



# Bargaining over Remedies in Merger Regulation

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**Abstract:** This paper provides a first attempt to understand how outcomes are determined by the standard institutions of merger control. In particular, we focus on the internationally standard 2-phase investigation structure and remedy negotiations of the form practiced by the EC. We find that there are inherent biases in remedy outcomes, and identifiable circumstances where offers will be excessive and where they will be deficient. In particular, we find clear circumstances in which firms offer excessive remedies, which goes against a possible intuition that firms should expect to extract an information rent for possessing superior information about competition in the market.

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Merger regulation is the *ex ante* part of competition policy that aims to prevent the creation of market power by acquisition. In order to appraise the competitive effects, most jurisdictions have adopted a variant of 2-phase investigation for qualifying mergers, with a first phase deciding on whether there is a case for more detailed investigation in a second phase.<sup>1</sup> The central concern is that a merger between firms with overlapping markets may lessen the degree of competition to the detriment of customers, even though other aspects of the acquisition may create the potential for efficiencies. In order to prevent expected abuse of market power due to such a merger, a competition agency usually has the power to prohibit a merger, either administratively or by going to court. Alternatively, it can condition its approval on the merging firms undertaking either to divest some of their assets or to accept a package of behavioural restrictions. Such agreed conditions are known as ‘remedies’.

Examples of remedies include the divestiture of certain products or, in the case of regional markets (or national markets within the EU), product rights in a particular geographic market. Product rights may include a brand, production facility, technology or access rights. In most jurisdictions, remedies can be agreed in either phase of investigation. In particular, both the EU and US merger regulations allow remedies to be agreed in either phase of the investigation. Either *de jure* (as in the EU) or *de facto*, the usual practice is for the firms to make the final remedy offer to the agency, who can either accept or reject it. Rejection in Phase I results in referral to Phase II, and rejection in Phase II results in prohibition.

Conditional approval is an order of magnitude more common as a means of eliminating competitive concerns than is outright prohibition of a merger. For example, the European Commission decided on 3,098 qualifying mergers between September 1990 (i.e. when the first cases were decided under the ECMR (1989)) and the end of 2006. During this time, 140 were remedied in Phase I, 79 were remedied in Phase II, and just 19 were prohibited. In this paper, we model the incentives for merging firms and a competition agency to agree remedies at alternative times during the investigation.

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<sup>1</sup> In the EU, there are explicit Phases I and II, and in the US, the equivalent of Phase II is associated with the issuing of a ‘Second Request’. There is often a prior sifting of cases such that only those of a qualifying combined size (or sometimes market share) reach Phase I. Further sifting may take place if there are clearly no significant market overlaps (e.g. the EC has a ‘simplified procedure’ in lieu of Phase I in such cases; EC (2005)). Such mergers fall outside the context of the present paper because they do not raise issues relating to remedies.

A fundamental problem for the agency is that, at least initially, it has less information than the merging firms do with respect to the conditions of competition in the market. Even if it has sufficient evidence to deduce that a significant lessening of competition would result from the initial merger proposal, it may be difficult to design appropriate remedies; i.e. remedies that adequately address the competition concern without unnecessarily restricting the freedom of firms to seek efficiencies and profitable opportunities. The purpose of an investigation is to discover relevant information, but such inquiries are costly. An immediate problem is that the merging parties do not have the incentive to reveal the truth because they would prefer to keep as many assets as possible, especially those that would enhance market power. Nevertheless, the agency, or whoever designs the rules that the agency has to follow, does have one advantage, in that it can design the framework in which firms have to negotiate.

The essential structure of our model is as follows. The agency would like to find the 'true remedy' that would resolve competition concerns with minimum intervention. As a first approximation, the firm knows this while the agency does not, so it conducts a Phase I investigation to obtain an independent estimate of the true remedy. One determinant of the accuracy of the agency's estimate is the amount of resources that it invests in the investigation (e.g. time, number of people, use of experts). The first phase of investigation is not perfect, so the agency can make a mistake.<sup>2</sup> Appreciating this, it compares the results of its own investigation with any remedy that is offered by the merging firms. If the offer is supported by the results obtained from its own investigation, then the agency approves the merger subject to the proposed remedy. Otherwise, it proceeds into the second phase. In Phase II, the agency conducts a more detailed investigation with more resources (e.g. four months in the EU) and consequently greater accuracy. Such a detailed investigation is costly for both the merging firms and the agency.

The questions that we try to answer are:

- How does the 2-phase inquiry structure affect negotiated outcomes?
- How efficient is the remedy negotiation process in revealing the truth?
- What types of error are more likely and under what conditions?
- Do merging firms get information rents in remedy negotiation?
- Should firms prefer a more or less well resourced agency?

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<sup>2</sup> In the EU, the agency has just one month to appraise the merger in Phase I.

There have been few attempts in the literature to address the issue of bargaining over remedies in merger regulation. Farrell (2003) was one of the first to discuss a bargaining model of remedy negotiation between an antitrust agency and merging parties. He applies Nash bargaining to determine the appropriate welfare standard of a competition agency which values efficiencies due to a merger. Glazer and Rubinstein (2004) look at the underlying issue more generically. They develop a persuasion game model in which one party (a speaker) wishes to persuade another party (a listener) to accept a certain request. The value of the request depends on two parameters. These are known only to the speaker, while the listener can check the value of only one. The authors derive an optimal rule that determines the listener's response: which parameter to check and whether to accept or reject the speaker's request. Although they do not apply their model to remedy negotiation, it is close in structure to the single phase version of our model.<sup>3</sup>

The next section sets out our characterisation of merger remedies and other main modelling assumptions. Our main propositions are in section 2, and section 3 briefly discusses some potential extensions. Section 4 concludes.

## **1. The Model**

### ***a) Sequence of Decision Making***

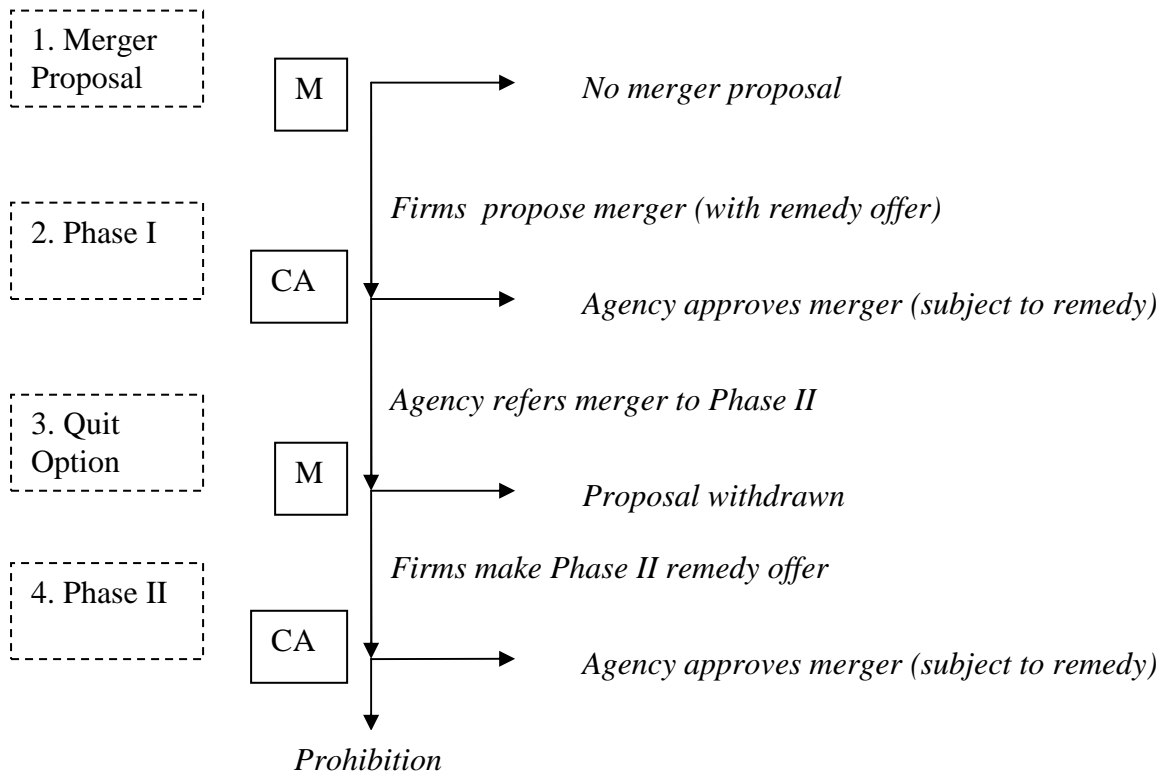
The timing of the game is shown in Figure 1:

- 1) *Merger decision*: firms decide whether to propose a merger in the knowledge that it would enter the merger approval process.
- 2) *Phase I*: the agency conducts a limited investigation of the merger; the firms can, if they wish, propose a remedy package before the agency makes its decision either to approve the merger (subject to any proposed remedy) or to refer the case to a full Phase II investigation.
- 3) *Quit option*: firms can either proceed to Phase II or opt to abandon the merger.
- 4) *Phase II*: the agency further investigates the merger, with more resources and precision; the firms can propose the same or a different remedy before the agency decides whether to approve the merger (subject to the remedy proposed by the merging firms) or to prohibit the merger.

