Collusion with Reporting and Monitoring of Sales

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- Firms agreed to a target price.
- Sales quotas were allocated to each firm.

Market Allocation (tons)

Company	Global	Europe
Ajinomoto	73,500	34,000
ADM	48,000	5,000
Kyowa	37,000	8,000
Sewon	20,500	13,500
Cheil	6,000	5,000

- Kanji Mimoto of Ajinomoto was assigned the task of preparing monthly "scorecards" for the cartel.

Kanji Mimoto
Ajinomoto

Terry Wilson
Archer Daniels Midland

  - Each company telephoned or mailed their sales volumes to Mimoto.
  - Mimoto prepared a spreadsheet that was distributed at the quarterly maintenance meetings.
- "Guaranteed buy-ins"
  - A company that sold more then its quota would have to buy product from producers who were below quota.

- Sales reports relative to quotas
  - For 1994, reported sales were only 1.4% above the targeted amount.
  - Sewon was farthest from its allotted share - selling 14.3% instead of 14.7%.
- Misreporting of sales
  - Cheil claims that it reported "misleading" sales information to the other companies.
  - Ajinomoto hid 3,500 tons of lysine from the cartel's auditors.
    - An internal memo read: "Hide 1,000 tons in Thailand internal business."

- Other cartels
  - Vitamins (1989-99)
  - Citric acid (1991-95)
  - Zinc phosphate (1994-98)
- Common features of these price-fixing cartels
  - Demand is largely from industrial buyers.
  - Price is set bilaterally between seller and buyer and is generally not public information.
  - Collusive agreement is monitored in terms of sales compared to quotas.
  - Punishment involved transfers.
- Source: J. Harrington, *How Do Cartels Operate?* (*Foundations and Trends in Microeconomics*, July 2006)

Objective of Research Project

- Develop a better theory of hard-core cartels which fits these facts.
- Collusion when prices are private information and sales are public information (*RAND Journal of Economics*, Summer 2007)
  - Impossibility result: Price wars cannot sustain collusion.
  - Possibility result: Asymmetric punishments (buy-backs) can sustain collusion.
- Collusion when prices and sales are private information (work in progress)
  - Characterization: Firms truthfully report sales and condition punishments on those reports.

Model

- Infinitely repeated game in which $n \geq 2$ firms make simultaneous price decisions.
- Common discount factor is $\delta \in (0, 1)$.
- Demand
  - Market demand is fixed at m units where m is a positive integer.
  - Set of feasible quantity vectors is:

$$\Delta \equiv \left\{ (q_1, \ldots, q_n) \in \{0, 1, \ldots, m\}^n : \sum_{i=1}^{n} q_i = m \right\}.$$

  - Firm demand is stochastic where

$$\psi\left(\underline{q} \,\middle|\, \underline{p}\right) : \Delta \times \Re^n \rightarrow [0, 1]$$

    is the probability of quantity vector $\underline{q}$ given price vector $\underline{p}$.

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  - A3: $\sum_{i=1}^{n}\left(\frac{\partial\psi\left(\underline{q}|p,\ldots,p\right)}{\partial p_i}\right)=0,\ \forall\left(\underline{q},p\right)$. [local invariance]

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  - when ψ depends only on the price differences.
  - with the discrete choice model (without an outside option)

Model

- Common constant marginal cost, c.
- Information structure
  - Imperfect monitoring as firms' prices are private information.
  - Firms' quantities are common knowledge.
- Perfect public equilibria - firms condition their price on the publicly observed history of quantities (and not on the privately observed history of prices).

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- A Nash equilibrium is *history-relevant* when: if $\underline{\widetilde{p}}^t\left(h^{t-1}\right) = \underline{p}^N$ then $v_i^{t+1}\left(h^{t-1}, \underline{q}'\right) = v_i^{t+1}\left(h^{t-1}, \underline{q}''\right)\ \forall \underline{q}\prime, \underline{q}'', \forall i.$

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  - $\underline{\widetilde{p}}^t\left(h^{t-1}\right)$ is the equilibrium price vector in period t after history h^{t-1}.
  - $\underline{p}^N$ is a static Nash equilibrium price vector.

Theorem

Assuming A1-A3, the set of strongly symmetric exchangeable history-relevant Nash equilibrium prices for the infinite horizon game coincides with the set of symmetric Nash equilibrium prices for the stage game.

- Example: duopoly
- Consider a strategy profile in which there is a low continuation payoff ("price war") if either firm has a market share exceeding some threshold, $\widehat{s}$.

