that for \( P_{A2} > P^B \), firm B would prefer to have a lower selling price than firm A but cannot attain it with A’s guarantee in place. Thus, A’s guarantee affects B’s incentives by penalizing it for posting a lower price—instead of gaining additional sales from having a lower selling price than firm A, firm B ends up triggering only an adverse price response.\(^{11}\) As such, it is easy to show that for all \( P_{A2} \in (P^B, P^M) \), the effect of A’s guarantee is essentially to transform B’s best reply in Figure 2 from \( BR_{B2}(P_{A2}) \) (which holds in the absence of A’s guarantee) to the boldface-dashed line (which holds in its presence). We can write firm B’s best response if firm A adopts a price-matching guarantee in period two as follows:

\[
BR_{B2}^{PM}(P_{A2}) \equiv \begin{cases} 
BR_{B2}(P_{A2}) & \text{if } P_{A2} \leq P^B \\
P_{A2} & \text{if } P^B < P_{A2} \leq P^M \\
P^M & \text{if } P_{A2} > P^M
\end{cases}
\]

Note that for all \( P_{A2} > P^M \), firm B will set a price of \( P^M \) knowing that A’s price will come down to match it and that the resulting outcome will maximize overall profit of which it receives half.

As shown in Hvid and Shaffer (1999), however, and as one can see from Figure 2, A’s guarantee turns out to be harmless, as it has no effect on equilibrium prices despite the effect on B’s incentives. The reason is that A’s guarantee does not affect its own incentive to undercut B’s price, nor does it affect B’s best reply at the Bertrand prices. This implies that there is no other pair of prices at

\(^{11}\)In contrast, for \( P_{A2} < P^B \), firm B prefers to have a higher price than firm A, and so A’s guarantee has no effect.
which B’s best reply, $BR_{B2}^{PM}(P_{A2})$, intersects A’s best reply, $BR_{A2}(P_{B2})$, and thus that unilateral adoption of price-matching by firm A in period two will have no effect on period-two prices.

**Most-favored-customer clauses**

Now suppose that firm A offered an MFC clause in period one but does not offer a price-matching guarantee in period two. What will be the effect on period-two prices? Recall that with its first-period MFC clause, firm A is obligated to refund any price difference between its period one and period-two prices to consumers in period one if its period-two price is lower. If, for example, firm A were to set a first-period price of $P_{A1}$ and a second-period price of $P_{A2} < P_{A1}$, then firm A would be obligated to refund a total of $(P_{A1} - P_{A2}) \cdot D_A(P_{A1}, P_{B1})$ to its first-period customers.

At first glance, one might think that consumers would benefit from this because first-period buyers gain if A’s second-period price is lowered. However, as with price-matching guarantees, first impressions can be misleading. In this case, B’s second-period incentives are not altered, nor are its first-period incentives, but A’s incentives to lower its second-period price are. As we shall see, by unilaterally offering to penalize itself, firm A is essentially able to commit to its second-period price in period one. It is this commitment effect that allows MFC clauses to facilitate higher prices.
To see this, note that A’s profit in period two if it offers a refund to period-one consumers is

\[ P_{A2} \cdot D_A(P_{A2}, P_{B2}) - (P_{A1} - P_{A2}) \cdot D_A(P_{A1}, P_{B1}), \]

and the associated first-order condition is

\[ P_{A2} \cdot \frac{\partial D_A(P_{A2}, P_{B2})}{\partial P_{A2}} + D_A(P_{A2}, P_{B2}) + D_A(P_{A1}, P_{B1}) = 0. \]  

(5)

Let the corresponding best reply of firm A be denoted by \( BR_{A2}^{pen}(P_{B2}; P_{A1}) \), where the superscript ‘pen’ stands for the refund penalty that A incurs in period two if \( P_{A2} < P_{A1} \). If \( P_{A2} \geq P_{A1} \), then A’s second-period profit is given in (1) and the associated first-order condition is given in (2).

In comparing (2) and (5), note that the slope of A’s best-reply is the same in both cases, and that the third term in (5) implies that \( BR_{A2}^{pen}(P_{B2}; P_{A1}) > BR_A(P_{B2}) \) for all \( P_{B2} \), provided that A has positive sales in period one. Thus, as illustrated in Figure 3 below, the graph of \( BR_{A2}^{pen}(P_{B2}; P_{A1}) \) is a parallel shift to the right of the graph of \( BR_A(P_{B2}) \). Letting \( P_{B2} = P_L(P_{A1}) \) solve (5) and \( P_{B2} = P_H(P_{A1}) \) solve (2) when each condition is evaluated at \( P_{A2} = P_{A1} \), it follows that for \( P_{B2} \in [P_L(P_{A1}), P_H(P_{A1})] \), firm A’s best reply to \( P_{B2} \) is to set \( P_{A2} = P_{A1} \). For \( P_{B2} < P_L(P_{A1}) \), firm A’s best reply to \( P_{B2} \) is to set \( P_{A2} = BR_{A2}^{pen}(P_{B2}; P_{A1}) \) and pay the refund, while for \( P_{B2} > P_H(P_{A1}) \), firm A’s best reply to \( P_{B2} \) is to set \( P_{A2} = BR_A(P_{B2}) \) and no refund is needed. Formally, we can write firm A’s best response to \( P_{B2} \) in period two if it had offered an MFC clause in period one as:

\[
BR_{A2}^{MFC}(P_{B2}; P_{A1}) = \begin{cases} 
BR_{A2}^{pen}(P_{B2}; P_{A1}) & \text{if } P_{B2} < P_L(P_{A1}) \\
\{ & \\
\{ & \\
\{ & \}
\end{cases}
\]

(5)

It is easy to see from this that \( BR_{A2}^{MFC}(P_{B2}; P_{A1}) \) is increasing and continuous in \( P_{B2} \), as shown by the boldface line in Figure 3. Moreover, it is easy to see that \( BR_{A2}^{MFC}(P_{B2}; P_{A1}) \) intersects \( BR_{B2}(P_{A2}) \) uniquely for any \( P_{A1} \) from period one. If \( P_{A1} \leq P^B \), the intersection occurs at \( P^B \) and the outcome in period two is Bertrand. For all other \( P_{A1} \) such that firm A has positive sales in period one, however, the intersection leads to higher prices. Figure 3 illustrates one such case.

