A Comparison of the Wholesale Structure and the Agency Structure in Differentiated Markets

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1. Introduction

Today individuals rarely purchase goods directly from the manufacturer. We shop at high street stores, supermarkets, as well as a variety of online retailers. With the application of advanced information technology, goods are now passing through more elaborate supply and distribution chains. These supply and distribution chains, not surprisingly, are not all structured in single and identical ways. Among the various vertical structures observed in practice there are two common ones, a wholesale structure and an agency structure. Under the wholesale structure, retailers buy from suppliers and resell to final consumers, e.g., florists buy roses at flower markets and resell them in store. Under the agency structure, suppliers set prices and retailers merely help to make transactions happen. In return, retailers receive shares of revenue specified by themselves, e.g., eBay sets a “final value fee” rate and receives a fraction of sellers’ total revenues.¹

Whilst the wholesale structure remains the common business format in a bricks-and-mortar environment, the agency structure becomes predominant in online markets. Some natural questions to ask then would be why different vertical structures are adopted in different markets, and which structure is beneficial to consumers. In this paper, we seek to analyse how answers to these two questions are related to the degrees of product differentiation in vertically-related markets. We do so by examining the two structures in a bilateral duopoly framework with product differentiation at both the supplier level and the retailer level.

Suppose that the vertical structure under which a market operates is determined by firms in the market. Suppliers’ and retailers’ considerations on the choice of structure would depend on, among other market circumstances, their preferences over different structures, which is driven by the relative profitability, and their power relations within the market. We focus on firms’ preferences, given which, a chosen business format could per se reveal interesting characteristics about the market.

Further, since many giant online retailers such as Amazon marketplace, Apple, eBay, Google and various booking websites have adopted the agency structure, one may question whether the agency structure is a means through which strong retailers abuse their power, at least the Department of Justice (DOJ)’s decision on forcing publishers to move away from the

¹ See eBay Seller Centre [http://sellercentre.ebay.co.uk/introduction-ebay-fees](http://sellercentre.ebay.co.uk/introduction-ebay-fees).
agency structure for e-books seems to send such a signal. The welfare conditions of different vertical structures are hence interesting and relevant issues to examine.

Recent papers studying the agency structure are overwhelmingly inspired by the (in)famous e-book case, therefore tend to address a more specific question: why was the agency structure adopted in the e-book market? There are several motivating focuses: i) Device, e.g. smartphones and tablets (Gaudin and White, 2014); ii) The Most Favoured Nation (MFN) clause (Boik and Corts, 2013); iii) Consumer lock-in (Johnson, 2013b); iv) Asymmetric information (Condorelli et al., 2013). Meanwhile, there is a lack of systematic analysis of the agency structure per se as well as detailed comparisons of the wholesale and agency structures, which constitutes the basis of understanding some transformations in vertical relations. We take the initial steps to fill the gap by comparing the equilibrium outcomes under the two structures. We first assess welfare implications and then examine firms’ preferences in relation to the choice of business format, in order to explain some aspects of the decision-making on business formats in vertically-related markets and the associated social effects.

In symmetric equilibrium we find that retail prices are always lower and quantities demanded are always higher under the agency structure relative to the wholesale structure. The intuition is simple: with supplier pricing, double marginalisation that exists under the wholesale structure disappears under the agency structure. To prove this, we show that the total mark-up under the agency structure is equal to the first mark-up (wholesale price over marginal cost) under the wholesale structure.

In our model, the equilibrium consumer surplus is always higher under the agency structure, whereas industry profits are higher under the wholesale structure for a wider range of degrees of product differentiation in the market. Summing the two up, we show that a social planner favours the wholesale structure when supplier goods and retailer services are approximately homogenous. As goods and services start to become more differentiated, her favoured business format quickly shifts to the agency structure. Furthermore, under each structure, the social planner prefers more differentiation in the market to less.

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2 In April 2010, five large book publishers in the US switched from the wholesale structure, which they used to have with Amazon, to selling e-books to the agency structure put forward by Apple. Following the price rise of e-books after the switch, the DOJ has lodged a complaint against Apple and publishers for their contractual agreements. For details see *Competition Law Journal Concurrences* No.3-2012 on “e-Books and the Boundaries of Antitrust” [http://ssrn.com/abstract=2140778](http://ssrn.com/abstract=2140778).

3 Johnson (2014) suggests that double marginalization exists also under the agency structure because of revenue sharing. Revenue sharing lowers suppliers’ per-unit profits, suppliers therefore perceive marginal costs to be higher, which act as the first mark-up. Findings from our model, however, show that there is only one mark-up under the agency structure.
In order to tell intuition from the choice of business format in vertically-related markets, we examine firms’ preferences over the two structures. If suppliers and retailers have conflicting preferences over vertical structures, then it must be that the chosen structure makes one party better off while makes the other worse off. Therefore, *ceteris paribus*, the chosen structure by itself could offer interesting conjectures on the power relations between suppliers and retailers.

Results show that suppliers always prefer the wholesale structure, whereas retailers prefer the agency structure for a wider range of degrees of product differentiation in the market. Furthermore, for retailers, the relative profitability of the alternative schemes is sensitive to the degrees of differentiation at the supplier level, which also decides whether the two parties can have consistent preference. The two parties would both wish to adopt the wholesale structure if the degrees of differentiation at the supplier level are sufficiently low, and they would have conflicting preferences, i.e., suppliers prefer the wholesale structure and retailers prefer the agency structure, otherwise. This would generally suggest that, *ceteris paribus*, suppliers are in a relatively stronger position if the vertically-related market operates under the wholesale structure, and that retailers are in a relatively stronger position if the market operates under the agency structure.

Immediately, this explains why in real life the agency structure is initiated by retailers – suppliers never have the incentive to switch away from the wholesale structure. For large online retailers with strong network and negotiation power, their preferred business format may be part of the “take it or leave it offer” they have for suppliers, while suppliers may be vulnerable and have no choice. For instance, it appears that it was Apple who persuaded publishers to adopt the agency structure.

The fact that retailers could benefit from the differentiation at the supplier level is interesting, as it contrasts with our conventional understanding of the relationships between firms’ profitability and the degrees of differentiation at different levels of the market. Given that they do not collude, firms in general would benefit from high degrees of differentiation at their own level and low degrees of differentiation at the other level of the market, such that they can exercise market power. This is true under the wholesale structure but not under the agency structure, meaning that the two parties’ incentives are better aligned under the agency structure.
The popularity of the agency structure in some markets in turn implies the powerful position of retailers in those markets.\textsuperscript{4} With regard to the concerns stated earlier on that strong retailers may abuse their power, while it may be true to suppliers, since suppliers are essentially “forced” to accept the agency structure and potentially other terms,\textsuperscript{5} it does not apply to consumers whose surplus are always higher under the agency structure. In fact, social surplus is maximized under the agency structure when goods and services are perfectly differentiated. Therefore adopting the agency structure is socially desirable when the degrees of differentiation in the market are high.