Collusion with Symmetric Punishments

Impossibility Theorem: Intuition

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- Locally, those two effects are of the same size.
- A firm's price has no effect on its expected continuation payoff.
- Equilibrium price maximizes expected current profit.

- If firms are in the collusive state then
  - a firm pays $z \geq 0$ for each unit it sells
  - the proceeds are shared equally among the remaining members of the cartel.
- State of the industry
  - Firms start in the collusive state.
  - Firms remain in the collusive state as long as transfers are paid.
  - Failure to make a transfer causes firms to switch to static Nash equilibrium forever.

Collusion with Asymmetric Punishments

Firm Maximization Problem

- Assume that transfers will be made.
- In the collusive state, firm i chooses p_i to maximize:

$$\sum_{\underline{q} \in \Delta} \psi\left(\underline{q}\,|\widetilde{p}, \ldots, p_i \ldots, \widetilde{p}\right)\left[\left(p_i - c\right) q_i + z\left(\frac{m - q_i}{n-1}\right) - zq_i\right] + \delta V$$

  where V is the collusive value.
- Equivalent representation:

$$\sum_{\underline{q} \in \Delta} \psi\left(\underline{q}\,|\widetilde{p}, \ldots, p_i \ldots, \widetilde{p}\right)\left[p_i - c - \left(\frac{n}{n-1}\right) z\right] q_i + \left(\frac{m}{n-1}\right) z + \delta V$$

- Equilibrium prices are the same as in a one-shot game in which firms have a cost of $c + \frac{n}{n-1} z$ per unit.

- A4: The one-shot game with cost c has, $\forall c \geq 0$, a symmetric Nash equilibrium price $p^N(c)$ that is increasing, continuous, and unbounded in c.

Theorem

If A4 holds then, for any price $p > p^N(c)$, there exists $\delta^* < 1$ such that for all $\delta \geq \delta^*$ there exists a perfect public equilibrium in which the cartel sets a price of p in every period.

- Equilibrium condition (price): For any $p > p^N$ choose the per-unit transfer z so that

$$p = p^N\left(c + \frac{n}{n-1}z\right)$$

- Equilibrium condition (transfer):
  - It is sufficient to verify the incentives of a firm that sells to all customers:

$$-mz + \delta V(p) \geq \delta V^N \Leftrightarrow \delta\left[V(p) - V^N\right] \geq mz$$

    - $V(p)$ is the collusive value.
    - V^N is the non-collusive (Nash) value.
  - As $\delta \to 1$, $\delta\left[V(p) - V^N\right] \to \infty$.

- Theoretical findings
    - Symmetric price wars cannot sustain collusion.
        - Robust to market demand being highly price-inelastic.
    - Asymmetric punishments in the form of transfers can sustain collusion.
        - Robust to when firms set customer-specific prices.
- A transfer can be consummated through inter-firm sales.
- Examples of cartels using inter-firm sales as a punishment device.
    - Citric acid (1991-95)
    - Graphite electrodes (1992-97)
    - Vitamins A and E (1989-99)

- Theoretical finding: If firms' sales are public information then collusion can be supported using asymmetric punishments.
- Cartel practice: Firms report their sales in cartel meetings but are these reports truthful?
- Theoretical exercise
    - Assume prices and quantities are private information.
    - Firms exchange messages (sales reports) prior to making transfers.
    - Characterize an equilibrium in which firms truthfully report their sales and collusion is sustained.

- Market demand
  - m^t is total sales and is *iid* over time.
  - $\rho(m) : \{\underline{m}, \underline{m}+1, \ldots, \overline{m}\} \rightarrow [0,1]$

$$\mu \equiv \sum_{m=\underline{m}}^{\overline{m}} \rho(m)\, m$$

  - Market demand does not depend on firms' prices.
- Firm demand
  - Support: $\{0, 1, \ldots, \overline{q}\}$.
  - $\psi_i(q; m, \underline{p})$ is the probability function on firm i's sales given total demand is m and the price vector.

- $\sigma_i\left(m; q, \underline{p}\right)$ is the probability that market demand is m given firm i's sales is q and firms' prices.

$$\sigma_i\left(m; q, \underline{p}\right) = \frac{\rho\left(m\right)\psi_i\left(q; m, \underline{p}\right)}{\sum_{m'=\underline{m}}^{\overline{m}}\rho\left(m'\right)\psi_i\left(q; m', \underline{p}\right)}.$$

- Assumption: $\sigma_i\left(\overline{m}; q, \underline{p}\right) > 0,\ \forall q,\ \forall \underline{p}.$
- Assumption: If $q' > q''$ then $\sigma\left(\cdot\,\middle|\,q', \underline{p}\right)$ first-order stochastically dominates $\sigma\left(\cdot\,\middle|\,q'', \underline{p}\right)$.