This result confirms what has previously been shown in the seminal works of Cooper (1986) and Neilson and Winter (1993). Intuitively, compared to the Bertrand outcome, adopting an MFC clause in period one enables firm A to shift a portion of its period-two best reply to the right, which is generally beneficial in a price-setting game. The length of the portion that shifts is determined
by A’s period-one price, which also locates the vertical segment of A’s best reply. By raising its period-one price, firm A can further shift its best reply to the right and away from its “MFC-less” Bertrand best reply. The gain in period two from the resulting higher equilibrium prices is tempered, however, by the loss A incurs in period one from raising $P_{A1}$ above $P_B^B$. At the margin, the latter concern is a second-order effect, but it figures more prominently as $P_{A1}$ increases. As Neilson and Winter (1992) show, when these countervailing effects are taken into account, A’s optimal period-one price will exceed Bertrand but fall short of the one-period Stackelberg price.

3 Results

We now show that combining a price-matching guarantee in period two with an MFC clause from period one can increase A’s profit and lead to higher equilibrium prices. Since a period-two price-matching guarantee is powerless on its own, it is not surprising that offering both an MFC clause and a price-matching guarantee does weakly better than offering only to match prices. Thus, what needs to be shown is why offering both may yield higher profit than offering only an MFC clause.

We first state the result and then illustrate the intuition with the help of Figure 4 below.
Proposition 1 Suppose firm A offers an MFC clause in period one and has a price of $P_{A1}$. Then, if $P_{A1} > P_B$, it is strictly profitable for firm A to offer a price-matching guarantee in period two.

Proof. See Appendix A. ■

By combining a price-matching guarantee in period two with an MFC clause from period one, firm A can reduce B’s incentive to lower its price. To illustrate, Figure 4 depicts the equilibrium in period two for a particular $P_{A1}$. As is clear from the diagram, although A’s equilibrium price in period two is the same in both cases, B’s equilibrium price is higher when A has a price-matching guarantee than when it does not. In the former case, B’s equilibrium price is $P_{A1}$. In the latter case, B’s equilibrium price is $BR_{B2}(P_{A1}) < P_{A1}$. Hence, firm A is better off with price-matching.

Figure 4: Illustration of period-two equilibrium

Intuitively, firm A’s MFC clause in period one allows it to act as a price leader by reducing its incentive to charge a lower price in period two. However, it fails to alter firm B’s incentives. In contrast, A’s price-matching guarantee reduces B’s incentive to charge a low price in period two, but fails to alter A’s incentives. By combining the two promises, firm A can alter both its own and its rival’s incentives, thereby reducing the intensity of the firms’ price competition in period two.
3.1 Solving for the equilibrium

Turning to firm A’s first-period pricing incentives when it has an MFC clause, it is helpful to begin by establishing two lemmata, one relating to whether refunds will be offered in period two, and the other bounding the range of first-period prices that can be supported in equilibrium for any $P_{B1}$.

**Lemma 1** In any equilibrium of the game in which firm A has an MFC clause, it must be that $P_{A2} = P_{A1}$ whether or not firm A offers a price-matching guarantee in period two.

**Proof.** See Appendix A. ■

Lemma 1 implies that it is never optimal for firm A to give refunds in period two to first-period customers. This means that, in any equilibrium, A’s second-period best reply must intersect B’s best reply where $P_{A2} = P_{A1}$, and thus A is effectively setting its second-period price in period one.

One might think, therefore, that firm A can induce any outcome along firm B’s best reply in period two by choosing $P_{A1}$ appropriately, and that it is constrained, for example, from reaching the Stackelberg outcome in period two only because of the loss in profit it would suffer in period one from choosing $P_{A1} > P_{B}$.\(^{12}\) However, as we now show, this reasoning is only partially true.

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\[^{12}\text{Neilson and Winter (1992) have shown that equilibrium prices with MFCs are bounded above by } \mathbf{P}^*\text{.}

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Consider the claim that firm A can induce any outcome it wants along firm B’s best reply in period two. Figure 5 is illustrative. In this case, there is an equilibrium in period two in which firm A chooses $P_{A2} = P_{A1} = \tilde{P}_A$ and firm B chooses $\hat{P}_B$ or $\bar{P}_B$ depending on whether firm A has a price-matching guarantee. For example, if firm A does not have a guarantee, then the equilibrium is $(\tilde{P}_A, \hat{P}_B)$, which occurs at the intersection of $BR_{A2}^{pen}(P_{B2}; P_{A1})$, $BR_{B2}(P_{A2})$, and $P_{A2} = P_{A1} = \tilde{P}_A$.

Could firm A have induced instead an outcome in which $P_B \leq P_{A2} \leq \tilde{P}_A$? The answer is yes because firm A can choose $P_{A1}$ in this range, thus ensuring that $P_{A2} = P_{A1}$. This follows because for $P_{A1} < \tilde{P}_A$, $BR_{B2}(P_{A2})$ intersects $P_{A2} = P_{A1}$ before $BR_{B2}(P_{A2})$ intersects $BR_{A2}^{pen}(P_{B2}; P_{A1})$ (the line $P_{A2} = P_{A1}$ shifts to the left relative to the diagram, while the curve $BR_{A2}^{pen}(P_{B2}; P_{A1})$ shifts to the right relative to the diagram). But notice that A could not have induced an outcome in which $P_{A2} > \tilde{P}_A$ because, for any $P_{A1} > \bar{P}_A$, $BR_{B2}(P_{A2})$ intersects $BR_{A2}^{pen}(P_{B2}; P_{A1})$ before $BR_{B2}(P_{A2})$ intersects $P_{A2} = P_{A1}$. This establishes that $\bar{P}_A$ is the upper bound on the price that firm A may profitably choose in period one when it does not have a price-matching guarantee in period two.

Now suppose A has a price-matching guarantee. Then the upper bound on the price that A may profitably choose in period one exceeds $\bar{P}_A$. This follows because at $P_{A1} = \tilde{P}_A$, $BR_{B2}^{pen}(P_{A2})$ intersects $P_{A2} = P_{A1}$ before it intersects $BR_{A2}^{pen}(P_{B2}; P_{A1})$, which means that for a small enough increase in $P_{A1}$, $BR_{B2}^{pen}(P_{A2})$ will continue to intersect $P_{A2} = P_{A1}$ before it intersects $BR_{A2}^{pen}(P_{B2}; P_{A1})$.