The paper proceeds as follows. Section 2 discusses the related literature. Section 3 presents the framework for our analysis, in which we characterize the vertical relation first by the wholesale structure and then by the agency structure. Section 4 compares the symmetric equilibrium outcomes under the two structures to assess welfare implications and examine firms’ preferences over the alternative schemes. Section 5 concludes.

\subsection*{2. Related Literature}

This paper is related to the extensive literature on vertical relations, especially those on vertical restraints such as resale price maintenance (RPM). RPM is generally viewed as a practice imposed by suppliers to exercise power over prices. Traditional and modern economic theory suggest various situations in which suppliers would desire to adopt RPM and the associated welfare implications (e.g., Telser, 1960; Marvel and McCafferty, 1984; Winter, 1993; Dobson and Waterson, 2007; Asker and Bar-Isaac, 2014). With the long existence of the argument that RPM may be sponsored by retailers to dampen downstream competition (Pickering, 1966), such “retailer-sponsored” RPM almost always generates higher retail prices and harms consumers (e.g., Dobson and Waterson, 1999).

Since one of the characteristics of the agency structure is supplier pricing (as opposed to retailer pricing under the wholesale structure), some studies regard RPM and the agency structure as similar contracts (e.g., Foros, \textit{et al}, 2013). We show that the agency structure could be seen as an extreme form of RPM under the wholesale structure, given the condition that the degree of differentiation at the retailer level of the market is approaching zero. In contrast to

\textsuperscript{4} Although not central to our analysis, we are aware that such popularity and strong positions of online retailers may also have been reinforced and accelerated by the rise of online markets, which retain some specific characteristics, e.g., concerns about inventory and asymmetric information tend to mitigate.

\textsuperscript{5} This might no longer hold if side payments are paid to suppliers by retailers.
the previous literature, this paper suggests that supplier pricing could be socially desirable even when retailer power is strong.

More recent studies focus on the different types of vertical structures, varying in both decision roles, i.e., retail prices are decided by suppliers or retailers, and contract forms, i.e., wholesale or revenue sharing. Foros, et al (2013) compare the agency structure to an alternative structure in which revenue sharing rates and retail prices are both set by retailers. They rationalize retailers’ decision to give up pricing in the cases where the degrees of differentiation are relatively higher at the supplier level. But they follow closely the e-book market and do not consider the wholesale structure.

Johnson (2014) compares the wholesale structure and the agency structure with a focus on clarifying the existence of double marginalisation under the latter. Different from our paper, he finds that retailer profits are always higher under the agency structure. The difference arises mainly because market coverage is assumed differently; he assumes full market coverage hence the changes in demand are omitted. Demand in our paper is decided by two countervailing forces – substitution effect and price effect. As goods and services become more substitutable, the average purchase goes down. Nevertheless retail prices are also lower, which lead to higher demand. In equilibrium, demand and the degrees of differentiation exhibit a U-shape relationship. Furthermore, while Johnson does not discuss social surplus, it delivers important implication in our paper.

The relevant comparisons are also made in the literature of intermediary, with a focus on asymmetric information – intermediaries have privileged information about consumers.\textsuperscript{6} Condorelli et al (2013) suggest that efficiency increases when intermediaries refer consumers for a fee (the agency structure), instead of buying and reselling (the wholesale structure). Besides the wholesale structure and the agency structure, Liu and Shuai (2015) consider two more, namely, the “reversed wholesale structure” and the “reversed agency structure”.\textsuperscript{7} They focus on the comparisons of welfare across regimes but do not discuss the preferences of suppliers and retailers over different regimes.

\textsuperscript{6} There are studies, e.g., Hagiu and Wright (2014), examining the two structures in the context of two-sided markets. In this paper, retailers under the agency structure obtain shares of revenues from suppliers only and do not charge consumers for their service. For example, eBay collects fees from sellers but do not charge buyers for their purchasing activities.

\textsuperscript{7} In Liu and Shuai (2015), the reversed wholesale structure characterizes a vertical relation in which retailers first set reversed wholesale prices and then suppliers set retail prices; under the reversed agency structure, for a given revenue sharing rate, retailers set retail prices.
3. Model

We employ a bilateral duopoly framework to incorporate differentiation at both the supplier level and the retailer level. In a vertically-related market, there are two suppliers, \( j = 1, 2 \) and two retailers, \( i = 1, 2 \). Each supplier produces a single good \( j \) and each retailer \( i \) presents final consumers with goods from both suppliers, i.e., \( q_i^j > 0 \), such that vertical contracts are nonexclusive. Correspondingly, consumers are able to choose from four “final goods”. We assume that goods produced by suppliers and services offered by retailers are symmetrically differentiated. For simplicity, we normalized all costs to zero.

We assume a continuum of consumers and the representative consumer maximizes \( V = U(y) + U(q) \) subject to the budget constraint \( I = y + \sum p q \), where \( p = \{ p_i^j, p_{-i}^j, p_{-i}^{-j}, p_{-i}^{-j} \} \), \( q = \{ q_i^j, q_{-i}^j, q_{-i}^{-j}, q_{-i}^{-j} \} \) and \( y \) is the quantity of composite goods consumed at price of one. Following Dobson and Waterson (1996, 2007) and Gabrielsen and Johansen (2015), this is equivalent to having consumers maximize

\[
U(q) = \sum_{i,j} q_i^j - \frac{1}{2} q_1^2 - \beta (q_1^1 q_2^1 + q_1^2 q_2^2) - \gamma (q_1^1 q_2^1 + q_1^2 q_2^2)
\]

\[-\beta \gamma (q_1^1 q_2^1 + q_1^2 q_2^2).\]  

(1)

This utility function gives rise to the downward sloping inverse demand function

\[
p_i^j = 1 - q_i^j - \beta q_{-i}^j - \gamma q_{-i}^{-j} - \beta \gamma q_{-i}^{-j}.\] 

(2)

The parameter \( \beta \in [0, 1] \) measures the degree of competition between retailer services and the parameter \( \gamma \in [0, 1] \) measures the degree of competition between goods.\(^8\) Retailer services are perceived to be perfectly differentiated when \( \beta = 0 \) and become closer substitutes as \( \beta \to 1 \). Likewise, when \( \gamma = 0 \), the two goods are viewed as perfectly differentiated and demand-unrelated, as \( \gamma \to 1 \), they become closer substitutes. We assume that neither suppliers nor retailers behave cooperatively in setting prices, and the direct demand function is then given by

\[
q_i^j = \frac{(1-\beta)(1-\gamma) - p_i^j + \beta p_{-i}^j - \gamma p_{-i}^j}{(1-\beta^2)(1-\gamma^2)}.
\] 

(3)

The quantity demanded for good \( j \) at retailer \( i \) is a function of own price, \( p_i^j \), prices of two relatively closer substitutes, \( p_{-i}^j \) and \( p_{-i}^{-j} \), and the price of another substitute that is further away

\[^8\] Dobson and Waterson (1996) assume \( \gamma \in (-1, 1) \) where a negative \( \gamma \) indicates that the goods are complements. We do not consider this case in this paper.
in the product space, \( p^{-1}_i \). Given the ranges of parameters, it can be easily verified that own-price effect dominates each cross-price effect.