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<sup>3</sup> In a similar vein, the bargaining process we describe could be applied to any situation with asymmetric information between two parties, and where the party with more information wants to retain the maximum possible amount of assets but the party with less information wants to learn and enforce a 'true' or ideal distribution of assets. For example, this may be relevant for certain divorce arrangements where out-of-court settlement would be cheaper but less informed than a fully litigated outcome.

**Figure 1: Sequence of Decisions in Merger Regulation**



Notes to Fig.1: M is an action taken by the merging firms; CA is an action taken by the competition authority.

Our model leaves the design of the remedy entirely in the hands of the merging firms, with the agency holding veto power. This is a reasonable characterisation of the EU system where firms must adhere to strict deadlines in making their final proposals in each Phase (i.e. stages 2 and 4 of the game) before the agency decides whether to accept or reject the offer. However, other institutions can have different negotiation structures. For example, remedy proposals may be left in the hands of the agency, with the firms choosing only to accept or to reject the agency's proposal.<sup>4</sup> In general, this would lead to a different outcome.<sup>5</sup>

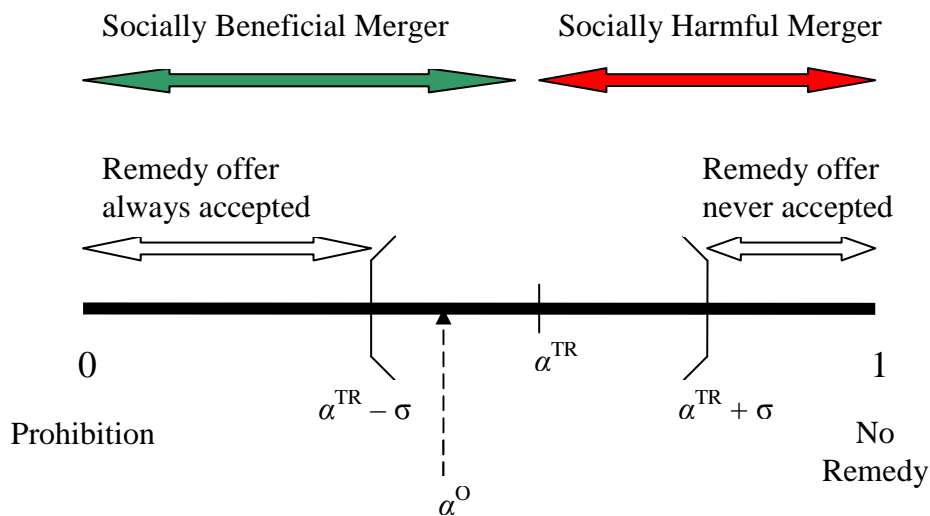
<sup>4</sup> For example, the UK Competition Commission makes final proposals (stage 4 in our game) in this way, with the firms having veto power in the sense that they can walk away from the merger proposal. Note that under either system, there can be communication and discussion about potential remedies and consequences prior to the final offers. In our model, this is subsumed within the agency's general information gathering.

<sup>5</sup> With such powers, the agency might condition its final decision on both the firms' offer and its own investigation. It may also try to restrict the information it receives. At one extreme, it could ignore any remedy proposal coming from the merging firms, and rely entirely on its own investigation. This would lead to different outcomes to our model. Alternatively, if the agency could credibly restrict itself to a choice between the firms' final offer and a prohibition, then this would be effectively the same as in our model.

**b) Characterisation of Remedies**

We suppose that the assets of the acquired firm can be represented by a unit line. Assets are arranged by competitive impact such that those with least competitive impact are located at zero, and those with most harmful competitive impact are located at one. We assume that there is a partition of assets such that those located at  $\alpha < \alpha^{TR}$  have no significantly adverse effect on competition, while  $\alpha > \alpha^{TR}$  would significantly impede competition if joined with the acquiring firm's assets.<sup>6</sup>  $\alpha^{TR} \in (0, 1)$  characterises the 'true' remedy in that if all assets labelled  $\alpha > \alpha^{TR}$  were divested, the merging parties could retain the maximum possible assets that would leave the degree of effective competition unchanged. For example, assets located at  $\alpha < \alpha^{TR}$  may be in non-competing markets, or the merger may lower costs such that customer price and service would be unchanged (or improved) despite the merged firm's larger market share. If the outcome is  $\alpha = 1$ , the merger proceeds without remedy, and if  $\alpha = 0$ , the merger is completely prohibited or abandoned. Any agreed  $\alpha \in (0, 1)$  represents a potential remedy.

**Figure 2: Characterisation of Harm and Phase I Remedy Offer**



Notes to Fig. 2:  $\alpha^{TR}$  is the 'true' remedy (i.e. the remedy that would result in the optimal merger such that the social benefits of the merger are maximised);  $\alpha^O$  is the optimal Phase I remedy offer made by the merging firms;  $\alpha^{TR} + \sigma$  measures the maximum inaccuracy of the agency's Phase I investigation.

<sup>6</sup> We refer to acquiring and acquired firms for clarity of exposition. It does not normally matter which firm's assets are being characterised on the unit line.

This simple characterisation allows us to focus on the negotiation process without having to make specific assumptions about underlying market structures or models of competition. While it is only an approximation to any particular set of circumstances, there are several cases for which this may be reasonable. For example, negotiation over remedies in the Cournot model with capacity constraints. The agency and the merging firms decide on the amount of divested assets ( $\alpha = \text{capacity}$ ) that would resolve competition concerns due to the merger.<sup>7</sup> Alternatively, negotiations may be over a threshold level of market concentration and/or pattern of market shares that would make collusion more difficult (e.g. Motta, 2004). In a differentiated products model of a merger with multiple brands, the divestiture of different portfolios of brands would lead to different market outcomes;  $\alpha$  would represent the content and range of the retained portfolio of brands.<sup>8</sup> Similarly,  $\alpha$  could represent a ranking of transport routes with varying degrees of overlap and substitutability. Another possible interpretation is that  $\alpha$  represents the amount and/or price of access to an essential facility.

### ***c) Information***

$\alpha^{\text{TR}}$  is known by the firms but not by the agency. As a result of its Phase I investigation, the agency obtains its own estimate of the required remedy  $x_1$ . This signal  $x_1$  is assumed to be unbiased in the sense that it is drawn out of the distribution of  $x$  with mean  $\alpha^{\text{TR}}$ . For simplicity, we assume the distribution to be uniform such that  $x \sim U[\alpha^{\text{TR}}, \sigma^2/3]$ , so  $[\alpha^{\text{TR}} - \sigma, \alpha^{\text{TR}} + \sigma]$  is the range. Both the agency and the firms know the interval of the distribution is  $2\sigma$ , but only the firms know the true mean. In other words, the agency knows the width of the interval out of which it draws  $x_1$  but not the position of this interval on  $[0, 1]$ . However, we do assume the agency knows that the merger is remediable, so  $0 < \alpha^{\text{TR}} - \sigma$  and  $\alpha^{\text{TR}} + \sigma < 1$ .<sup>9</sup>

$\sigma$  is exogenous to any particular merger. It depends on the prior allocation of resources to the agency and the inherent complexity of the competition appraisal. For ease of interpretation in deriving results, we focus on the availability of Phase I resources or effort, such that the variance of the distribution of  $x$  is determined by the amount of resources available to the agency,  $e$ . The

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<sup>7</sup> In this paper, we do not address some important practical problems of implementing agreed remedies. For example, merging firms and the buyer of divested assets have an anticompetitive incentive to cripple them in order to reduce capacity in an industry, compensating the buyer with a lower purchase price (Farrell, 2003). We assume that divestitures can be made to work as long as the agency is sufficiently careful (e.g. appointing changeover trustees).