We can summarize this discussion as follows:

**Lemma 2** Let $\bar{P}_A$ be the upper bound on the price that firm A may profitably choose in period one when it has an MFC clause in period one but does not anticipate having a price-matching guarantee in period two, and let $\tilde{P}_A$ be the corresponding upper bound on the price that firm A may profitably choose when it does anticipate having a price-matching guarantee in period two. Then, $\bar{P}_A > \tilde{P}_A$.

Lemma 2 is a useful result in that it establishes that the set of prices over which firm A maximizes its profit in period one, such that it will not want to offer a refund in period two, is larger when firm A anticipates having a price-matching guarantee in period two than when it does not. It is a direct consequence of B’s best reply under price-matching by A being weakly above its best reply when A does not engage in price matching, and the fact that $BR_{A2}^{pen}(P_{B2}; P_{A1})$ is upward sloping.

Using Lemmata 1 and 2, we can solve for A’s constrained profit-maximizing choice of $P_{A1}$ for any given $P_{B1}$. In the benchmark case in which firm A has no MFC clause in period one, the two periods are independent and A’s optimal choice in period one is given by $P_{A1} = BR_{A1}(P_{B1})$. 

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In the case in which firm A has an MFC clause in period one but does not anticipate having a price-matching guarantee in period two, A’s problem is to choose $P_{A1}$ to solve

$$\max_{0 \leq P_{A1} \leq \bar{P}_A} \left[ P_{A1} \cdot D_A(P_{A1}, P_{B1}) + P_{A1} \cdot D_A(P_{A1}, BR_{B2}(P_{A1})) \right].$$

(6)

In this case, it is easy to see that A’s profit-maximizing choice of $P_{A1}$ depends on the sum of the ‘Bertrand’ profits it receives in period one and the ‘Stackelberg’ profits it receives in period two.

In the case in which firm A has an MFC clause in period one and also anticipates having a price-matching guarantee in period two, A’s problem is to choose $P_{A1}$ to solve

$$\max_{0 \leq P_{A1} \leq \bar{P}_A} \left[ P_{A1} \cdot D_A(P_{A1}, P_{B1}) + P_{A1} \cdot D_A(P_{A1}, P_{A1}) \right].$$

(7)

In this case, A’s profit-maximizing choice of $P_{A1}$ depends on the sum of the ‘Bertrand’ profits it receives in period one and its share of the ‘monopoly’ profits it receives in period two.

Let $P_{A1} = BR_{A1}^{MFC}(P_{B1})$ denote the solution to (6), and let $P_{A1} = BR_{A1}^{Both}(P_{B1})$ denote the solution to (7). Then, it is easy to show that $BR_{A1}^{MFC}(P_{B1}) > P^B$ and $BR_{A1}^{Both}(P_{B1}) > P^B$ for all $P_{B1} \geq P^B$. Since B’s best reply in period one is upward sloping, it follows that $P_{A1} > P^B$ in all equilibria. This implies that having an MFC clause in period one is strictly profitable for A (otherwise, without an MFC clause, firm A would earn Bertrand profits in both periods), and thus, from Proposition 1, that A will offer a price-matching guarantee in period two in all equilibria.

It remains to consider whether firm A will also want to offer a price-matching guarantee in period one. If A does not offer a price-matching guarantee in period one, then equilibrium prices in period one are given by the intersection of A’s best response, $BR_{A1}^{Both}(P_{B1})$, and B’s best response, $BR_{B1}(P_{A1})$. On the other hand, if A does offer a price-matching guarantee in period one, then equilibrium prices in period one are given by the intersection of $BR_{A1}^{Both}(P_{B1})$ and $BR_{B1}^{PM}(P_{A1})$, where $BR_{B1}^{PM}(P_{A1})$ is defined analogously to $BR_{B2}^{PM}(P_{A2})$. Note that since A’s best reply given $P_{B1}$ is the same in both cases, and since $BR_{B1}^{PM}(P_{A1}) > BR_{B1}(P_{A1})$ for all $P_{A1} > P^B$, it follows that it is strictly profitable for firm A also to offer a price-matching guarantee in period one.

We can summarize our main result in this subsection as follows:

**Proposition 2** In all equilibria, firm A offers an MFC clause and a price-matching guarantee in period one, and a price-matching guarantee in period two.\(^\text{13}\)

\(^{13}\)Offering an MFC clause in period two is, of course, of no consequence to firm A since the game lasts only two periods. With more than two periods, we would expect firm A also to offer an MFC clause in period two.
This completes one of the main tasks we set out to accomplish. In reality, one often observes firms offering both price-matching guarantees and MFC clauses in a single promise to consumers. Previous literature has noted that each promise can facilitate higher prices but has considered these promises separately. Our analysis suggests, however, that a strategy of offering both can be more profitable, and that the joint effect on prices may exceed what either could have induced alone.

3.2 Comparing equilibrium prices and profits

We now turn to a comparison of equilibrium prices and profits. We have shown that it is profitable for A to offer an MFC clause in period one, as this allows it to commit credibly to a first-period price that exceeds $P_B$. And we have shown that it is profitable for A to offer price-matching guarantees in both periods, as this softens B’s incentives to compete in each period, conditional on A in fact having a first-period price that exceeds $P_B$. Since Lemma 1 implies that firm A’s price in period two will equal its price in period one, these observations establish that all prices will exceed $P_B$.

What we have not yet established, however, is whether A’s price-matching guarantee in period two does more than simply induce B to charge a higher price for a given $P_{A2}$. For example, will it also affect A’s first-period choice? Relatedly, we have not yet established whether A’s offering of price-matching guarantees in both periods in addition to its MFC clause leads to higher or lower equilibrium prices for both firms than would have occurred had A only offered an MFC clause.