One feature of the above direct demand system is that, the cross-price effect between different goods presented by different retailers is negative; \( \partial q^1_i / \partial p^{-1}_j = -\beta \gamma / (1 - \beta^2)(1 - \gamma^2) < 0 \), which contrasts to our understanding that as the price of one good increases, the demand of its substitute usually increases. Gabrielsen and Johansen (2015) suggest that the negative cross-price effect can be explained as a “second-order effect”. More specifically, as \( p^{-1}_i \) increases, consumers would optimally switch away from good \((-i, -j)\) and demand more \((i, -j)\) and \((-i, j)\), as these two are relatively closer substitutes to \((-i, -j)\). However, as more \((i, -j)\) and \((-i, j)\) are consumed, being relatively closer substitutes to them, \((i, j)\) would consequently be demanded less. This negative “second-order effect” dominates the direct substitution effect, leading to a negative cross-price effect.

Overall this demand system allows for differentiation at different levels of the vertically-related market to be parameterised in a convenient form, offering clear and tractable solutions. We are now going to model the two vertical structures. Suppose that suppliers cannot directly reach final consumers, then the manner in which the vertical relation is characterized differs between the two structures. We focus on symmetric equilibrium throughout the analysis.

### 3.1 The Wholesale Structure

We continue using the florist business as an example to describe a simple form of the wholesale structure. Assume that we have the flower wholesalers or florist suppliers who grow their own flowers and plants and do not buy from anyone else. They set wholesale prices and sell to florists who then set retail prices and deal with final consumers. Therefore the timing under the wholesale structure is as follows

1. Suppliers set wholesale prices simultaneously. The wholesale price set by supplier \( j \) to retailer \( i \) is \( w^1_i \).

2. Retailers set retail prices simultaneously. The price set by retailer \( i \) for good \( j \) is \( p^1_i \).

Retailer \( i \), denoted as \( R_i \), faces the following optimization problem

\[
\max_{p^1_i, p^2_i} \pi_{R_i} = \max_{p^1_i, p^2_i} [ (p^1_i - w^1_i)q^1_i + (p^2_i - w^2_i)q^2_i ]. \tag{4}
\]

The corresponding first-order condition is given by
\[
\frac{\partial \pi_R}{\partial p_i} = q_i^1 + (p_i^1 - w_i^1) \frac{\partial q_i^1}{\partial p_i} + (p_i^2 - w_i^2) \frac{\partial q_i^2}{\partial p_i} = 0. \tag{5}
\]

The second-order condition is fulfilled; \(\frac{\partial^2 \pi_R}{\partial (p_i^j)^2} = -2/(1 - \beta^2)(1 - \gamma^2) < 0\). Since we focus on symmetric equilibrium, we write the best response function as

\[
p_i^j = \frac{(1 - \beta)(1 - \gamma) + \beta p_i^j + (2 - \beta)\gamma p_i^j - w_i^j - \gamma w_i^j}{2}. \tag{6}
\]

Solving (6) we get

\[
p_i^j = \frac{1 - \beta + w_i^j}{2 - \beta}. \tag{7}
\]

Inserting (7) into (3) we can write

\[
q_i^j = \frac{(1 - \gamma) - w_i^j + \gamma w_i^j}{(1 + \beta)(2 - \beta)(1 - \gamma^2)}. \tag{8}
\]

Supplier \(j\), denoted as \(S_j\), faces the following optimization problem

\[
\max_{w_1^j, w_2^j} \pi^{S_j} = \max_{w_1^j, w_2^j} (w_1^j q_1^j + w_2^j q_2^j), \tag{9}
\]

which, given symmetry, becomes

\[
\max_{w_i^j} \pi^{S_i} = \max_{w_i^j} 2w_i^j q_i^j. \tag{10}
\]

The corresponding first-order condition is given by

\[
\frac{\partial \pi^{S_i}}{\partial w_i^j} = 2w_i^j \frac{\partial q_i^j}{\partial w_i^j} + 2q_i^j = 0, \tag{11}
\]

from which we have the following best response function

\[
w_i^j = \frac{(1 - \gamma) - \gamma w_i^j}{2}. \tag{12}
\]

From (12) we obtain the symmetric equilibrium wholesale prices, denoted as \(w^*\)

\[
w^* = \frac{1 - \gamma}{2 - \gamma}. \tag{13}
\]

The second-order condition is fulfilled; \(\frac{\partial^2 \pi^{S_i}}{\partial (w_i^j)^2} = -4/(1 + \beta)(2 - \beta)(1 - \gamma^2) < 0\). Due to the nature of the sequential game, \(w^*\) depends only on \(\gamma\), the degrees of differentiation at the supplier level.

Inserting (13) into (7) and (8), the symmetric equilibrium retail prices and demand, denoted as \(p^*\) and \(q^*\) respectively, are given by

\[
p^* = \frac{(1 - \beta)(2 - \gamma) + 1 - \gamma}{(2 - \beta)(2 - \gamma)}, \tag{14}
\]

\[
q^* = \frac{(1 - \gamma) - \gamma w^* + \gamma w^*}{(1 + \beta)(2 - \beta)(1 - \gamma^2)}. \tag{15}
\]
They are determined by the degrees of differentiation at both levels of the market. We present the complete set of symmetric equilibrium under the wholesale structure including firms’ profits in the following lemma, which will be further useful in the analysis.

**Lemma 1.** Under the wholesale structure, there exists a symmetric equilibrium in which wholesale prices $w^* = (1 - \gamma)/(2 - \gamma)$, retail prices $p^* = [(1 - \beta)(2 - \gamma) + 1 - \gamma]/(2 - \beta)(2 - \gamma)$, quantities demanded (per good at per retailer) $q^* = 1/(1 + \beta)(2 - \beta)(1 + \gamma)(2 - \gamma)$, supplier $j$’s profits $\pi^{s_j} = 2(1 - \gamma)/(1 + \beta)(1 + \gamma)(2 - \beta)(2 - \gamma)^2$ and retailer $i$’s profits $\pi^{r_i} = 2(1 - \beta)/(1 + \beta)(1 + \gamma)(2 - \beta)^2(2 - \gamma)^2$.

### 3.2 The Agency Structure

Now we consider the agency structure. For example, eBay as an online retailer, does not buy goods from sellers and resell to consumers. Instead, eBay specifies a revenue sharing rate and is in a role of distributing sellers’ listings to potential consumers, whereas prices are set directly by sellers. Every time a good is sold, eBay gets 10% of the total revenue (including postage) generated. Therefore the timing under the agency structure is as follows

1. Retailers declare the revenue sharing rates simultaneously. The revenue sharing rate set by retailer $i$ is $\alpha_i \in [0,1)$.

2. Suppliers set prices simultaneously. The price set by supplier $j$ to retailer $i$ is $p^j_i$.

Revenue generated is split according to $\alpha_i$, with suppliers retaining $1 - \alpha_i$ of it. When $\alpha_i = 0$, the share of revenue obtained by retailer $i$ is zero and when $\alpha_i \rightarrow 1$, it is becoming 100%.