- Expected profit:

$$\pi_i\left(p_1,\ldots,p_n\right)=\sum_{m=\underline{m}}^{\overline{m}}\left(p_i-c\right)q_i\rho\left(m\right)\psi_i\left(q;m,\underline{p}\right).$$

- Static Nash equilibrium:

$$p^N\left(c\right)\in\arg\max\sum_{m=\underline{m}}^{\overline{m}}\left(p_i-c\right)q_i\rho\left(m\right)\psi_i\left(q;m,p^N,\ldots,p_i,\ldots,p^N\right)$$

- Assumption: The one-shot game with cost c has, $\forall c\geq 0$, a symmetric Nash equilibrium price $p^N\left(c\right)$ that is continuous, increasing, and unbounded in c.

- Stage 1 (price): Each firm chooses price.
- Stage 2 (demand): With prices being private information, market demand is realized and each firm learns its sales.
- Stage 3 (report): With prices and quantities being private information, each firm submits a publicly observed costless message (where a message is to be interpreted as a sales report).
- Stage 4 (transfer): With prices and quantities being private information but reports being public information, each firm makes a payment to the other $n - 1$ firms.

- In the price stage:
  - if in the collusive phase then price at $\widehat{p}$
  - if in the non-collusive phase then price at p^N
- In the report stage:
  - if in the collusive phase then report q_i^t
  - if in the non-collusive phase then do not report
- In the transfer stage:
  - if in the collusive phase,
    - and all firms reported then make a payment of $r_i^t z$
    - and one or more firms did not report then make a zero payment
  - if in the non-collusive phase then make a zero payment

Collusion with Credible Reporting of Sales

Strategy Profile: Transition from Collusive Phase to Non-collusive Phase

- In the transition stage (public randomization device):
  - if in the collusive phase,
    - all firms reported and $x_j^t = r_j^t z \ \forall j$ (where x_j^t is firm j's payment), then firms remain in the collusive phase with probability $1 - \phi\left(\sum_{j=1}^{n} r_j^t\right)$ and shift to the non-collusive phase with probability $\phi\left(\sum_{j=1}^{n} r_j^t\right)$
    - otherwise, go to the non-collusive phase with probability one
  - if in the non-collusive phase then remain in the non-collusive phase with probability one.

Theorem

For any $\varepsilon > 0$ and $\widehat{p} > p^N$, if δ is sufficiently close to one and $\frac{\mu}{\overline{m}-\mu}$ is sufficiently high then the lysine strategy profile with collusive price $\widehat{p}$ is a semi-public perfect equilibrium and the probability of a price war is less than ε.

- μ is average market sales.
- $\overline{m}$ is maximal market sales.

Collusion with Credible Reporting of Sales

Equilibrium Condition: Price

- Firm 1's payoff in the collusive phase:

$$\sum_{m=\underline{m}}^{\overline{m}} \rho(m) \sum_{q=0}^{m} \psi_1(q; m, \underline{p}) \left\{ \left[(p_1 - c) q + z \left(\frac{m-q}{n-1} \right) - zq \right] \right.$$
$$\left. + \phi(m) \delta V^N + (1 - \phi(m)) \delta V \right\}$$

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- Equilibrium condition:

$$\widehat{p} \in \arg\max \sum_{m=\underline{m}}^{\overline{m}} \rho(m) \sum_{q=0}^{m} \psi_1(q; m, p_1, \widehat{p}, \ldots, \widehat{p}) \times \left(p_1 - c - \left(\frac{n}{n-1} \right) z \right) q$$

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$$\left(p_1 - c - \left(\frac{n}{n-1} \right) z \right) q$$

- Equilibrium price: $\widehat{p} = p^N \left(c + \left(\frac{n}{n-1} \right) z \right)$.