<sup>8</sup> It is possible that there are more than one true  $\alpha$  that would eliminate competition concerns, in which case the agency should consider the Pareto efficient set determined by potential merger efficiencies.

more resources invested, the smaller the range:  $\sigma'(e) < 0$ , with  $\sigma''(e) > 0$ . Finally, if there is a second round (Phase II), then the agency and the firm bear exogenous costs  $K_A$  and  $K_F$  respectively, and the agency receives a more accurate signal,  $x_2$ . For the formal results of this paper, we further assume that this signal is perfectly accurate, so the agency would learn the true remedy following a Phase II inquiry. We leave it to later to discuss the implications of this assumption.

#### ***d) Firms' Objective***

We assume that the expected profit enhancement from the merger is proportional to the fraction of assets that the firms retain after remedies are approved by the antitrust agency. Profits would increase by  $\pi$  if no remedy was required (i.e. if  $\alpha = 1$ ). Clearly, the firms want the agency to agree as high an  $\alpha$  as possible.

The objective of the merging firms is to choose a remedy offer so as to maximise expected profit (as compared with the pre-merger situation).  $\alpha^O \in [0,1]$  is the remedy offered by the merging firms in Phase I, and  $\alpha^{OO} \in [0,1]$  is offered in Phase II (if reached). The merging firms are risk neutral so their objective is:

$$\text{Max}_{\alpha^O, \alpha^{OO}} \left\{ \Pr(\text{Approval in Phase I}) \alpha^O \pi + [1 - \Pr(\text{Approval in Phase I})] [\Pr(\text{Approval in Phase II})] [\alpha^{OO} \pi - K_F] \right\} \quad (1)$$

Note that the probabilities of approval will depend on the remedy offers. Given our assumption that the agency knows  $\alpha^{TR}$  after Phase II, the problem is reduced to the choice of an optimal offer in Phase I:

$$\text{Max}_{\alpha^O} \left\{ \Pr(\text{Approval in Phase I}) \alpha^O \pi + [1 - \Pr(\text{Approval in Phase I})] [\alpha^{TR} \pi - K_F] \right\} \quad (1a)$$

#### ***e) Agency Objective and Errors***

Many agencies, including the EU, USA and UK, are given a delegated objective to ensure that mergers do not result in a 'substantial lessening of competition' or some similar wording.<sup>10</sup> This is widely interpreted by competition agencies to mean that the merger should: a) not create any consumer harm; and b) subject to this priority, that firms should be allowed to maximise profits

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<sup>9</sup> We assume a uniform distribution only for simplicity, and similar results are obtained if we use a triangle or any other distribution with a finite support within (0, 1).

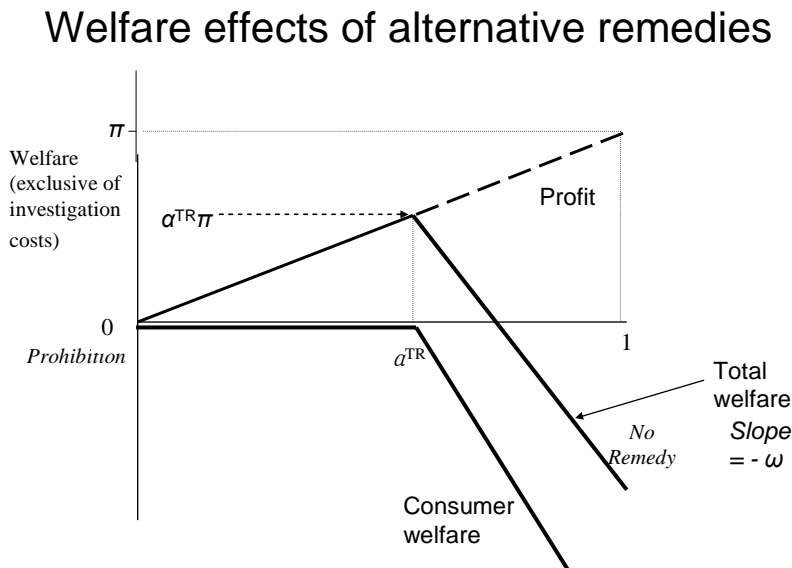
<sup>10</sup> The ECMR (1989) had an objective based on the concept of dominance, which was revised in 2004 to ensure that there was no 'significant impediment to effective competition' (known as the SIEC test).



as they see fit. This lexicographic objective is what we assume for the agency in our model. The agency’s decision should also be supported by evidence such that it can be justified in court. Thus, the agency is concerned that if it has evidence that  $\alpha > \alpha^{TR}$ , enhanced market power would harm consumers (relative to pre-merger) so it should reject such a remedy offer; but if the evidence suggests that  $\alpha \leq \alpha^{TR}$ , there is no expected harm to consumers so such a remedy should be agreed.

While this is the objective delegated to the agency, it is appropriate to appraise the benefits and costs of merger control (as distinct from the success of the agency in meeting its objectives) by a broader welfare function. This may weight profits equal to consumer surplus, or possibly somewhat less. We characterise this welfare function by the parameter  $\omega$ , such that any  $\alpha > \alpha^{TR}$  would reduce welfare by  $[\alpha - \alpha^{TR}] \omega$  compared with the optimum remedy. These welfare effects are summarised in Figure 3.<sup>11</sup>

**Figure 3**



In addition, society should be interested in the agency’s costs of investigation,  $K_A$ , and the firms’ compliance costs,  $K_F$ . Once again, there may or may not be different social weights attached to such costs. We simply assume that the appropriately weighted sum of these costs is  $K$ . Note that Phase I investigation costs are inevitable given the basic 2-phase system for all qualifying

<sup>11</sup> It would be straightforward to adapt the model to allow the agency some trade-off between profits and consumer surplus.

mergers and exogenous resourcing for Phase I.<sup>12</sup> For this reason, we take no account of them here. However, Phase II costs are avoidable in that they only arise due to a ‘mistake’ resulting in failure to agree remedies in Phase I.

Given that there is a discrete possibility that a potentially beneficial merger may be abandoned, there are four qualitatively different types of error in remedy selection. There may be: Type 1 errors associated with the agency accepting an excessive remedy that prevents some profitable opportunities that would not harm consumers; Type 1D ‘drastic’ errors, arising if the merger is prohibited or abandoned, despite there being desirable features that could be remedied; Type 2 errors associated with allowing mergers that are harmful to consumers; and Type 3 errors associated with incurring Phase II investigation and compliance costs when these could have been avoided by reaching agreement in Phase I. We describe Type 1 and Type 2 errors as ‘incremental’ in the sense that they arise from a choice of remedy that is somewhat too strong ( $0 < \alpha < \alpha^{\text{TR}}$ ) or somewhat too limited ( $0 < \alpha^{\text{TR}} < \alpha$ ), but which do not result in the discrete events of no merger or a Phase II investigation.<sup>13</sup> The welfare cost of these errors can be measured relative to the ideal remedy of  $\alpha = \alpha^{\text{TR}}$ . Table 1 summarises the costs and occurrence of each type of error.

**Table 1: Errors and Welfare Losses (Excluding Phase I Investigation Costs)**

<i>\ Firms' offer</i>	$\alpha^{\text{O}} < \alpha^{\text{TR}}$	$\alpha^{\text{O}} > \alpha^{\text{TR}}$
<i>Agency decision \</i>		
<i>Approve</i>	$[\alpha^{\text{TR}} - \alpha^{\text{O}}] \pi$ <i>Type 1</i>	$[\alpha^{\text{O}} - \alpha^{\text{TR}}] \omega$ <i>Type 2</i>
<i>Prohibit (or firms abandon merger)</i>	$\alpha^{\text{TR}} \pi$ <i>Type 1D</i>	$\alpha^{\text{TR}} \pi$ <i>Type 1D</i>
<i>Enter Phase II</i>	$K$ <i>Type 3</i>	$K$ <i>Type 3</i>

<sup>12</sup> It would be a separate question to look at the rules for a merger to qualify for scrutiny and the level of Phase I funding.