To address these questions, it is useful to consider how firm A’s best reply in period one is affected by its second-period guarantee. For any $P_{B1}$, let $P_{A1} = P_A^{MFC}(P_{B1})$ solve the unconstrained first-order condition to A’s maximization problem in (6). Then it follows that

$$BR_{A1}^{MFC}(P_{B1}) = \min\{\bar{P}_A, P_A^{MFC}(P_{B1})\}.$$

Now let $P_{A1} = P_A^{Both}(P_{B1})$ solve the first-order condition to A’s problem in (7):

$$P_{A1} \cdot \frac{\partial D_A(P_{A1}, P_{B1})}{\partial P_{A1}} + D_A(P_{A1}, P_{B1}) + P_{A1} \cdot \frac{\partial D_A(P_{A1}, P_{A1})}{\partial P_{A2}} + P_{A1} \cdot \frac{\partial D_A(P_{A1}, P_{A1})}{\partial P_{B2}} + D_A(P_{A1}, P_{A1}) = 0. \quad (8)$$

Evaluating the left-hand-side of (8) at $P_{A1} = P_A^{MFC}(P_{B1})$, and using the definition of $P_A^{MFC}(P_{B1})$ and Assumption 3, we find that the left-hand side is positive. Given that A’s profit in (7) is concave from our assumptions, this result implies that $P_A^{Both}(P_{B1}) > P_A^{MFC}(P_{B1})$ for all $P_{B1} \geq 0$. Since

$$BR_{A1}^{Both}(P_{B1}) = \min\{\bar{P}_A, P_A^{Both}(P_{B1})\},$$

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it then follows immediately from Lemma 2 and the fact that $P_{A}^{Both}(P_{B1}) > P_{A}^{MFC}(P_{B1})$, that

$$BR_{A1}^{Both}(P_{B1}) > BR_{A1}^{MFC}(P_{B1}).$$

This condition is sufficient to establish that prices are higher in period one (than if A only had an MFC clause) because B’s best reply is upward sloping. And since A and B’s period-two prices are the same as in period-one, prices are also higher in the second period. Thus, all prices (and profits) are higher when A combines an MFC clause with price-matching guarantees than when A only has an MFC clause. Since in both cases prices exceed Bertrand levels, we have shown the following:

**Proposition 3** Equilibrium prices and profits are strictly higher for both firms than they would have been had firm A only offered an MFC clause in period one, only offered a price-matching guarantee in each period, or offered neither a price-matching guarantee or an MFC clause.

Proposition 3 implies that A’s price-matching guarantees do more than simply dampen B’s incentive to charge low prices (recall that in the absence of A’s MFC clause, this dampening by itself does not lead to higher prices)—they also increase A’s incentive at the margin to charge a higher price in period one. The prospect of earning a higher margin in period two shifts out A’s best reply in period one (for any $P_{B1}$, firm A wants to have a higher price), and this is then reinforced in equilibrium by A’s first-period price-matching guarantee, which softens competition with B.

### 3.3 Linear Example

To get a sense of how much prices can rise when MFC clauses and price-matching guarantees are jointly offered, we let demand in each period be given by

$$D_{i}(P_{it}, P_{jt}) = \alpha - \beta \cdot P_{it} + \gamma \cdot P_{jt}, \quad i, j = A, B, \quad i \neq j,$$

where $\alpha$ is the intercept term, $\beta$ is a measure of own-price sensitivity, and $\gamma$ is a measure of cross-price sensitivity. Setting up the two-period profits in each case and solving, we derive in Appendix B the Bertrand, Stackelberg, and monopoly prices, as well as A’s equilibrium price, and the price that A would charge if it only offered an MFC clause. These prices can be ranked as follows:

**Lemma 3** For all $\beta > \gamma > 0$, firm A’s prices in the various cases can be ranked as follows:

$$P^{M} > P_{A}^{Both} > P_{A}^{S} > P_{A}^{MFC} > P^{B}.$$
Proof. See Appendix B. ■

Thus, firm A’s price when it only has an MFC clause exceeds its Bertrand price but, as Neilson and Winter (1992) show in general, is below the Stackelberg-leader price. On the other hand, A’s equilibrium price (i.e., when A offers an MFC clause and a price-matching guarantee in period one, and a price-matching guarantee in period two) exceeds the Stackelberg-leader price but falls short of the monopoly price. Figure 6 illustrates the various rankings based on a numerical example.

![Diagram of price points](image)

Parameter values: \( \alpha = 1; \beta = 1; \gamma = 3/4 \)

Figure 6: Numerical example based on linear demand

The numerical example provides visual confirmation of the intuition that the power of an MFC clause to induce higher prices by itself may be quite limited whereas the combined power of MFC clauses and price-matching guarantees may be considerable even though they stop short of yielding monopoly prices. Firm A’s prices fall short of the Stackelberg-leader price and the monopoly price, respectively, because the power of an MFC clause does not fully take effect until period two. As the number of periods increases, we would expect A’s MFC-only price to approach the Stackelberg-leader price in the limit and A’s equilibrium price to approach the monopoly price in the limit. Thus, as we discuss more fully in the next section, the facilitating power of combining an MFC clause with price-matching guarantees is most likely understated when there are only two-periods.

4 Discussion and Extensions

The classical papers beginning with Salop (1986) demonstrate that in a symmetric, duopoly setting, if both firms adopt price-matching guarantees, then all prices between the Bertrand prices and the monopoly prices can be supported in equilibrium. Three criticisms have been raised against the theory, namely that it requires both firms to offer price-matching guarantees; that it requires some equilibrium refinement to predict the actual outcome given the multiplicity of equilibria; and that, as shown in Hviid and Shaffer (1999), even infinitesimally small costs of invoking the guarantees
destroy any ability of the guarantees to raise prices above the Bertrand prices.

As an empirical matter, there are certainly industry sectors in which adoption is not universal. For example, the analysis in Arbatskaya et al (2006), which tests whether observed prices are consistent with an aim of low-price guarantees to facilitate higher prices, relies on non-universal adoption in the retail-tire market. Also, universal adoption requires some degree of tacit coordination among firms, particularly when it comes to the details of the guarantees, and as demonstrated in Arbatskaya et al (2004), these details may matter greatly for the effectiveness of the guarantees.¹⁴

There may also be a legal distinction between a single-firm adopting price guarantees and price guarantees that are adopted by all firms in the industry. In the former case antitrust law cannot currently reach the guarantees even if they were deemed to be anti-competitive. This is so in both the US (see Gavil et al, 2008, p 344) and in the EU. In the latter case, it may be possible to argue in the EU that the guarantees are part of a concerted (parallel) action which in theory could be remedied, in particular if the aims of the unilateral conduct could not be achieved without the participation of others (see Odudu, 2006, p. 65). The unilateral adoption by a single firm of both a price-matching guarantee and an MFC clause thus has a distinct advantage over the universal adoption of only price-matching guarantees in that in the former case no coordination among firms is required about the details of the guarantee itself, nor do the firms run any antitrust risks.