- Given report r_i, firm i makes a transfer of zr_i iff

$$\sum_{m=\underline{m}}^{\overline{m}} \sigma_i\left(m \,|\, q_i, \underline{p}\right)\left[z\left(\frac{m-q_i}{n-1}\right) - zr_i + \phi\left(m+r_i-q_i\right)\delta V^N + \left(1-\phi\left(m+r_i-q_i\right)\right)\delta V\right]$$

$$\geq \sum_{m=\underline{m}}^{\overline{m}} \sigma_i\left(m \,|\, q_i, \underline{p}\right)\left[z\left(\frac{m-q_i}{n-1}\right) + \delta V^N\right]$$

$$\sum_{m=\underline{m}}^{\overline{m}} \sigma_i\left(m \,|\, q_i, \underline{p}\right)\left(1-\phi\left(m+r_i-q_i\right)\right)\delta\left(V-V^N\right)$$

$$\geq zr_i$$

Collusion with Credible Reporting of Sales

Equilibrium Condition: Report

- Given q_i, firm 1's expected payoff from reporting r_i is

$$\sum_{m=\underline{m}}^{\overline{m}} \sigma_i \left(m \middle| q_i, \underline{p}\right) \left\{ \left[(p_i - c) q_i + z \left(\frac{m - q_i}{n-1} \right) - z r_i \right] \right.$$
$$\left. + \phi (m - q_i + r_i) \delta V^N + (1 - \phi (m - q_i + r_i)) \delta V \right\}.$$

- Reporting q_i is preferred to reporting r_i $(\neq q_i)$ iff

$$\sum_{m=\underline{m}}^{\overline{m}} \sigma_i \left(m \middle| q_i, \underline{p}\right) \left[\phi (m - q_i + r_i) - \phi (m) \right] \times$$
$$\delta \left(V - V^N\right) \quad \geq z (q_i - r_i)$$

Collusion with Credible Reporting of Sales

Construction of Probability of Price War Function

- Discourage under-reporting

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  - Assumption: $\phi\left(\sum_{j=1}^{n} r_j^t\right)$ is decreasing in $\sum_{j=1}^{n} r_j^t$, $\forall \sum_{j=1}^{n} r_j^t \in \{\underline{m}, \ldots, \overline{m}\}$.

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- Avoid inefficiencies from price wars.
  - Assumption: $\lim_{\delta \to 1} \max\{\phi(m) : \underline{m} \leq m \leq \overline{m}\} = 0$.

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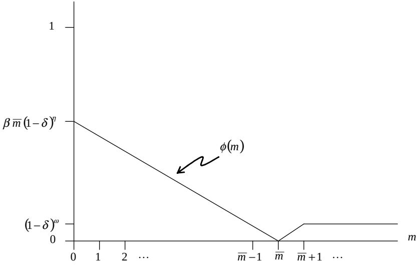
  - Assumption: For $\sum_{j=1}^{n} r_j^t > \overline{m}$, $\phi\left(\sum_{j=1}^{n} r_j^t\right)$ is large relative to $\max\{\phi(m) : m \leq \overline{m}\}$.
- Avoid inefficiencies from price wars.
  - Assumption: $\lim_{\delta \to 1} \max\{\phi(m) : \underline{m} \leq m \leq \overline{m}\} = 0$.
- Assumption: $\phi\left(\sum_{j=1}^{n} r_j^t\right)$ is weakly convex in $\sum_{j=1}^{n} r_j^t$, $\forall \sum_{j=1}^{n} r_j^t \in \{\underline{m}, \ldots, \overline{m}\}$.

- Probability of punishment function, $\phi : \{0, 1, 2, \ldots\} \to [0, 1]$

$$\phi(m) = \begin{cases} \beta(\overline{m} - m)(1-\delta)^{\eta} & \text{if } m \leq \overline{m} \\ (1-\delta)^{\omega} & \text{if } \overline{m} < m \end{cases}$$

where $\beta > 0$ and $0 < \omega < \eta < 1$.



- Assumptions
  - Market demand is 0 or 1; ρ is the probability $m = 1$.
  - Assume $p^N = \frac{1}{bn} + c$.
- Properties of an optimal equilibrium in this class:

$$\phi(0) = \frac{(n-1)(1-\delta)}{\rho - \delta(n-1)(1-\rho)}, \phi(1) = 0, \phi(2) = 1$$

- Probability of a price war, $(1-\rho)\phi(0)$, is
  - decreasing in the discount factor, δ
  - increasing in the number of firms, n
  - decreasing in the probability of the high demand state, ρ

Concluding Remarks

- Summary of main findings
  - For a common class of demand systems, symmetric punishments are unable to sustain any collusion for any discount factor.
  - Collusion can be sustained through
    - accurate self-reporting of sales
    - asymmetric punishments conditional on sales.
- Further work
  - weaken demand restriction: $\frac{\underline{\mu}}{\overline{m}-\underline{\mu}}$ is required to be sufficiently high
  - characterize optimal equilibria - is there a better collusive mechanism?
  - allow market demand to be sensitive to firms' prices
  - explore the optimal amount of delay in communication