<sup>13</sup> There is no equivalently drastic Type 2 error because even if the agency wrongly accepted an offer of  $\alpha = 1$ , this would be qualitatively the same as what we have called an ‘incremental’ Type 2 error. This serves to illustrate that the cost of an ‘incremental’ Type 2 error may be larger than a ‘drastic’ Type 1D error, depending on  $\alpha^{\text{TR}}$  and the relative size of  $\omega$  and  $\pi$ .

### f) Agency Approval Rule

After obtaining its own estimate of the true (ideal) remedy, the agency compares its preliminary finding,  $x_1$ , with the offer made by the firm,  $\alpha^O$ , and applies an approval rule. In this paper, we consider the following *approval rule*: if  $x_1 \geq \alpha^O$ , then approve the remedy, otherwise proceed to the next phase. This is a special case of the linear approval rule: approve *iff*  $x_1 \geq \theta\alpha^O + \varepsilon$ . In general, the process would be repeated in Phase II; i.e. the firm makes an offer  $\alpha^{OO}$ , and simultaneously the agency picks up its own signal  $x_2$  out of the distribution of  $x$ , where in Phase II,  $x \sim U[\alpha^{TR}, \sigma_2^2/3]$  and  $\sigma_2 < \sigma$ . As already stated, in this paper we assume that the agency learns the value of the true remedy for sure in the second phase so,  $x_2 = \alpha^{TR}$  and  $\sigma_2 = 0$ .<sup>14</sup>

## 2. Optimal Offers with Exogenous Agency Effort

It is convenient to define the following terms:

- Size of the remedy offer =  $1 - \alpha^O$
- Socially optimal remedy =  $1 - \alpha^{TR}$
- Excessive (deficient) remedy offer by the firms if  $\alpha^{TR} - \alpha^O > 0$  ( $< 0$ )
- Accuracy of the agency's investigation =  $1 / \sigma$
- Relative inaccuracy of the agency's Phase I investigation =  $\frac{\sigma}{\alpha^{TR}}$ 
  - The investigation is relatively inaccurate if  $3\sigma > \alpha^{TR}$
  - The investigation is relatively accurate if  $3\sigma \leq \alpha^{TR}$
- Marginal profit of asset retention =  $\pi$
- Relative cost of Phase II to the firms =  $\frac{K_F}{\alpha^{TR} \pi}$
- 'The firms' Phase II cost relative to the agency's Phase I inaccuracy' is defined as the ratio of the relative cost to the firms in Phase II to the relative inaccuracy of the agency's Phase I investigation. We define this as:
  - High if  $3\sigma \leq \frac{K_F}{\pi}$
  - Intermediate if  $\sigma \leq \frac{K_F}{\pi} < 3\sigma$

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<sup>14</sup> In this case, it is clearly always optimal for the firms to offer  $\alpha^{OO} = \alpha^{TR}$  if they get to Phase II.

- Low if  $\frac{K_F}{\pi} < \sigma$

In order to identify the underlying mechanisms, we proceed by considering a single phase investigation before extending to the second phase.

### Case 1: Single Phase Investigation

The agency has a simple choice: either approve the offered remedy; or prohibit the merger. Lemma 1 provides the merging firms' optimal offer.

**Lemma 1** (*Optimal offer in 1-phase investigations*):

If there is single phase merger control, the firms' optimal offer is:

$$\alpha^* = \frac{\alpha^{TR} + \sigma}{2} \text{ if } \alpha^{TR} < 3\sigma, \text{ and } \alpha^* = \alpha^{TR} - \sigma \text{ if } \alpha^{TR} \geq 3\sigma.$$

*Proof*

The firms' objective function reduces to:

$$\text{Max}_{\alpha^o} \{ \Pr(\text{Approval in Phase I}) \alpha^o \pi \} \quad (1b)$$

The interval from which the agency's evidence,  $x$ , is drawn is  $[\alpha^{TR} - \sigma, \alpha^{TR} + \sigma]$ . If  $\alpha^o$  were to fall outside this range, then the probability of approval would be either 1 (in which case  $\alpha^o$  could be profitably increased) or zero (in which case  $\alpha^o$  could be profitably reduced). See Fig. 1. Thus,  $\alpha^o$  must be chosen within this interval, and the probability of approval of the offered remedies is the following:

$$\Pr(\text{Approval in Phase I}) = \Pr(\alpha^o \leq x) = 1 - \Pr(x < \alpha^o) = \Pr(\alpha^o) = \frac{1}{2} \left[ 1 - \frac{\alpha^o - \alpha^{TR}}{\sigma} \right] \quad (2)$$

From (1b) and (2), the objective of the merging firms is  $\text{Max}_{\alpha^o} \{ \Pr(\alpha^o) \alpha^o \pi \}$ . If there is an internal solution, then the first order condition can be rearranged to give the optimal offer:

$$\alpha^* = \frac{\alpha^{TR} + \sigma}{2} \quad (3)$$

The condition that this offer lies within the feasible range (i.e.  $\alpha^o > \alpha^{TR} - \sigma$ ) is:

$$\alpha^{TR} < 3\sigma \quad (4)$$

If this condition does not hold, there is a corner solution with

$$\alpha^* = \alpha^{TR} - \sigma \quad (3a)$$

*QED*

Note that the size of the remedy offer by the firms (e.g. assets offered for divestiture) is always increasing in the socially optimal offer, but not always one-to-one. However, there is a non-monotonicity in  $\sigma$ . If the investigation is relatively inaccurate, the size of the offer is increasing in the accuracy of the investigation; but if it is already relatively accurate, then the size of offer declines as the agency's accuracy improves. The intuition behind this non-monotonicity lies in the quadratic form of the profit function with respect to  $\alpha^O$ . This arises because a larger  $\alpha^O$  raises the profitability of an approved merger, but also the chances of a prohibition. Consequently, the optimal offer lies between maximum (i.e.  $\alpha^{TR} + \sigma$ ) and minimum (i.e. 0) amount of assets that the merging parties might feasibly retain after negotiations. A higher  $\sigma$  raises the maximum without affecting the minimum. However, if the agency is sufficiently accurate, the firms find it more profitable to cut out any risk of prohibition and tailor their offer only to the worst selection of evidence (from their perspective) that the agency might find (i.e.  $\alpha^{TR} - \sigma$ ). Proposition 1 sets out the consequences for agency errors.

**Proposition 1** (*Expected errors in 1-phase investigations*):

If there is only a single phase of merger control:

- a) The optimal remedy offer made by the firms is excessive (i.e. creates a Type I error). This error is decreasing in the size of the socially optimal remedy unless the investigation is relatively accurate (in which case the error is independent of  $\alpha^{TR}$ ). The error is increasing in the accuracy of the agency's investigation if the investigation is relatively inaccurate, but decreasing if it is relatively accurate.
- b) The probability of prohibition is strictly positive (i.e. there is an expected Type II error) unless the investigation is relatively accurate, in which case it is zero. For relatively inaccurate investigations, the prohibition probability is increasing in the size of socially optimal remedy and decreasing in the accuracy of the agency's investigation.

*Proof*

From (3) and (3a), the difference between the true offer and the optimal offer is:

$$\alpha^{TR} - \alpha^* = \frac{1}{2}[\alpha^{TR} - \sigma] > 0 \quad \text{if } \alpha^{TR} < 3\sigma, \text{ and} \quad (5)$$

$$\alpha^{TR} - \alpha^* = \sigma \quad \text{if } \alpha^{TR} \geq 3\sigma \quad (5a)$$

(5) and (5a) are always positive by our assumption that the agency knows the merger is remediable so  $\alpha^{TR} - \sigma > 0$ .

From (2) and (3), the probability of failure to agree and so also of prohibition is:

$$1 - \Pr(\alpha^*) = 1 - \left[ \frac{1}{2} - \frac{\frac{1}{2}\alpha^{TR} + \frac{1}{2}\sigma - \alpha^{TR}}{2\sigma} \right] = \frac{1}{4} \left[ 3 - \frac{\alpha^{TR}}{\sigma} \right] > 0 \quad \text{if } \alpha^{TR} < 3\sigma, \text{ and} \quad (6)$$

$$1 - \Pr(\alpha^*) = 0 \quad \text{if } \alpha^{TR} \geq 3\sigma \quad (6a)$$

Thus, this probability lies in the range [0, 1] and is strictly positive when condition (4) holds.