The concerns over multiple equilibria are well known and have been eloquently expressed in Edlin (1997).¹⁵ While one could argue that the monopoly price, which under symmetry is unique and the same for both firms, may well be a focal point, this still relies on a refinement to pick the particular equilibrium. Moreover, there is nothing in the standard story to explain how the adoption of a price-matching guarantee drives up prices. Indeed the guarantee can be seen as insurance in case no one else follows a price increase. This is in contrast to the unilateral adoption of an MFC clause, alone or with a price-matching guarantee, where the adopter has a unilateral reason for raising its price even if no rivals were to follow. Moreover the equilibrium price is unique.

Yet another critique of the standard Salop story is that it implicitly assumes that consumers face no hassle costs when activating a guarantee.¹⁶ To see why this matters, assume that both

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¹⁴ The view that firms can easily copy each other’s price guarantee is undermined by guarantees that require the customer to see or ask “in store for details” even in cases where providing such details would be virtually costless. For example Plush, an Australian sofa retailer advertises on the web “Lowest Price Guarantee terms, conditions and limitations apply. See in-store for full details.” http://www.plushleather.com.au/html/terms.php.

¹⁵ The problem is that in the simple Salop story in which both (all) firms adopt price-matching guarantees, any price between the firms’ marginal cost and the monopoly price can be supported in a subgame-perfect equilibrium.

¹⁶ Arbatskaya et al. (2004) provide examples of the source of such costs. Restrictions whose role is to reduce
firms have price-matching guarantees and let \( z > 0 \) be the cost to a consumer of activating a guarantee. The consequence of this is that for \( P_{it} - P_{jt} \in [0, z] \), firm \( i \)'s price-matching guarantee is not triggered. We illustrate this in Figure 7, where the lines \( P_{B2} = P_{A2} - z \) and \( P_{B2} = P_{A2} + z \) are added to the usual Bertrand best-reply diagram. Compared to Figure 2, the part of firm \( i \)'s best reply that was on the 45 degree line, is now parallel to it but off-set by an amount \( z \).

![Figure 7: Price-matching guarantees and hassle costs](image)

It is evident that the best replies only intersect at the Bertrand price pair. Intuitively, at any price pair above the Bertrand prices, at least one firm can find a deviation that will take it toward its Bertrand-best reply without triggering the other firm’s price-matching guarantee. Hence, positive hassle costs, however small, have the effect of eliminating the ability of price matching guarantees on their own, and when adopted by both firms, to increase prices above the Bertrand prices.

Potential risks to the firm from the actions of either naive or devious rivals are common as are restrictions that are caused by the difficulty of firms to monitor rivals’ prices to ensure that guarantees are not activated. Some of these restrictions have to do with the product actually being available at the rival store and that it is really the same good. Other restrictions have to do with the proximity of rivals, where the further apart the firms are, the more difficult it is to monitor each other’s prices and also the more likely the outlets are to be affected by different price shocks.
4.1 The effect of hassle costs

In this subsection, we consider the robustness of our analysis to the introduction of hassle costs. First, consider the effect of a small amount of hassle costs on an MFC clause. For simplicity, assume that the hassle costs of invoking an MFC clause is $z > 0$, the same as for invoking a price-matching guarantee.\footnote{In reality one might expect hassle costs to be lower for activating an MFC clause, since firms should be able to assess more easily whether a consumers’ request for a refund is warranted. For a variety of reasons, consumers are also less likely to be tolerant of high levels of hassle costs on MFC clauses than they are on price-matching guarantees.} This implies that for all $P_{A1}, P_{A2}$, if $P_{A1} - P_{A2} \leq z$ a first-period consumer would not be willing to request a refund in period two. Thus, the vertical part of the best reply in Figure 3 is moved to the left by an amount $z$. Otherwise, A’s best reply is unchanged, as illustrated below.

![Figure 8: Most-favored-customer clauses and hassle costs](image)

As Figure 8 illustrates, equilibrium prices will be lower in period two for any $P_{A1} > P^B$ that firm A commits to in period one, when consumers incur hassle costs. Although this dampens the ability of MFC clauses to act as commitment devices, the effect is not nearly as dramatic as in the case of price-matching guarantees only, as equilibrium prices will still be above Bertrand prices.

The same is true for the overall equilibrium in which firm A offers both a price-matching guarantee in each period and an MFC clause in period one, as illustrated in Figure 9.

In Figure 9, we have chosen the same $P_{A1}$ as in Figure 5 to get an understanding of how a small
amount of hassle costs will affect the equilibrium. Compared to Figure 5 the two relevant best replies have been partly displaced. Firm A’s best reply has had its vertical part shifted to the left by an amount \(z\), so that given \(P_{A1}\), the corresponding period-two price is \(P_{A2} = P_{A1} - z\). This will have the effect of reducing the price that can be supported slightly irrespective of whether or not firm A also offers a price guarantee. Note that the effect is not as strong as indicated in the figure because firm A has an incentive to increase \(P_{A1}\) so that \(P_{A2}\) is increased towards the intersection of \(BR_{A2}(P_{A2})\) and \(BR_{A2}^{pen}(P_{A2}; P_{A1})\). However this increase is less than \(z\), because increasing \(P_{A1}\) shifts \(BR_{A2}^{pen}(P_{A2}; P_{A1})\) to the left. Secondly, B’s best reply is shifted down to reflect its ability to undercut A by \(z\) without activating A’s guarantee. This reduces B’s equilibrium price in the case in which A has also adopted a guarantee, but only by the amount of the hassle cost. Thus, the overall effect of a marginal increase in \(z\) from any point, but in particular from zero is to reduce equilibrium prices marginally. One does not see the dramatic effect illustrated in Figure 7 above.

A second implication of hassle costs is that, in equilibrium, B’s prices are lower than A’s prices (because B has the ability to undercut A’s prices by an amount \(z > 0\) without activating A’s guarantee). This means that even if firm B were to offer a price-matching guarantee in each period, its guarantees would be redundant. As pointed out by Neilson and Winter (1993), the same is true.
for MFC clauses, as at most one firm can be a price leader. Thus, our focus on the case of single-firm adoption is without loss of generality if one allows for positive hassle costs, as summarized below.