As there is symmetry between firms at each level of the market, we allow retailers to specify one sharing rate that applies to both suppliers, and this does not conflict with what we observe in real life. It is common that retailers specify one sharing rate and regard it as universal knowledge. For example, Apple claims the same rate to all book publishers, eBay claims the same rate to all sellers and Google claims the same rate to all apps developers.

Under this structure, $S^j$ obtains the price control and faces the following optimization problem

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9 This describes the fixed price listing. eBay also offers an auction-style listing under which prices are the highest bids when auctions end. For details see [http://sellercentre.ebay.co.uk/auction-style-or-fixed-price-listing](http://sellercentre.ebay.co.uk/auction-style-or-fixed-price-listing).
\[
\max \pi_i^{s_i} = \max_{p_i^1, p_i^2} [(1 - \alpha_1)p_i^1 q_i^1 + (1 - \alpha_2)p_i^2 q_i^2].
\]  
(16)

The corresponding first-order condition is given by
\[
\frac{\partial \pi_i^{s_i}}{\partial p_i^1} = (1 - \alpha_1)(q_i^1 + p_i^1 \frac{\partial q_i^1}{\partial p_i^1}) + (1 - \alpha_2)p_i^2 \frac{\partial q_i^2}{\partial p_i^1} = 0,
\]  
(17)
from which we obtain the following best response function
\[
p_i = \frac{(1-\beta)(1-\gamma)(1-\alpha_i)+\beta p_{i-1}(1-\alpha_i)+(1-\gamma)(1-\alpha_i)}{(2-\gamma)(1-\alpha_i)}.
\]  
(18)

The second-order condition is fulfilled; \(\frac{\partial^2 \pi_i^{s_i}}{\partial (p_i^1)^2} = -2(1 - \alpha_1)/(1 - \beta^2)(1 - \gamma^2) < 0\).

Solving (18) we get
\[
p_i = \frac{(1-\beta)(1-\gamma)[(2-\gamma)+\beta(1-\gamma)+\beta(1-\gamma)]}{L(1-\alpha_i)}.
\]  
(19)
where \(L = (2 - \gamma)^2 - \beta^2 \left[1 + (1 - \gamma)^2 + \frac{(1-\gamma)(1-\gamma) + (1-\gamma)}{(1-\alpha_i)(1-\alpha_i)}\right]\). Inserting (19) into (3) we can write
\[
q_i = \frac{L(1-\gamma)[(1-\beta)+(1-\gamma)^2(1-\gamma)+\beta(1-\gamma)]}{L(1+\beta)(1+\gamma)}.
\]  
(20)

\(R_i\) faces the following optimization problem
\[
\max \pi_{R_i} = \max_{\alpha_i} \alpha_i (p_i^1 q_i^1 + p_i^2 q_i^2).
\]  
(21)

Since \(R_i\) rationally anticipating \(S^j\)'s decision, we can write
\[
\max \pi_{R_i} = \max_{\alpha_i} 2\alpha_i p_i q_i.
\]  
(22)

\(R_i\)'s first-order condition is given by
\[
\frac{\partial \pi_{R_i}}{\partial \alpha_i} = 2p_i q_i + 2\alpha_i \left[p_i \frac{\partial q_i}{\partial \alpha_i} + q_i \frac{\partial p_i}{\partial \alpha_i}\right] = 0.
\]  
(23)

Solving the corresponding best response function for the symmetric equilibrium revenue sharing rate,\(^10\) denoted as \(\alpha^*\)
\[
\alpha^* = \frac{(2-\gamma)(1-\beta^2)}{2-\gamma(1+\beta)}.
\]  
(24)

\(^10\) The stability of this symmetric equilibrium has been proved by Foros et al. (2013). For details see their Proposition 4.
Inserting (24) into (19) and (20), the symmetric equilibrium retail prices and demand under the agency structure, denoted as \( p_A^* \) and \( q_A^* \) respectively, are given by

\[
p_A^* = \frac{1 - \gamma}{2 - \gamma}, \tag{25}
\]

and

\[
q_A^* = \frac{1}{(1 + \beta)(1 + \gamma)(2 - \gamma)}. \tag{26}
\]

The following lemma summarizes the complete set of symmetric equilibrium under the agency structure.

**Lemma 2.** Under the agency structure, there exists a symmetric equilibrium in which revenue sharing rates \( \alpha^* = (2 - \gamma)(1 - \beta^2)/(2 - \gamma(1 + \beta)) \), retail prices \( p_A^* = (1 - \gamma)/(2 - \gamma) \), quantities demanded (per good at per retailer) \( q_A^* = 1/(1 + \beta)(1 + \gamma)(2 - \gamma) \), supplier j’s profit \( \pi_{ij}^S = 2\beta(1 - \gamma)[2\beta - \gamma(1 + \beta)]/(1 + \beta)(1 + \gamma)(2 - \gamma)(2 - \gamma)(2 - \gamma(1 + \beta)) \) and retailer i’s profits \( \pi_{RiA} = 2(1 - \beta)(1 - \gamma)/(1 + \gamma)(2 - \gamma)(2 - \gamma)(2 - \gamma(1 + \beta)). \)

Before we go into details on the comparisons between the two sets of equilibrium outcomes, we draw a link among the wholesale structure, RPM and the agency structure.

**Corollary 1.** As the degree of differentiation at the retailer level of the market is approaching zero, i.e., \( \beta \rightarrow 1 \), the outcomes under the agency structure and the wholesale structure are approximately the same.

**Proof.** Appendix A.

**Corollary 1** says that some degrees of differentiation at the retailer level are crucial in generating differences between the equilibrium outcomes under the two structures. The key reason is that the agency structure may be seen as an extreme form of RPM under the wholesale structure. RPM usually involves suppliers imposing certain price restrictions, an extreme form of which would be that suppliers directly set retail prices. Under the wholesale structure, when \( \beta \rightarrow 1 \), suppliers can easily impose contractual obligations including directly setting retail prices, hence the two structures become the same with regard to pricing. This is reflected by the convergence in equilibrium outcomes under the two structures as the degrees of differentiation at the retailer level decrease.

We close this section by commenting on the relationship between demand and the two parameters measuring the degrees of differentiation at each level of the market, \( \beta \) and \( \gamma \), which will be useful later in the analysis.
Lemma 3. Under the wholesale structure, the equilibrium quantities demanded decrease in $\beta$ and $\gamma$ for $\beta, \gamma \in [0, 1/2)$, and increase in $\beta$ and $\gamma$ for $\beta, \gamma \in (1/2, 1)$. Under the agency structure, the equilibrium quantities demanded always decrease in $\beta$ whereas they decrease in $\gamma$ for $\gamma \in [0, 1/2)$ and increase in $\gamma$ for $\gamma \in (1/2, 1)$.