*QED*

Observe that Proposition 1a does *not* arise from risk aversion – we have explicitly assumed that firms are risk neutral. The excessive remedy offer is due to the drastic downside of a failure to agree, which creates an asymmetry of payoffs in balancing an offer that is slightly too big compared with one that is slightly too small. Note that because there is never a Type 2 error, consumers always benefit, or at least never lose, from mergers with agreed remedies. Note also that as long as condition (4) continues to hold, the size of Type 1 error is higher, the *lower* is the socially optimal remedy (see equation 5) – there is higher error with less problematic mergers. Proposition 2 evaluates the benefit side of increased agency funding.

**Proposition 2** (*Agency resources and the cost of errors in 1-phase investigations*):

An exogenous increase in the resources available for single phase appraisal leads to:

- a) A lower welfare cost of error; in particular, although the cost of Type 1 error rises when the investigation is relatively inaccurate (excessive offers), this is more than offset by a reduction in Type 1D error (risk of prohibition).
- b) Higher expected profit for merging firms.

*Proof:* See Appendix.

Inasmuch as agencies are judged on either their accuracy or cost of error, and because firms would prefer to have a proficient and well-funded competition authority rather than a weak one, both have an incentive to lobby for more resources to be invested in merger investigations. The socially optimal level of resourcing, of course, would depend on the shadow price of the taxes that would have to fund any increase.<sup>15</sup>

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<sup>15</sup> Up to a point, the firms may be willing to fund some increase in accuracy themselves.

## Case 2: Two-Phase Investigation

Next, allow firms a second chance to agree remedies, but at the cost of having to take part in a Phase II investigation. We begin by noting that if the compliance costs in Phase II outweigh the expected private benefits of the merger, firms would abandon the merger if referred:

**Corollary** (*Mergers abandoned on referral to Phase II*):

If  $\alpha^{TR} \pi < K_F$ , firms would exercise their quit option if referred to Phase II. In such cases, Lemma 1 and Propositions 1 and 2 hold also for 2-phase investigations.

The remainder of this section focuses on cases where Phase II is not prohibitively costly.

**Lemma 2** (*Optimal offer in 2-phase investigations*):

If there are two phases of merger control, complete revelation of the optimal remedy in Phase II, and the firms do not face prohibitive costs when referred to Phase II:

$$\alpha^* = \alpha^{TR} + \frac{1}{2} \left[ \sigma - \frac{K_F}{\pi} \right] \text{ if } \frac{K_F}{\pi} < 3\sigma \quad \text{and} \quad \alpha^* = \alpha^{TR} - \sigma \text{ if } 3\sigma \leq \frac{K_F}{\pi}.$$

*Proof*

The firms know that the true remedy would be agreed if they go into Phase II, so their objective is:

$$\text{Max}_{\alpha^0} \left\{ \Pr(\alpha^0) \alpha^0 \pi + [1 - \Pr(\alpha^0)] [\alpha^{TR} \pi - K_F] \right\} \quad (7)$$

Suppose there is an interior solution. From (2) and (7), the first order condition can be rearranged to give:

$$\alpha^* = \alpha^{TR} + \frac{1}{2} \left[ \sigma - \frac{K_F}{\pi} \right] \quad (8)$$

The condition that this is interior to the interval from which the agency draws its information (i.e.  $\alpha^0 > \alpha^{TR} - \sigma$ ) is:

$$\frac{K_F}{\pi} < 3\sigma \quad (9)$$

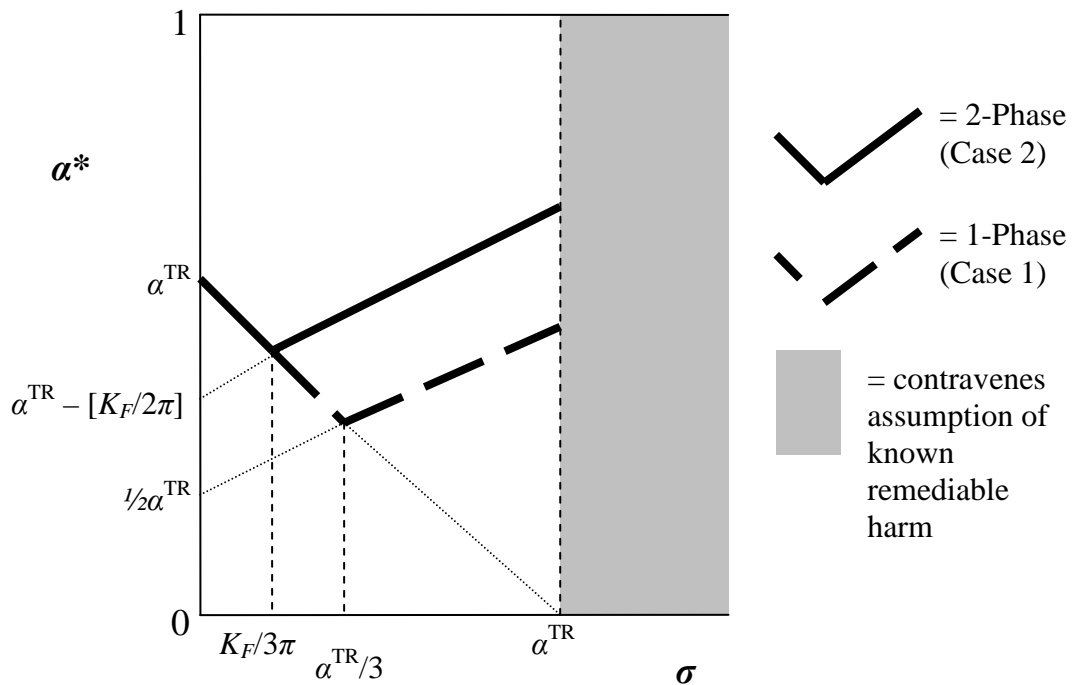
If condition (9) does not hold, then

$$\alpha^* = \alpha^{TR} - \sigma \quad (8a)$$

Thus, from (8), (9), (8a) and  $K_F \geq 0$ ,  $\alpha^* \in [\alpha^{TR} - \sigma, \alpha^{TR} + \frac{1}{2}\sigma]$ . *QED*

The size of the Phase I remedy offer by the firms now always increases one-to-one with the size of the optimal remedy. However, the non-monotonicity in  $\sigma$  continues: if the investigation is relatively inaccurate, the size of the offer is increasing in the accuracy of the investigation; but if it is already relatively accurate, then the size of offer declines as the agency's accuracy improves. This effect, and a comparison with Case 1, is illustrated in Figure 4. A new feature introduced by the second phase is the compliance cost of firms relative to the marginal profit of asset retention. For high Phase II cost to the firms relative to the agency's Phase I inaccuracy, the size of Phase I remedy offer increases with the compliance cost of firms relative to the marginal profit of asset retention.

**Figure 4: Optimal Remedy Offers in Single and 2-Phase Inquiries**



**Proposition 3 (Expected errors in 2-phase investigations):**

If there are two phases of merger control, complete revelation of the optimal remedy in Phase II, and the firms do not face prohibitive costs when referred to Phase II:

- a) The optimal Phase I remedy offer may be either excessive or deficient: if the Phase II cost to the firms relative to the agency's Phase I inaccuracy is *low*, there will be a Type 1 error (deficient remedy); otherwise the error will be Type 2 (excessive remedy). Only in



the ‘knife-edge’ case of  $\frac{K_F}{\pi} = \sigma$  will the firms offer exactly the true remedy (truthful revelation).

- b) If the Phase II cost to the firms is *high* relative to the agency’s Phase I inaccuracy, then the firms’ costs relative to marginal profits do not affect the error. Otherwise, the error is increasing in  $K_F/\pi$  when there is a Type 1 error, and decreasing when the error is Type 2.
- c) If the Phase II cost to the firms is *intermediate* relative to the agency’s Phase I inaccuracy, then the Type 1 error increases with the accuracy of the agency. Otherwise (i.e. either *high* or *low*), the (respective Type 1 or Type 2) error decreases with agency accuracy.
- d) The probability of a Phase II investigation is strictly positive, unless the Phase II cost to the firms is *high* relative to the agency’s Phase I inaccuracy, in which case it is zero. In the *low* and *intermediate* cases, the referral probability is decreasing in both the accuracy of the agency’s investigation, and the firms’ costs of Phase II investigation relative to marginal profits.