**Proposition 4** With non-zero hassle costs, only one firm’s set of guarantees will provide an active restraint on price setting. Any guarantee adopted by the other firm will be redundant in equilibrium.

### 4.2 Adding more periods

Some firms have had MFC clauses for a considerable period of time and thus it is worth speculating on the effects of adding more periods. Suppose that instead of two periods, there are \( n > 2 \) periods. In the last period, it must still be true that firm A (the firm with the MFC clause) will not deviate from \( P_{An} = P_{An-1} \). Thus, the relevant upper bound on price identified in Lemma 2 must still be respected. If it is, it must also be respected in the penultimate period, in which case \( P_{An-1} = P_{An-2} \), and so on. The objective function of firm A in period 1 if A only has an MFC clause is given by

\[
\max_{0 \leq P_{A1} \leq P_A} \left[ P_{A1} \cdot D_A \left( P_{A1}, P_{B1} \right) + (n - 1) \cdot P_{A1} \cdot D_A \left( P_{A1}, BRB_2 \left( P_{A1} \right) \right) \right],
\]

while its objective function if it has both an MFC clause and a price-matching guarantee is

\[
\max_{0 \leq P_{A1} \leq \bar{P}_A} \left[ P_{A1} \cdot D_A \left( P_{A1}, P_{B1} \right) + (n - 1) \cdot P_{A1} \cdot D_A \left( P_{A1}, P_{A1} \right) \right]
\]

Consider first A’s problem in (9). Ignore for the moment the constraint on prices, \( P_{A1} \leq \bar{P}_A \). It is evident that the first-order condition will be a convex combination of a Bertrand and a Stackelberg first-order condition with the weight on the latter increasing in \( n \). Thus, unless the constraint binds, A’s period-one price will get arbitrarily close to the Stackelberg-leader price but never exceed it. The only remaining issue is whether \( \bar{P}_A \) is above or below the Stackelberg leader price, \( P^S_A \).

Turning to A’s problem in (10), the same reasoning as above implies that as \( n \) increases, A’s optimal period-one price (and hence all prices) will get closer and closer to the monopoly price. Only the constraint that firm A must not want to deviate in the last period (that is, \( P_{A1} \leq \bar{P}_A \)), may prevent this. We now show that \( \bar{P}_A \) may be larger or smaller than the monopoly price, \( P^M \).

**Lemma 4** The monopoly price, \( P^M \), is less than (greater than) the upper bound on the period \( n \) price, \( \bar{P}_A \), if the cross-price elasticity of demand evaluated at \( P^M \) is less than (greater than) unity.

**Proof.** See appendix A. ■

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When $\tilde{P}_A > P^M$, the constraint that firm A must not want to deviate in the last period by lowering its price is always slack, and with enough periods, A’s period-one price will get arbitrarily close to the monopoly price but never exceed it. We can summarize this result as follows.

**Proposition 5** The equilibrium prices are increasing in the number of periods in which firm A has adopted an MFC clause. Where the cross-price elasticity of demand, evaluated at the monopoly price, is less than unity, the equilibrium prices can get arbitrarily close to the monopoly price.

Note that what matters is the number of periods in which firm A offers an MFC clause, and not the number of periods for which an MFC clause offered to a particular consumer remains valid.

## 5 Conclusion

This paper has demonstrated how the unilateral adoption of a pricing policy that covers not just rivals’ but also own future price differences is capable of raising prices above the Bertrand level. This offers a competition-related motivation for the joint adoption of these guarantees observed in a large number of cases. Moreover, we have shown that the result is robust to the presence of hassle costs, and that as the number of periods increase, equilibrium prices may become arbitrarily close to the monopoly prices. A further advantage of these joint guarantees is that they lead to uniqueness and hence predictability. A firm that adopts a price-matching guarantee with most-favored customer protection clearly signals to its rival that it is about to raise its price above the Bertrand level. Thus, the adoption is an invitation to the other firm of following it in raising prices.

The underlying intuition for our results is that a firm’s MFC clause essentially provides a substitute for rivals’ price-matching guarantees. With this in mind, it is clear that the main results will carry over to more than two firms. Building on the insights in Hviid and Shaffer (1999), we would also expect the model to be qualitatively robust to small degrees of asymmetry in demand.

From a competition-policy perspective, the results in this paper present a worry. Since the high prices that are facilitated result from the unilateral adoption of a guarantee to consumers, and is not necessarily carried out by a dominant firm, no conventional competition law can reach this.

From an empirical perspective, this paper adds weight to the argument that to get a clearer assessment of the anti-competitive effects of the various forms of price promises, data on sectoral patterns of adoption combined with the precise details of the promises themselves is needed. Both price-matching guarantees and most-favored-customer clauses can be used by firms as means to
different ends. The precise details of the guarantees, including any restrictions imposed on them, together with information of patterns of adoption would, even absent information about prices or the incidence of activation by customers, enable some discrimination between competing theories.
Appendix A

Proof of Proposition 1: At the beginning of period two, firm A’s MFC clause and $P_{A1}$ are exogenous, and the decision facing firm A is whether to offer a price-matching guarantee. If A does not offer a price-matching guarantee, then the equilibrium prices in period two are given by the intersection of A’s best response, $BR_{A2}^{MFC}(P_{B2}; P_{A1})$, and B’s best response, $BR_{B2}(P_{A2})$. On the other hand, if A does offer a price-matching guarantee at the start of period two, then the equilibrium prices in period two are given by the intersection of $BR_{A2}^{MFC}(P_{B2}; P_{A1})$ and $BR_{B2}^{PM}(P_{A2})$.

Note that for all $P_{A1} > P^B$, both intersections occur at prices for which $P_{A2} > P^B$. Since A’s best reply given $P_{B2}$ is the same in both cases, and since $BR_{B2}^{PM}(P_{A2}) > BR_{B2}(P_{A2})$ for all $P_{A2} > P^B$, it follows that if $P_{A1} > P^B$, then firm A is better off having a price-matching guarantee. Q.E.D

Proof of Lemma 1: Suppose an equilibrium of the game exists where firm A has an MFC clause and $P_{A2} \neq P_{A1}$, and consider first the case in which firm A does not have a price-matching guarantee in period two. Then there are two subcases to consider: either $P_{A2} > P_{A1}$ or $P_{A2} < P_{A1}$.