**Proof.** Appendix B.

Under the wholesale structure, $\beta$ and $\gamma$ affect demand through two effects: a direct substitution effect and an indirect price effect. As $\beta$ and $\gamma$ increase, on one hand, goods and services become less differentiated hence average demand falls; on the other hand, retail prices decrease which lead to higher demand. Whether demand increases or decreases in $\beta$ and $\gamma$ thus depends on which effect is stronger. In equilibrium, demand and each of the two parameters exhibit a U-shape relationship. Under the agency structure, the U-shape relationship between $\gamma$ and demand stays whereas demand is strictly decreasing in $\beta$. Since $\beta$ does not affect retail prices under the agency structure, it affects demand only through the negative substitution effect.

4. Analysis

In this section we first compare the two sets of symmetric equilibrium following Lemmata 1 and 2 to assess welfare implications, and then examine firms’ preferences over alternative schemes. Results would enable us to discuss scenarios in which the two parties have preferences that are consistent or conflicting, as well as given ranges of degrees of product differentiation, whether the market’s chosen business format is socially desirable.

4.1 Welfare Comparison

We start with the following proposition.

**Proposition 1.** In equilibrium, retail prices are lower, quantities demanded are higher and consumer surplus is higher under the agency structure than under the wholesale structure.

**Proof.** Appendix C.

The intuition from Proposition 1 is simple: double marginalisation is eliminated under the agency structure and consumers benefit from lower prices and higher consumption. To show

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11 See Figure 1 illustrating the two countervailing effects in Appendix B.
this, we compare the equilibrium mark-ups under the two structures. The sum of two mark-ups under the wholesale structure is\(^{12}\)

\[
\frac{w^*}{1st} + \frac{p^* - w^*}{2nd} = \frac{1-\gamma}{2-\gamma} + \frac{1-\beta}{(2-\beta)(2-\gamma)} \tag{27}
\]

With supplier pricing, the total mark-up under the agency structure is equivalent to \(p_A^* = (1 - \gamma)/(2 - \gamma)\). This is identical to the first mark-up under the wholesale structure, which means that the second mark-up stated in (27) is avoided under the agency structure. Hence if a social planner cares only about consumers, she will prefer the agency structure to the wholesale structure.

Since demand expands as retail prices fall, we do not know a priori which structure yields higher profits, and thereafter, higher social surplus. Figure 2 illustrates the comparisons of industry profits and social surplus between the two structures. On both plots, the agency outcome is relatively lower in the grey area and is relatively higher in the white area.

\[\text{Figure 2. Comparisons of industry profits (left panel) and social surplus (right panel)}\]

**Observation 1.** When goods and services are approximately homogenous, i.e., \(\beta \to 1\) and \(\gamma \to 1\), industry profits and social surplus are both relatively higher under the wholesale structure.\(^{13}\) When goods and services are perfectly differentiated, i.e., \(\beta = 0\) and \(\gamma = 0\), industry profits and social surplus are both relatively higher under the agency structure.

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\(^{12}\) Here we assume that firms incur zero marginal costs, therefore the cost component does not appear in the equation. With positive marginal costs, the first mark-up, i.e., the wholesale mark-up under the wholesale structure becomes \(w^* - c\).

\(^{13}\) Following **Corollary 1**, the difference between equilibrium outcomes under the two structures are very small when goods and services are approximately homogenous, but as long as \(\beta < 1\), we are able to rank them.
Observation 1 describes two polar scenarios: when final goods presented in the market are close substitutes and when they are demand-unrelated. However for a social planner, the shift in favoured business format from the wholesale structure to the agency structure takes place long before $\beta$ and $\gamma$ both approaches the origin. Furthermore, such shift is more responsive to the increase in product differentiation at the supplier level, i.e., a lower $\gamma$, than the increase in product differentiation at the retailer level, i.e., a lower $\beta$. As shown on Figure 2 (right panel), when $\gamma$ remains high at point $h$, i.e., supplier goods are approximately homogenous, social surplus becomes relatively higher under the agency structure only when $\beta$ gets close to the origin. That is, huge increase in product differentiation at the retailer level is required. But when retailer services are approximately homogenous, at point $k$, a much smaller increase in product differentiation at the supplier level would be sufficient for the social planner to shift her preference. Moreover, as long as the $\gamma$ is no higher than that at point $k$, a social planner who wishes to maximise social surplus would prefer the agency structure to the wholesale structure, regardless of the value of $\beta$.

Then we seek to explain what we have observed: why does the agency structure generate relatively higher social surplus as the degrees of product differentiation in the market, especially at the supplier level, increase? We suggest there are two reasons. First, at each given combination of $\beta$ and $\gamma$, the agency structure offers relatively lower retail price which leads to relatively higher total consumption. Second, with supplier pricing, not only suppliers but also retailers can benefit from, to some extent, the differentiation at the supplier level, therefore under the agency structure such differentiation is “appreciated” by both parties, as well as enjoyed by consumers. Consumers gain higher utility not only from consuming more goods, but also from consuming goods that are more differentiated. The second reason essential says that the incentives of suppliers and retailers are better aligned under the agency structure, which will be discussed in more details in Section 4.2. Now we summarize the broader picture that Figure 2 presents as below to complement Observation 1.

Observation 2. Suppose that both $\beta$ and $\gamma$ are independently and equally likely to take up any value over the interval $[0,1)$. Then there are more combinations of $\beta$ and $\gamma$ such that industry profits are lower and social surplus is higher under the agency structure than under the wholesale structure.

Observations 1 and 2 are useful in telling the social planner’s preferences over the two structures, given combinations of $\beta$ and $\gamma$. It is not explicitly clear, however, whether she prefers more differentiation to less.
Proposition 2. Industry profits and social surplus increase as supplier goods and retailer services become more differentiated, and they are maximised at the point of perfect differentiation at both levels of the vertically-related market characterized by the agency structure.

Proof. Appendix D.

Proposition 2 suggests that, for society as a whole, the benefits from higher degrees of differentiation always outweigh the benefits from lower prices. For example, following Lemma 3, we know that under the wholesale structure, total consumption is maximized at the two polar scenarios, i.e., at $\beta, \gamma \to 1$ and at $\beta, \gamma = 0$. Despite that retail prices are lower at the former, a social planner would favour the latter.

So far we have focused the analysis more on social surplus, which is higher under the agency structure for a wider range of degrees of differentiation. With regard to industry profits, although they are more likely to be relatively higher under the wholesale structure, as the degrees of differentiation increase, they eventually become higher under the agency structure. A better understanding on the industry profits requires the study on supplier profits and retailer profits respectively, which we focus on next.

4.2 Firms’ Preferences

In the presence of two vertical structures, we rule out side payments and other strategic considerations, and assume that firms always wish to choose the one that offers them higher profits. Given the symmetry at each level of the market, it follows that firms at the same level of the market will have preferences that are consistent, whereas preferences may differ across market levels.

Proposition 3. In equilibrium, in the absence of side payments, suppliers always prefer the wholesale structure to the agency structure, whereas there exists a $\gamma' \in [0,1)$ such that retailers always prefer the agency structure to the wholesale structure if $\gamma \in [0, \gamma')$.

Proof. Appendix E.