*Proof*

From (8), the difference between the true remedy and the optimal offer is:

$$\alpha^{TR} - \alpha^* = \frac{1}{2} \left[ \frac{K_F}{\pi} - \sigma \right] \quad \text{if} \quad \frac{K_F}{\pi} < 3\sigma \quad \text{and} \quad (10)$$

$$\alpha^{TR} - \alpha^* = \sigma \quad \text{if} \quad 3\sigma \leq \frac{K_F}{\pi} \quad (10a)$$

There is no restriction for (10) to be either positive or negative, and the offer is deficient if  $\frac{K_F}{\pi} < \sigma$  (i.e. Type 2 error). On the other hand, if the offer is excessive it cannot be as excessive as in single phase merger appraisal (case 1) because the merger would be abandoned before going into Phase II if  $\alpha^{TR} \pi < K_F$ . Comparing (5) and (10), if there is a Type 1 error in case 2, it must be less than in case 1.

For Propositions 3b) and 3c), see proof of Proposition 4a) in the Appendix.

The probability of failure to agree in Phase I, and so of referral into Phase II, is:

$$1 - \Pr(\alpha^*) = 1 - \left[ \frac{\alpha^{TR} + \frac{1}{2} \left( \sigma - \frac{K_F}{\pi} \right) - \alpha^{TR}}{2\sigma} \right] = \frac{1}{4} \left[ 3 - \frac{K_F}{\sigma\pi} \right] > 0 \quad \text{if} \quad \frac{K_F}{\pi} < 3\sigma \quad \text{and} \quad (11)$$

$$1 - \Pr(\alpha^*) = 0 \quad \text{if } \frac{K_F}{\pi} \geq 3\sigma \quad (11a)$$

The probability in (11) lies in the range  $(0, \frac{3}{4})$ . Also, because  $\alpha^{TR}\pi > K_F$ , the probability of referral to Phase II in case 2 must exceed the probability of prohibition in case 1 (holding all parameter values except the bargaining procedure constant). *QED*

Proposition 3 identifies another bias in Phase I remedy offers, which works in the opposite direction to the asymmetry identified in single phase investigations (Proposition 2). Consider one extreme as  $K_F/\pi \rightarrow 0$ , so firms can ignore the costs they would incur in Phase II. This removes the downside of a rejected offer, so the merging firms will try to keep a larger share of assets. They have an incentive to bluff in order to win approval of a deficient offer because this would be better than being left with a guaranteed  $\alpha^{TR}$ . If the agency's market testing turns out to be lucky for the firms, they win Phase I approval and the agency incurs a Type 2 error. At the other extreme, as  $K_F \rightarrow \alpha^{TR}\pi$ , so it would hardly be worth proceeding to Phase II, the bias from case 1 dominates (i.e. the large downside of a rejected offer) and the agency incurs a Type 1 error.

Another way to interpret Proposition 3 is if we define the 'size of merger' as  $\pi / K_F$ . This defines size by the potential to increase profit relative to compliance costs, which are likely to exhibit economies of scale. Proposition 3 then shows that firms involved in bigger mergers will offer less generous remedies and are more likely to go into Phase II. Note that the latter prediction is due to the bargaining stance taken by the merging firms, and is *not* due to a judgement by the agency that it is 'not worth' pursuing small mergers or that large mergers are more complex.

It is the interaction between this incentive to bluff (i.e. to make deficient offers) and the costs of not reaching agreement in Phase I that creates the complex effects set out in Proposition 3. For example, the intuition behind Proposition 3c) is as follows. Start from a very inaccurate agency, such that  $\frac{K_F}{\pi} < \sigma$  and there is a Type 2 error. A reduction in  $\sigma$  induces the firms to make an offer closer to the optimum. However, as the agency's accuracy improves further, the 'knife-edge' optimum offer is passed and the error switches to Type 1, which increases (as in Case 1) with agency accuracy. Throughout both regions, there is a continuous reduction in Type 3 error (i.e. probability of referral). Once agency accuracy gets sufficiently high, firms pitch their offer such that the Type 3 error is eliminated. The firms' balance of incentives switches to the Type 1

error, which they reduce by offering closer to the true remedy. Proposition 4 focuses on the benefits associated with a better resourced agency.

**Proposition 4** (*Agency resources and the cost of errors in 2-phase investigations*):

If there are two phases of merger control, complete revelation of the optimal remedy in Phase II, and the firms do not face prohibitive costs when referred to Phase II, then an exogenous increase in the resources available in Phase I leads to:

- a) A lower overall welfare cost of error for *low* and *high* Phase II cost to the firms relative to the agency's Phase I inaccuracy, and for *intermediate* cases where  $K_F < K$ . However, a sufficient condition for an increase in agency resources to *increase* overall welfare cost of error in the *intermediate* case is  $K < \frac{5}{9} K_F$ .
- b) Higher profits except for *low* Phase II cost to the firms relative to the agency's Phase I inaccuracy, in which case expected profits fall.

*Proof:* See Appendix.

Proposition 4a) shows that, unlike in the single phase case, the reduction in Type 1D error may not always outweigh the increasing Type 2 error as agency resources are increased in a 2-phase investigation. It suggests that there may be a difficult lacuna for an intermediately resourced agency, in which extra resources may be counterproductive. This arises when the agency has low costs relative to the firms and there is a low social weighting on compliance costs of firms relative to the agency's costs. An implication of Proposition 4b) is that firms operating in a developed country with a reasonably well-resourced agency should support an increase in resources for the agency to get it even more accurate, while firms operating in a developing or transition country, with a very poorly resourced and inaccurate agency, might prefer it to be even more handicapped so that there is more opportunity to bluff a deficient remedy in Phase I.

### 3. Potential Extensions

One of the assumptions in the model is that the agency discovers the true remedy in Phase 2. The optimal offer by the merging firms is then determined by the maximum and minimum remedies that the firms can obtain as a result of the bargaining process. Suppose, however, that Phase II does not bring truthful revelation. Following the logic of Case 1, the optimal Phase II

offer will be either  $\alpha^{**} = \alpha^{TR} - \sigma_2$  or  $\alpha^{**} = \frac{1}{2}[\alpha^{TR} + \sigma_2]$ . Suppose the Phase II accuracy is sufficient to make the former offer optimal. It is straightforward to show that the Phase I offer will be either  $\alpha^* = \alpha^{TR} - \sigma$  or  $\alpha^* = \alpha^{TR} + \frac{1}{2}\left[(\sigma - \sigma_2) - \left(\frac{K_F}{\pi}\right)\right]$ . In terms of Figure 4, the optimal offer lies between Case 1 and Case 2. Thus, there can still be either Type 1 or Type 2 errors, though the likelihood of the latter is reduced by an inaccurate Phase II investigation. Similarly, the firms may either raise or reduce their remedy offer in Phase II (if reached). The firms may either increase or decrease their offer in Phase II compared with Phase I, but the direction of change no longer maps neatly into whether the Phase I offer was excessive or deficient.<sup>16</sup> The probability of referral is increased as the expected Phase II outcome worsens for the firms. The comparative statics on  $\sigma$  remain the same, but it is as if  $\sigma$  is reduced by  $\sigma_2 > 0$ . Thus, whilst modifying some significant detail, this more general case does not add new insight into the underlying mechanisms and nature of remedy outcomes.<sup>17</sup>

Another of our assumptions is that the agency uses a simple, evidence-based rule for remedy appraisal. Alternatively, it might try to infer the true remedy from the size of the firms' offer. This would imply a signalling game in which the firms may recognise how their offer will be interpreted, and try credibly to persuade the agency to accept the minimum remedy. We have chosen not to pursue this modelling route because, apart from the sophisticated calculations required of both sides, it would likely fall foul of the appeals process. In practice, when agencies are tempted to require remedies without evidential support from their own investigations, the remedies are likely to be dismissed by a judge at the appeal stage.

A third variant of the model would be to endogenise agency effort, allowing it to decide how deeply to investigate the merger in Phase I, conditional on the information it has available at the time. This would require extending the model to include certain easily observed parameters (e.g. firm size) on which the effort level could be conditioned.

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<sup>16</sup> Which is the case in our 'full revelation in Phase II' model.