If $P_{A2} > P_{A1}$, then it must be that the intersection of $BR_{B2}(P_{A2})$ and $BR_{A2}^{MFC}(P_{B2}; P_{A1})$ occurs where $BR_{B2}(P_{A2})$ and $BR_{A2}(P_{B2})$ intersect. Since these latter two curves intersect only at $(P^B, P^B)$, it follows that $P_{A2} = P^B$ and hence $P_{A1} < P^B$. But if $P_{A1} < P^B$, then at least one firm will have a profitable deviation in period one. Hence there is no equilibrium in which $P_{A2} > P_{A1}$.

If $P_{A2} < P_{A1}$, then it must be that the intersection of $BR_{B2}(P_{A2})$ and $BR_{A2}^{MFC}(P_{B2}; P_{A1})$ occurs where $BR_{B2}(P_{A2})$ and $BR_{A2}^{gen}(P_{B2}; P_{A1})$ intersect. In this case, firm A’s profit is

$$\hat{P}_{A1}D_A(\hat{P}_{A1}, \hat{P}_{B1}) + \hat{P}_{A2}D_A(\hat{P}_{A2}, \hat{P}_{B2}) - \left(\hat{P}_{A1} - \hat{P}_{A2}\right)D_A(\hat{P}_{A1}, \hat{P}_{B1}),$$

where the superscript ‘hat’ denotes equilibrium quantities. This expression simplifies to

$$\hat{P}_{A2}D_A(\hat{P}_{A1}, \hat{P}_{B1}) + \hat{P}_{A2}D_A(\hat{P}_{A2}, \hat{P}_{B2}).$$

(11)

Now consider a deviation in which firm A chooses $P_{A1} = \hat{P}_{A2}$ and note that $\frac{\partial BR_{A2}^{gen}(P_{B2}; P_{A1})}{\partial P_{A1}} < 0$, which implies that $BR_{A2}^{gen}(P_{B2}; \hat{P}_{A2}) > BR_{A2}^{gen}(P_{B2}; \hat{P}_{A1})$. It follows that the equilibrium quantities in period two will be unchanged, and thus that A’s profit under the deviation is

$$\hat{P}_{A2}D_A(\hat{P}_{A2}, \hat{P}_{B1}) + \hat{P}_{A2}D_A(\hat{P}_{A2}, \hat{P}_{B2}).$$

(12)

Comparing (11) and (12), it is easy to see that A’s profit is higher under the proposed deviation because it has higher first-period sales in (12). Hence there is no equilibrium in which $P_{A2} < P_{A1}$.
The case in which firm A has a price-matching guarantee in period two is analogous.  

**Proof of Lemma 4.** Recall that $P^M$ is the solution to (3), while $\tilde{P}_A$ solves (5) when evaluated at $P_{A2} = P_{A1}$ and $P_{A2} = P_{B2}$. That is, $\tilde{P}_A$ solves

$$D_A(P_{A2}, P_{A2}) + P_{A2} \cdot \frac{\partial D_A(P_{A2}, P_{A2})}{\partial P_{A2}} + D_A(P_{A2}, P_{B1}) = 0.$$ 

Evaluating the left-hand-side at $P_{A2} = P^M$, we get

$$\xi(P^M, P^M) \equiv D_A(P^M, P^M) + P^M \cdot \frac{\partial D_A(P^M, P^M)}{\partial P_{A2}} + D_A(P^M, P_{B1}). \quad (13)$$

Using that (3) holds, we get

$$\xi(P^M, P^M) = D_A(P^M, P_{B1}) - P^M \cdot \frac{\partial D_A(P^M, P^M)}{\partial P_{B2}}.$$ 

Note that

$$\text{Sign} \{ \xi(P^M, P^M) \} = \text{Sign} \left\{ \frac{D_A(P^M, P_{B1})}{D_A(P^M, P^M)} - \frac{P^M}{D_A(P^M, P^M)} \cdot \frac{\partial D_A(P^M, P^M)}{\partial P_{B2}} \right\}.$$ 

The first term is unity at $P_{B1} = P_m$ and the second term is the cross-price elasticity of demand at $P^M$, which may be greater or less than one. Where $\xi(P^M, P^M)$ is positive, $\tilde{P}_A > P^M$.  

**Q.E.D.**
Appendix B

Let demand in a given period be given by

\[ D_i(P_i, P_j) = \alpha - \beta \cdot P_i + \gamma \cdot P_j, \]

and normalize costs to zero. The Bertrand-best reply of firm \( i \) is found by maximizing \( P_i \cdot D_i(P_i, P_j) \) with respect to \( P_i \), and is given implicitly by

\[ \alpha - 2\beta \cdot P_i + \gamma \cdot P_i = 0, \]

from which we get

\[ BR_i^B(P_j) = \frac{\alpha}{2\beta} + \frac{\gamma}{2\beta} \cdot P_j. \]  \hspace{1cm} (14)

The key prices needed for our comparisons are the Bertrand, Stackelberg, and monopoly prices. In Bertrand, the best replies are given by (14) and the equilibrium price is given by:

\[ P^B = \frac{1}{2\beta - \gamma} \cdot \alpha. \]

In Stackelberg, the leader maximizes \( P_i \cdot D_i(P_i, BR_j^B(P_i)) \) with respect to \( P_i \), yielding

\[ P^S_A = \frac{2\beta + \gamma}{4\beta^2 - 2\gamma^2} \cdot \alpha. \]

Finally, the monopoly price is found by maximizing \( P_i \cdot D_i(P_i, P_i) \) and is given by:

\[ P^M = \frac{1}{2(\beta - \gamma)} \cdot \alpha. \]

**Period two**

Assume that firm \( A \) adopts an MFC. If \( P_{A1} > P_{A2} \), the period-two profits of firm \( A \) are given by

\[ P_{A2} \cdot (\alpha - \beta \cdot P_{A2} + \gamma \cdot P_{B2}) - (P_{A1} - P_{A2}) \cdot (\alpha - \beta \cdot P_{A1} + \gamma \cdot P_{B1}), \]

for which the first-order condition is

\[ \alpha - 2\beta \cdot P_{A2} + \gamma \cdot P_{B2} + (\alpha - \beta \cdot P_{A1} + \gamma \cdot P_{B1}) = 0. \]