Proposition 3 offers a few useful insights. First it says that supplier profits are always relatively higher under the wholesale structure, meaning that the effect of relatively higher per-unit supplier profits under the wholesale structure always outweigh the effect of relatively higher demand under the agency structure. It is easy to verify the comparison of suppliers’ per-unit profits. Under the wholesale structure, suppliers’ per-unit profits are precisely the
equilibrium wholesale prices, \( (1 - \gamma)/(2 - \gamma) \); under the agency structure, suppliers’ per-unit profits are their shares of equilibrium retail prices, \( (1 - \alpha)(1 - \gamma)/(2 - \gamma) \), which are a fraction of the former.

Second, as illustrated in Figure 3, retailer profits are relatively higher under the agency structure if and only if \( \gamma \) is below a ceiling of \( \gamma' \), meaning that sufficient differentiation at the supplier level is a crucial condition when retailers rank the two structures. It is intuitive because when the degrees of differentiation at the supplier level are low, suppliers under the agency structure are unable to set high retail prices. Consequently for any given revenue sharing rate, retailers will receive low shares of revenues.\(^{14}\)

![Figure 3. The comparison of retailer profits](image)

Given that firms in a vertically-related market would usually benefit from high degrees of differentiation at their own level and low degrees of differentiation at the other level of the market, such that they can exercise market power, e.g., as under the wholesale structure, it is interesting to see that under the agency structure retailers could benefit from high degrees of differentiation at the supplier level. That is, the relationships between profitability and degrees of product differentiation in the market are different under the two structures: while it is conflicting under the wholesale structure, it is more aligned under the agency structure.

Then it is clear when suppliers and retailers would have consistent or conflicting preferences over the two vertical structures. Again we start with the scenarios when final goods are close substitutes or when they are perfectly differentiated.

\(^{14}\) Here we focus on the comparison of retailer profits between the two structures. Within the agency structure, \( \gamma \) affects retailer profits through two channels – a negative effect on retail prices and a positive effect on revenue sharing rates, and if we do not restrict the range of \( \gamma \), the combined effect on retailer profits is non-monotonic.
Observation 3. When goods and services are approximately homogenous, i.e., $\beta \to 1$ and $\gamma \to 1$, suppliers and retailers both prefer the wholesale structure to the agency structure. When goods and services are perfectly differentiated, i.e., $\beta = 0$ and $\gamma = 0$, and in the absence of side payments, suppliers prefer the wholesale structure whereas retailers prefer the agency structure.

Observation 3 is straightforward given Proposition 3. When final goods are close substitutes, the associated degrees of differentiation at the supplier level do not satisfy the condition $\gamma \in [0, \gamma')$, thus retailers cannot benefit from the agency structure and the two parties have consistent preference for the wholesale structure. Such condition, however, is satisfied when final goods are perfectly differentiated, retailers are better off under the agency structure, therefore the two parties have conflicting preferences. Beyond the polar cases, a more general observation is given below.

Observation 4. Suppose that both $\beta$ and $\gamma$ are independently and equally likely to take up any value over the interval $[0,1)$. Then in the absence of side payments, there are more combinations of $\beta$ and $\gamma$ such that suppliers and retailers have conflicting preferences, rather than consistent preference over business formats.

Observation 4 is driven by that retailer profits are relatively higher under the agency structure for more combinations of $\beta$ and $\gamma$. With combinations of $\beta$ and $\gamma$ that fall in the grey area in Figure 3, both parties obtain relatively higher profits under the wholesale structure, therefore they have consistent preference for it. With combinations of $\beta$ and $\gamma$ fall in the white area, however, retailer profits are relatively higher under the agency structure thus the two parties have conflicting preferences. Note that since suppliers are always better off under the wholesale structure, when side payments are not allowed, the two parties would never have consistent preference for the agency structure.

4.3 Further Discussion

Suppose that the vertical structure under which a market operates is determined by firms in the market. Then firms’ preferences over business formats matter, primarily because they are crucial in the decision-making, which thereafter affects the welfare conditions. Further, if one structure is chosen over the other, then the knowledge on firms’ preferences may be useful in revealing some characteristics of the market, as well as making conjectures on the power relations between suppliers and retailers. To be more specific, when suppliers and retailers have
conflicting preferences, then it must be that the chosen structure makes firms at one level of the market better off and at the other level worse off. It implies that, ceteris paribus, suppliers are in a relatively stronger position if the market operates under the wholesale structure, and retailers are in a relatively stronger position if the market operates under the agency structure.

Figure 5. Structure choice and social surplus

Now we discuss when the market’s chosen business format is consistent with the social planner’s preferences. Figure 5 put together firms’ preferences and the social planner’s preferences.\textsuperscript{15} If the wholesale structure is adopted because both suppliers and retailers prefer it, then the associated effects on social surplus are desirable given combinations of $\beta$ and $\gamma$ in area a), and are adverse given those in area b). If the wholesale structure is adopted because suppliers are in relatively stronger bargaining position when preferences are different, the associated effects on social surplus are desirable given combinations of $\beta$ and $\gamma$ in area c), and are adverse given those in area d). Finally, if the agency structure is adopted, this must be that retailers are in relatively stronger bargaining position, the associated effects on social surplus are desirable given combinations of $\beta$ and $\gamma$ in area d), and are adverse given those in area c).

Overall this suggests that whether a chosen business format is optimal from the social planner’s perspective depends on, ceteris paribus, the degrees of product differentiation in the market. However, if the degrees of differentiation are endogenous and can increase, say, with investment on R&D or advertising, then when cost of such investment is trivial, following Proposition 2, a social planner clearly favours area d). Since the agency structure is more

\textsuperscript{15} Suppliers and retailers have consistent preference for the wholesale structure in the grey area, a) and b), and have conflicting preferences outside the grey area, c) and d). A social planner prefers the wholesale structure above the solid line in the checked area, a) and c), and prefers the agency structure below the solid line, b) and d).
efficient when goods and services are sufficiently differentiated, when suppliers and retailers have conflicting preference over business formats, the social planner would prefer when retailers can make the decision. This in turn implies that the loss of social surplus would be large if the degrees of differentiation in the market are high but suppliers are in a relatively stronger position, as suppliers will choose the wholesale structure which maximises their own profits.

5. Conclusion

This study is motivated by the prevalence of the agency structure in some supply and distribution chains in the recent years. We seek to provide a baseline comparison of the wholesale structure and the agency structure, with focuses on how welfare conditions and firms’ preferences over business formats are related to the degrees of differentiation in vertically-related markets. We do so by employing a bilateral duopoly model with product differentiation at both the supplier level and the retailer level of the market.

We find that, relative to the wholesale structure, the agency structure leads to lower retail prices, higher demand and higher consumer surplus. Industry profits and social surplus are both relatively higher under the wholesale structure when final goods presented in the market are close substitutes, but are both maximised under the agency structure when final goods are perfectly differentiated. With regard to firms’ preferences, suppliers always prefer the wholesale structure whereas retailers prefer the agency structure if the degrees of differentiation at the supplier level are not too low. It follows that suppliers and retailers have consistent preferences for the wholesale structure if the degrees of differentiation at the supplier level are sufficiently low, and they have conflicting preferences otherwise.