<sup>17</sup> The qualitative results remain the same if conditions are such that the optimal Phase II offer still leaves room for a prohibition (i.e. when  $\alpha^{**} = \frac{1}{2}[\alpha^{TR} + \sigma_2]$ ). In this case, the optimal Phase I offer is

$$\alpha^* = \frac{5}{8}\alpha^{TR} + \frac{1}{2}\left[\sigma - \frac{K_F}{\pi}\right] + \frac{\sigma_2}{16}\left[1 + \left(\frac{\alpha^{TR}}{\sigma_2}\right)^2\right],$$

which is decreasing in  $\sigma_2$  in the relevant range. The probability of a prohibition is the same as in equation (6), but with  $\sigma_2$  replacing  $\sigma$ .

## 4. Conclusions

This paper provides a first attempt to understand how outcomes are determined by the standard institutions of merger control. In particular, we focus on the internationally standard 2-phase investigation structure and remedy negotiations of the form practiced by the EC (i.e. firms have the last chance to make offers in each phase, and the Commission can only accept or reject, but not modify these last offers). We find that there are inherent biases in remedy outcomes, and identifiable circumstances where offers will be excessive, and where they will be deficient. In particular, we find clear circumstances in which firms offer excessive remedies. This goes against a possible intuition that firms should expect to extract an information rent for possessing superior information about the true remedy.

In principle, some of these predictions could be confirmed or rejected empirically. For example, our model predicts that a Phase II investigation is determined by factors affecting the incentive of the merging parties to offer more or less generous remedies early in the investigation (e.g. compliance costs of Phase II, inaccuracy of the agency in Phase I) and not necessarily by the potential harm of an unremedied merger – plenty of potentially harmful mergers are remedied in Phase I.

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## Appendix

As we assumed earlier in the paper, the more resources invested,  $e$ , the greater the accuracy of the agency,  $\sigma'(e) < 0$ . For ease of exposition, we take the derivative of different functions with respect to  $\sigma$ , rather than  $e$ , noting that if  $\frac{\partial(\cdot)}{\partial\sigma} > 0$ , then  $\frac{\partial(\cdot)}{\partial e} < 0$ , and *vice versa*.

### Proposition 2a:

a) The cost of Type 1D error is non-increasing:

i) From Lemma 1, if  $\alpha^{TR} < 3\sigma$ , the firms offer  $\alpha^* = \frac{\alpha^{TR} + \sigma}{2}$ . Also, the probability of

approval is  $\Pr(\alpha^*) = \frac{1}{2} \left[ 1 - \frac{\alpha^* - \alpha^{TR}}{\sigma} \right]$ . The cost of Type 1D error is  $[1 - \Pr(\alpha^*)]\alpha^{TR}\pi$ .

$$\frac{\partial([1 - \Pr(\alpha^*)]\alpha^{TR}\pi)}{\partial\sigma} = \frac{\partial\left(\left[\frac{3}{4} - \frac{\alpha^{TR}}{4\sigma}\right]\alpha^{TR}\pi\right)}{\partial\sigma} = \frac{(\alpha^{TR})^2\pi}{4\sigma^2} > 0.$$

Thus,  $\frac{\partial(\text{Type 1D Error Cost})}{\partial e} < 0$ .

ii) If  $\alpha^{TR} > 3\sigma$  the firms offer  $\alpha^* = \alpha^{TR} - \sigma$  and  $\Pr(\alpha^*) = 1$ .

Thus,  $\frac{\partial(\text{Type 1D Error Cost})}{\partial e} = 0$

b) The cost of Type 1 error is increasing when the investigation is relatively inaccurate

( $\alpha^{TR} < 3\sigma$ ) and decreasing when it is relatively accurate ( $\alpha^{TR} > 3\sigma$ ):

i) If  $\alpha^{TR} < 3\sigma$ :

$$\frac{\partial(\text{Type 1 Error Cost})}{\partial\sigma} = \frac{\partial([\alpha^{TR} - \alpha^0]\Pr(\alpha^*)\pi)}{\partial\sigma} = \frac{\partial\left(\frac{\pi}{8}\left(\frac{(\alpha^{TR})^2}{\sigma} - \sigma\right)\right)}{\partial\sigma} = \frac{\pi}{8}\left(-\left(\frac{\alpha^{TR}}{\sigma}\right)^2 - 1\right) < 0$$

Thus,  $\frac{\partial(\text{Type 1 Error Cost})}{\partial e} > 0$ .

ii) If  $\alpha^{TR} > 3\sigma$ :

$$\frac{\partial(\text{Type 1 Error Cost})}{\partial\sigma} = \frac{\partial([\alpha^{TR} - \alpha^*]\Pr(\alpha^*)\pi)}{\partial\sigma} = \frac{\partial(\sigma\pi)}{\partial\sigma} = \pi > 0$$

Thus,  $\frac{\partial(\text{Type 1 Error Cost})}{\partial e} < 0$ .

c) The total cost of error decreases throughout.

i) If  $\alpha^{TR} < 3\sigma$ :

$$\frac{\partial(\text{Total Cost of Error})}{\partial\sigma} = \frac{\partial([1 - \Pr(\alpha^*)]\alpha^{TR}\pi + [\alpha^{TR} - \alpha^*]\Pr(\alpha^*)\pi)}{\partial\sigma} = \frac{\pi}{8} \left( \left( \frac{\alpha^{TR}}{\sigma} \right)^2 - 1 \right) > 0$$

This holds because  $\alpha^{TR} > \sigma$  by our assumption that the agency knows the merger is remediable.

Thus,  $\frac{\partial(\text{Total Cost of Error})}{\partial e} < 0$ .

ii) If  $\alpha^{TR} > 3\sigma$ :  $\frac{\partial(\text{Total Cost of Error})}{\partial e} = \frac{\partial(\text{Type1 Error Cost})}{\partial e} < 0$

**Proposition 2b:**

a) If  $\alpha^{TR} < 3\sigma$ , the firms offer  $\alpha^* = \frac{\alpha^{TR} + \sigma}{2}$ ,  $\Pr(\alpha^*) = \frac{1}{2} \left[ 1 - \frac{\alpha^* - \alpha^{TR}}{\sigma} \right]$ :

$$\frac{\partial(\text{Expected Profit})}{\partial\sigma} = \frac{\partial(\Pr(\alpha^*)\alpha^*\pi)}{\partial\sigma} = \frac{\partial}{\partial\sigma} \left( \frac{\pi}{8} \left( 1 + \frac{\alpha^{TR}}{\sigma} \right) (\alpha^{TR} + \sigma) \right) = \frac{\pi}{8} \left( 1 - \left( \frac{\alpha^{TR}}{\sigma} \right)^2 \right) < 0.$$

This holds because  $\alpha^{TR} > \sigma$  by our assumption that the merger is remediable.

Thus,  $\frac{\partial(\text{Expected Profit})}{\partial e} > 0$ .

b) If  $\alpha^{TR} > 3\sigma$ , the firms offer  $\alpha^* = \alpha^{TR} - \sigma$ ,  $\Pr(\alpha^*) = 1$ :

$$\frac{\partial(\text{Expected Profit})}{\partial\sigma} = \frac{\partial(\Pr(\alpha^*)\alpha^*\pi)}{\partial\sigma} = -\pi < 0. \text{ Thus, } \frac{\partial(\text{Expected Profit})}{\partial e} > 0$$

**Proposition 4a:**

i) The cost of Type 3 error is non-increasing:

a) If  $\frac{K_F}{3\pi} < \sigma$ , the firms offer  $\alpha^* = \alpha^{TR} + \frac{1}{2} \left( \sigma - \frac{K_F}{\pi} \right)$ , and  $\Pr(\alpha^*) = \frac{1}{2} \left[ 1 - \frac{\alpha^* - \alpha^{TR}}{\sigma} \right]$

$$\frac{\partial(\text{Type3 Error Cost})}{\partial\sigma} = \frac{\partial([1 - \Pr(\alpha^*)]K)}{\partial\sigma} = \frac{\partial}{\partial\sigma} \left( \frac{3K}{4} \left[ 1 - \frac{K_F}{3\pi\sigma} \right] \right) = \frac{K_F K}{4\pi\sigma^2} > 0$$