The solution gives us \( P_{A2} = BR_A^{Pens}(P_{B2}; P_{A1}) \). Consider the point on this best reply where given the period-one price, the penalty in period two is exactly zero, \( P_{A1} = P_{A2} \),

\[ 2\alpha - 3\beta \cdot P_{A2} + \gamma \cdot P_{B2} + \gamma \cdot P_{B1} = 0. \]
This defines $P_{B2}$ as a function of $P_{A2}$ and $P_{B1}$:

$$P_{B2} = \frac{3\beta}{\gamma} \cdot P_{A2} - \frac{2\alpha}{\gamma} - P_{B1}. \quad (15)$$

Consider first the case where $A$ has an MFC clause but not a price-matching guarantee. In this case, $B$ can get on its best reply given by (14). Solving (15) and (14) we get

$$\bar{P}_A = \frac{(4\beta + \gamma)}{(6\beta^2 - \gamma^2)} \cdot \alpha + \frac{2\beta \gamma}{(6\beta^2 - \gamma^2)} \cdot P_{B1}. \quad (16)$$

This is the highest possible period-two price that can be supported without firm $A$ preferring to set a period-two price that triggers a refund. By Lemma 1, equation (16) thus provides a constraint on the period-one price that can be chosen by firm $A$ in any equilibrium.

Consider secondly the case where firm $A$ has both an MFC clause and a price-matching guarantee. firm $B$ cannot now get on its best reply given by (14), but for $P_{A1} = P_{A2} > P^B$ is constrained to the forty-five degree line given by $P_{B2} = P_{A2}$. Using this in (15), we find

$$\bar{P}_A = \frac{2\alpha}{(3\beta - \gamma)} + \frac{\gamma}{(3\beta - \gamma)} \cdot P_{B1}. \quad (17)$$

**Period one**

In deriving the period-one prices, we initially assume that the no-refund constraint is not violated and then subsequently check that either (16) or (17) is not violated.

In the case where firm $A$ has an MFC clause but not a price-matching guarantee, $P_{A1} = P_{A2}$ and firm $A$ will choose $P_{A1}$ to maximize

$$P_{A1} \cdot (\alpha - \beta \cdot P_{A1} + \gamma \cdot P_{B1}) + P_{A1} \cdot (\alpha - \beta \cdot P_{A1} + \gamma \cdot BR_{B2}(P_{A1})).$$

Using (14), maximization yields the following first-order condition

$$\alpha - 2\beta \cdot P_{A1} + \gamma \cdot P_{B1} + \frac{1}{2\beta} (2\beta + \gamma) \cdot \alpha - 2 \left(2\beta^2 - \gamma^2\right) \cdot P_{A1} = 0.$$

Solving this we get

$$2\beta \gamma \cdot P_{B1} + (4\beta + \gamma) \cdot \alpha - 2 \left(4\beta^2 - \gamma^2\right) \cdot P_{A1} = 0. \quad (18)$$

Solving (18) together with $B$’s best reply given in (14) yields:

$$P_{A}^{MFC} = \frac{(4\beta + 2\gamma)}{(8\beta^2 - 3\gamma^2)} \alpha$$

(19a)
and
\[ P_B^{MFC} = \frac{(8\beta^2 + 4\beta\gamma - \gamma^2)}{2\beta (8\beta^2 - 3\gamma^2)} \alpha. \]  \hspace{1cm} (19b)

To ensure that the constraint given in (16) is not violated, we require that:
\[ P = \frac{(4\beta + \gamma)}{(6\beta^2 - \gamma^2)} \cdot \alpha + \frac{2\beta \gamma}{(6\beta^2 - \gamma^2)} \cdot \frac{(8\beta^2 + 4\beta\gamma - \gamma^2)}{2\beta (8\beta^2 - 3\gamma^2)} \alpha \geq P_A^{MFC}, \]
which holds, confirming that (19a) and (19b) will indeed be the equilibrium prices in both periods if firm A were to adopt an MFC clause but not a price-matching guarantee.

In the case where firm A has an MFC clause in period one and price-matching guarantees in both periods, \( P_{A1} = P_{A2} \) and \( P_{B1} = P_{B2} \) and firm A will choose \( P_{A1} \) to maximize
\[ P_{A1} \cdot (\alpha - \beta \cdot P_{A1} + \gamma \cdot P_{B1}) + P_{A1} \cdot (\alpha - \beta \cdot P_{A1} + \gamma \cdot P_{A1}), \]
with corresponding first-order condition
\[ \alpha - 2\beta \cdot P_A + \gamma \cdot P_B + (\alpha - 2\beta \cdot P_A + 2\gamma \cdot P_A) = 0. \]

The equilibrium is found by setting \( P_{A1} = P_{B1} \) and the solution is given by:
\[ P_A^{Both} = P_B^{Both} = \frac{2\alpha}{(4\beta - 3\gamma)}. \]  \hspace{1cm} (20)

It is easily verified that the constraint given in (17) is not violated, as the following holds
\[ \bar{P} = \frac{2\alpha}{(3\beta - \gamma)} + \frac{\gamma}{(3\beta - \gamma)} \cdot \frac{2\alpha}{(4\beta - 3\gamma)} \geq P_A^{Both}. \]

This confirms that the equilibrium prices in both periods will indeed be given by (20) if A were to adopt an MFC clause in period one and price-matching guarantees in both periods.

To verify that Lemma 3 holds, note first that \( P_A^{Both} < P_M \) as \( \frac{2}{(4\beta - 3\gamma)} \alpha < \frac{1}{2(\beta - \gamma)} \alpha \). Note second that \( P_A^{Both} > P_A^S \) as \( \frac{2}{(4\beta - 3\gamma)} \alpha > \frac{2\beta + \gamma}{4\beta^2 - 2\gamma} \alpha \). Note third that \( P_A^{MFC} < P_A^S \) as \( \frac{(4\beta + 2\gamma)}{(8\beta^2 - 3\gamma^2)} \alpha < \frac{2\beta + \gamma}{4\beta^2 - 2\gamma} \alpha \).

Note finally that \( P_A^{MFC} > P_B \) follows from \( \frac{(4\beta + 2\gamma)}{(8\beta^2 - 3\gamma^2)} \alpha > \frac{1}{4\beta - \gamma} \alpha \).

The numerical example confirms this. For \( \alpha = 1, \beta = 1, \gamma = 3/4 \), we find

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BIBLIOGRAPHY