Our results suggest that if the degrees of differentiation at both levels of the market are high, the agency structure is effectively a more efficient business format, therefore a social planner would wish retailers to choose their preferred structure when suppliers and retailers have conflicting preferences. While competition authorities in general believe that competition benefits consumers, we show that, although firms engaging in the agency structure may have incentives that are better aligned, they can offer not only cheaper prices but also final goods that are more differentiated, which benefit consumers.

Unlike the traditional views on vertical relations where retailers are often considered to be perfectly competitive and possess little market power, and vertical restraints including RPM are seen as contractual obligations imposed by suppliers who exercise their powers and keep
retailer margins low, the rise of the agency structure implies that, ceteris paribus, retailers are in strong position. Since suppliers are always better off under the wholesale structure thus have no incentive to switch, retailers wanting to impose the agency contracts have to possess relatively higher bargaining power. This suggests a role for revisit with respect to the understanding on the relative power relations in supply and distribution chains, especially with the rise of online markets.

In this paper we seek to show that, sometimes, knowing which structure a market has chosen is per se useful, as a market’s chosen structure can reveal interesting characteristics about itself. These results can be applied to examine and understand the choice of business formats. The typical values of parameters can be calibrated for the purpose of empirical studies.

This simple framework can be extended to include industry-specific issues such as complementary goods and side payments. While side payments are ruled out in this paper, they may be the remedy for social loss caused by the lack of incentives of suppliers to adopt the agency structure. When side payments are possible, since industry profits are higher under the agency structure when degrees of differentiation are high, retailer can make credible offers.

Further research can hopefully add bargaining component and have business format as the endogenous outcome of bargaining. Also, this paper is silent about how investment interacts with different business formats and its welfare implications, further research can focus on the role of investment in increasing product differentiation.

References


**Appendices**

**A. Proof of Corollary 1, Section 3.2**

If $\beta$ was 1, then given *Lemmata 1* and 2, it is easy to verify that
and

It is straightforward that

That is, $\pi^{S^I} = \pi^A$ and $\pi_{R_i} = \pi_{R_iA}$. The equilibrium outcomes under the two structures are the same at $\beta = 1$. However, this scenario is ruled out given $\beta \in [0, 1)$. It follows that, as $\beta \to 1$, the equilibrium outcomes under the two structures are approximately the same.

B. Proof of Lemma 3, Section 3.2

We take the partial derivatives of $q^*$ and $q^*_A$ with respect to $\beta$ and $\gamma$ respectively

$$
\frac{\partial q^*}{\partial \beta} = \frac{-1 + 2\beta}{(1 + \gamma)(2 - \gamma)(1 + \beta)^2(2 - \beta)^2},
$$

$$
\frac{\partial q^*}{\partial \gamma} = \frac{-1 + 2\gamma}{(1 + \beta)(2 - \beta)(1 + \gamma)^2(2 - \gamma)^2},
$$

$$
\frac{\partial q^*_A}{\partial \beta} = \frac{1}{(1 + \gamma)(2 - \gamma)(1 + \beta)^2},
$$

$$
\frac{\partial q^*_A}{\partial \gamma} = \frac{-1 + 2\gamma}{(1 + \beta)(1 + \gamma)^2(2 - \gamma)^2}.
$$

It is straightforward that $\partial q^*/\partial \beta < 0$ if $\beta < 1/2$ and $\partial q^*/\partial \gamma < 0$ if $\gamma < 1/2$; $\partial q^*_A/\partial \beta < 0$, and $\partial q^*_A/\partial \gamma < 0$ if $\gamma < 1/2$.

Figure 1 illustrates the countervailing effects that $\beta$ has on equilibrium demand under the wholesale structure, $q^*$, as an example.
Figure 1. The U-shape relationship between $\beta$ and $q^*$

Holding $\gamma$ constant, when $\beta \in [0, 1/2)$, the direct substitution effect dominates and demand is decreasing in $\beta$; when $\beta \in (1/2, 1)$, the indirect price effect dominates and demand is increasing in $\beta$. Therefore we observe a U-shape relationship, the lowest point of which is at $\beta = 1/2$. □

C. Proof of Proposition 1, Section 4.1

The comparisons of equilibrium retail prices and demand under the two structures are straightforward. Given Lemma 1 and 2, $p^* > p_A^*$ if $[(1 - \beta)(2 - \gamma) + (1 - \gamma)]/(2 - \beta)(2 - \gamma) - (1 - \gamma)/(2 - \gamma) > 0$, that is if $(1 - \beta)/(2 - \beta)(2 - \gamma) > 0$, which always holds given $\beta, \gamma \in [0, 1)$. $q^* = 1/(1 + \beta)(2 - \beta)(1 + \gamma)(2 - \gamma)$ and $q_A^* = 1/(1 + \beta)(1 + \gamma)(2 - \gamma)$. It is clear by comparing the denominators that $q^* < q_A^*$ always holds given $\beta, \gamma \in [0, 1)$.

We denote the symmetric equilibrium level of consumer surplus under the wholesale structure as $CS$ and that under the agency structure as $CS_A$, then

$$CS = \frac{4}{(1 + \beta)(1 + \gamma)(2 - \beta)(2 - \gamma)} - \frac{1}{2(1 + \beta)^2(1 + \gamma)^2(2 - \beta)^2(2 - \gamma)^2}$$

$$- \frac{2\beta}{(1 + \beta)^2(1 + \gamma)^2(2 - \beta)^2(2 - \gamma)^2} - \frac{2\gamma}{(1 + \beta)^2(1 + \gamma)^2(2 - \beta)^2(2 - \gamma)^2}$$

$$- \frac{2\beta \gamma}{(1 + \beta)^2(1 + \gamma)^2(2 - \beta)^2(2 - \gamma)^2}$$
\[ CS_A = \frac{4}{(1 + \beta)(1 + \gamma)(2 - \gamma)} - \frac{1}{2(1 + \beta)^2(1 + \gamma)^2(2 - \gamma)^2} \]

\[ \quad - \frac{2\beta}{(1 + \beta)^2(1 + \gamma)^2(2 - \gamma)^2} - \frac{2\gamma}{(1 + \beta)^2(1 + \gamma)^2(2 - \gamma)^2} \]

\[ - \frac{2\beta\gamma}{(1 + \beta)^2(1 + \gamma)^2(2 - \gamma)^2}. \]

We show \( CS_A - CS > 0 \) as below.

\[ CS_A - CS = \frac{(1 - \beta)(8\beta^2\gamma^2 - 4\beta^2\gamma - 8\beta\gamma^2 - 12\beta^2 - 16\gamma^2 + 5\beta + 4\gamma + 29)}{2(1 + \beta)^2(1 + \gamma)^2(2 - \beta)^2(2 - \gamma)^2}. \]

Since \((1 - \beta)/2(1 + \beta)^2(1 + \gamma)^2(2 - \beta)^2(2 - \gamma)^2\) is positive, \( CS_A - CS > 0 \) if

\[ 8\beta^2\gamma^2 - 4\beta^2\gamma - 8\beta\gamma^2 - 12\beta^2 - 16\gamma^2 + 5\beta + 4\gamma + 29 > 0. \]