Thus,  $\frac{\partial(\text{Type3 Error Cost})}{\partial e} < 0$



b) If  $\frac{K_F}{3\pi} > \sigma$ , the firms offer  $\alpha^* = \alpha^{TR} - \sigma$  and  $\Pr(\alpha^*) = 1$

$$\text{Thus, } \frac{\partial(\text{Type3 Error Cost})}{\partial e} = 0$$

ii) The cost of incremental errors:

a) Is decreasing if the firms' Phase II cost relative to agency's Phase I inaccuracy is *low*

(i.e.  $\sigma > \frac{K_F}{\pi}$ ); the firms offer  $\alpha^* = \alpha^{TR} + \frac{1}{2}\left(\sigma - \frac{K_F}{\pi}\right)$ , which is deficient ( $\alpha^* > \alpha^{TR}$ ).

$$\frac{\partial(\text{Type2 Error Cost})}{\partial \sigma} = \frac{\partial((\alpha^* - \alpha^{TR})\Pr(\alpha^*)\omega)}{\partial \sigma} = \frac{\partial\left(\frac{\omega\sigma}{8}\left(1 - \left(\frac{K_F}{\pi\sigma}\right)^2\right)\right)}{\partial \sigma} = \frac{\omega}{8}\left(1 + \left(\frac{K_F}{\pi\sigma}\right)^2\right) > 0$$

$$\text{Thus, } \frac{\partial(\text{Type2 Error Cost})}{\partial e} < 0.$$

b) Is increasing if the firms' Phase II cost relative to agency's Phase I inaccuracy is

*intermediate*

(i.e.  $\frac{K_F}{3\pi} < \sigma < \frac{K_F}{\pi}$ ); the firms offer  $\alpha^* = \alpha^{TR} + \frac{1}{2}\left(\sigma - \frac{K_F}{\pi}\right)$ , which is excessive ( $\alpha^* < \alpha^{TR}$ );

$$\frac{\partial(\text{Type1 Error Cost})}{\partial \sigma} = \frac{\partial((\alpha^{TR} - \alpha^*)\Pr(\alpha^*)\pi)}{\partial \sigma} = \frac{\partial\left(\frac{\pi}{8}\left(\frac{K_F^2}{\pi^2\sigma} - \sigma\right)\right)}{\partial \sigma} = \frac{\pi}{8}\left(-\frac{K_F^2}{\pi^2\sigma^2} - 1\right) < 0$$

$$\text{Thus, } \frac{\partial(\text{Type1 Error Cost})}{\partial e} > 0.$$

c) Is decreasing if the firms' Phase II cost relative to agency's Phase I inaccuracy is *high*

(i.e.  $\sigma < \frac{K_F}{3\pi}$ ); the firm offer  $\alpha^* = \alpha^{TR} - \sigma$ , which is excessive ( $\alpha^* < \alpha^{TR}$ );  $\Pr(\alpha^*) = 1$ ;

$$\frac{\partial(\text{Type1 Error Cost})}{\partial \sigma} = \frac{\partial((\alpha^{TR} - \alpha^*)\Pr(\alpha^*)\pi)}{\partial \sigma} = \frac{\partial(\sigma\pi)}{\partial \sigma} = \pi > 0.$$

$$\text{Thus, } \frac{\partial(\text{Type1 Error Cost})}{\partial e} < 0$$

iii) The total cost of error:

a) Is decreasing if the firms' Phase II cost relative to agency's Phase I inaccuracy is *low*

$$\frac{\partial(\text{Total Cost of Error})}{\partial \sigma} = \frac{\partial((1 - \Pr(\alpha^*))K + \Pr(\alpha^*)(\alpha^* - \alpha^{TR})\omega)}{\partial \sigma} = \frac{K K_F}{4\pi\sigma^2} + \frac{\omega}{8}\left(1 + \left(\frac{K_F}{\pi\sigma}\right)^2\right) > 0$$

Thus,  $\frac{\partial(\text{Total Cost of Error})}{\partial e} < 0$ .

b) Is decreasing if the firms' Phase II cost relative to agency's Phase I inaccuracy is *high*

(i.e.  $\sigma < \frac{K_F}{3\pi}$ ); the firm offer  $\alpha^* = \alpha^{TR} - \sigma$ , which is excessive;  $\Pr(\alpha^*) = 1$ ;

$$\frac{\partial(\text{Total Cost of Error})}{\partial \sigma} = \frac{\partial}{\partial \sigma} \left( (1 - \Pr(\alpha^*))K + \Pr(\alpha^*)(\alpha^{TR} - \alpha^*)\pi \right) = \frac{\partial}{\partial \sigma} (\sigma\pi) = \pi > 0,$$

Thus,  $\frac{\partial(\text{Total Cost of Error})}{\partial e} < 0$ .

c) May be either increasing or decreasing if the firms' Phase II cost relative to agency's Phase I inaccuracy is *intermediate*.

(i.e.  $\frac{K_F}{3\pi} < \sigma < \frac{K_F}{\pi}$ ); the firms offer  $\alpha^* = \alpha^{TR} + \frac{1}{2} \left( \sigma - \frac{K_F}{\pi} \right)$ , which is deficient ( $\alpha < \alpha^{TR}$ ):

$$\begin{aligned} \frac{\partial(\text{Total Cost of Error})}{\partial \sigma} &= \frac{\partial \left( [1 - \Pr(\alpha^*)]K + \Pr(\alpha^*)(\alpha^{TR} - \alpha^*)\omega \right)}{\partial \sigma} = \\ &= \frac{\partial}{\partial \sigma} \left( \frac{3K}{4} \left( 1 - \frac{K_F}{3\pi\sigma} \right) + \frac{\pi\sigma}{8} \left( \left( \frac{K_F}{\pi\sigma} \right)^2 - 1 \right) \right) = \frac{\pi}{8} \left( \frac{K_F}{\pi\sigma} \right)^2 \left( \frac{2K}{K_F} - 1 - \left( \frac{\pi\sigma}{K_F} \right)^2 \right). \end{aligned}$$

This may be either positive or negative. However, since  $\left( \frac{\pi\sigma}{K_F} \right)^2 \in (1/9, 1)$  in the intermediate

range, a sufficient condition for the above expression to be positive is  $K > K_F$ , and sufficient for it to be negative is  $K < 5/9 K_F$ .

**Proposition 4b:**

i) If  $\sigma > \frac{K_F}{3\pi}$ ,

$$\begin{aligned} \frac{\partial(\text{Expected Profit})}{\partial \sigma} &= \frac{\partial \left( \Pr(\alpha^*)\alpha^*\pi + [1 - \Pr(\alpha^*)](\alpha^{TR}\pi - K_F) \right)}{\partial \sigma} = \\ &= \frac{\partial}{\partial \sigma} \left( \frac{\pi}{8} \left( 2\alpha^{TR} + \frac{2\alpha^{TR}K_F}{\pi\sigma} + \sigma - \left( \frac{K_F}{\pi} \right)^2 \frac{1}{\sigma} \right) + \frac{1}{4} \left( 3\alpha^{TR}\pi - \frac{\alpha^{TR}K_F}{\sigma} - 3K_F + \frac{K_F^2}{\pi\sigma} \right) \right) = \\ &= \frac{\pi}{8} \left( 1 - \left( \frac{K_F}{\pi\sigma} \right)^2 \right) > 0, \text{ if } \sigma > \frac{K_F}{\pi}, \text{ otherwise } (<) < 0 \end{aligned}$$

Thus,  $\frac{\partial(\text{Expected Profit})}{\partial e} < 0$ , if  $\sigma > \frac{K_F}{\pi}$  (i.e. *low* Phase II cost to the firms relative to the agency's Phase I inaccuracy)

and  $\frac{\partial(\text{Expected Profit})}{\partial e} > 0$ , if  $\sigma < \frac{K_F}{\pi}$  (i.e. *intermediate* Phase II cost to the firms relative to the agency's Phase I inaccuracy)

ii) If  $\sigma < \frac{K_F}{3\pi}$  (i.e. *high* Phase II cost to the firms relative to the agency's Phase I inaccuracy),

$$\frac{\partial(\text{Expected Profit})}{\partial \sigma} = \frac{\partial(\text{Pr}(\alpha^*)\alpha^*\pi)}{\partial \sigma} = \frac{\partial((\alpha^{TR} - \sigma)\pi)}{\partial \sigma} = -\pi < 0.$$

Thus,  $\frac{\partial(\text{Expected Profit})}{\partial e} > 0$ .