Simplify the above, we get

\[ -8\beta\gamma^2(1 - \beta) + 4\gamma(1 - \beta^2) + 12(1 - \beta^2) + 16(1 - \gamma^2) + 5\beta + 1 > 0. \]

We look at the first two terms on the left-hand side, \(-8\beta\gamma^2(1 - \beta) + 4\gamma(1 - \beta^2) = (1 - \beta)[-8\beta\gamma^2 + 4\gamma(1 + \beta)] = (1 - \beta)[4\gamma(1 - \beta\gamma) + 4\beta\gamma(1 - \gamma)] > 0 \) given \( \beta, \gamma \in [0, 1] \). The other terms, i.e., \( 12(1 - \beta^2) + 16(1 - \gamma^2) + 5\beta + 1, \) can easily be shown to be positive. Therefore, \( CS_A - CS > 0 \), consumer surplus is relatively higher under the agency structure.

\[ \square \]

**D. Proof of Proposition 2, Section 4.1**

We first show that industry profits decrease in \( \beta \) and \( \gamma \) under both vertical structures. Under the wholesale structure, industry profits \( \pi^I = 2\pi^S_j + 2\pi_{R_j} \), and under the agency structure, industry profits \( \pi^A = 2\pi^S_j + 2\pi_{R_jA} \). We take the partial derivatives of each with respect to \( \beta \) and \( \gamma \) respectively,

\[ \frac{\partial \pi^I}{\partial \beta} = \frac{-4(2\beta^2\gamma - 5\beta\gamma - 4\beta^2 + 2\gamma + 7\beta - 4)}{(1 + \beta)^2(1 + \gamma)(\beta - 2)^3(2 - \gamma)^2}, \]

\[ \frac{\partial \pi^I}{\partial \gamma} = \frac{-4(2\beta\gamma^2 - 5\beta\gamma - 4\gamma^2 + 2\beta + 7\gamma - 4)}{(1 + \beta)(1 + \gamma)^2(2 - \beta)^2(\gamma - 2)^3}, \]

\[ \frac{\partial \pi^A}{\partial \beta} = \frac{4(\gamma - 1)}{(1 + \beta)^2(1 + \gamma)(2 - \gamma)^2}, \]

\[ \frac{\partial \pi^A}{\partial \gamma} = \frac{8(\gamma^2 - \gamma + 1)}{(1 + \beta)(1 + \gamma)^2(\gamma - 2)^3}. \]
\(\partial \pi^I / \partial \beta\) is negative if \(2\beta^2 \gamma - 5\beta \gamma - 4\beta^2 + 2\gamma + 7\beta - 4 < 0\). This is equivalent to if \(\beta < (4\gamma^2 + 4 - 7\gamma)/(2\gamma^2 + 2 - 5\gamma)\), which holds given \(\beta, \gamma \in [0, 1)\). Similarly, \(\partial \pi^I / \partial \gamma < 0\). \(\partial \pi_A^I / \partial \beta < 0\) because \(\gamma - 1 < 0\) and \(\partial \pi_A^I / \partial \gamma < 0\) because \((\gamma - 2)^3 < 0\).

This means that industry profits always increase in the degrees of differentiation in the market. It follow that, to find the maximum of industry profits, we simply need to compare \(\pi^I\) and \(\pi_A^I\) at \(\beta, \gamma = 0\), which are \(3/4\) and \(1\), respectively. Therefore industry profits are maximized at the point of perfect differentiation under the agency structure.

With regard to social surplus, it is denoted as \(S = \pi^I + CS\) under the wholesale structure and \(S_A = \pi_A^I + CS_A\) under the agency structure. Under the agency structure, the partial derivatives are

\[
\frac{\partial S_A}{\partial \beta} = \frac{16\beta \gamma^2 - 6\beta \gamma^2 + 16\gamma^3 - 6\beta \gamma - 6\gamma^2 + 16\beta - 12\gamma + 19}{(1 + \beta)^3(1 + \gamma)^2(\gamma - 2)^3},
\]

\[
\frac{\partial S_A}{\partial \gamma} = \frac{8\beta \gamma^2 - 2\beta \gamma + 8\gamma^2 - 10\beta - 2\gamma - 13}{(1 + \beta)^2(1 + \gamma)^3(2 - \gamma)^2},
\]

which can be easily verified to be negative. Hence \(S_A\) maximises at \(\beta, \gamma = 0\).

Under the wholesale structure, the partial derivatives are

\[
\frac{\partial S}{\partial \beta} = \frac{16\beta^3 \gamma^2 - 24\beta^2 \gamma^2 - 16\beta^3 \gamma - 24\beta \gamma^2 + 6\beta^2 \gamma - 32\beta^3}{-(1 + \beta)^3(1 + \gamma)^2(\beta - 2)^2(2 - \gamma)^2},
\]

\[
\frac{\partial S}{\partial \gamma} = \frac{16\beta^2 \gamma^3 - 24\beta^2 \gamma^2 - 16\beta^3 \gamma - 24\beta^2 \gamma + 6\beta \gamma^2 - 32\beta^3}{-(1 + \beta)^2(1 + \gamma)^3(2 - \beta)^2(\gamma - 2)^3}.
\]

\(\partial S / \partial \beta\) is maximized at \(\beta = 0, \gamma \rightarrow 1\), and \(\partial S / \partial \gamma\) is maximized at \(\beta \rightarrow 1, \gamma = 0\), where they both take the value \(-1/32 - \varepsilon\). Since the highest value they can take over \(\beta, \gamma \in [0, 1)\) is negative, \(S\) is decreasing in \(\beta\) and \(\gamma\).

At point \(\beta = 0, \gamma = 0\), \(S = 55/32\) and \(S_A = 23/8\), social surplus is maximized at the point of perfect differentiation under the agency structure. \(\square\)

**E. Proof of Proposition 3, Section 4.2**

Given Lemmata 1 and 2, \(\pi^S_I > \pi_A^S_I\) if

\[
\frac{2(1 - \gamma)}{(1 + \beta)(1 + \gamma)(2 - \beta)(2 - \gamma)^2} - \frac{2\beta(1 - \gamma)[2\beta - \gamma(1 + \beta)]}{(1 + \beta)(1 + \gamma)(2 - \gamma)^2[2 - \gamma(1 + \beta)]} > 0,
\]

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and that is if the following is positive

\[
\frac{2(1 - \gamma)}{(1 + \beta)(1 + \gamma)(2 - \beta)(2 - \gamma)(2 - \beta - \gamma \beta)} \times [2 - \gamma(1 + \beta) - \beta(2 - \beta)(2\beta - \gamma - \beta\gamma)].
\]

It is easy to verify that the first part is positive given $\beta, \gamma \in [0,1)$, it remains to be checked whether the term inside the square brackets is positive. It is positive if $\gamma < 2(1 - \beta^2 + \beta)/(1 - \beta^2)$, which always holds given $\beta, \gamma \in [0,1)$. Therefore, $\pi^j > \pi_A^j$, the equilibrium supplier profits are relatively higher under the wholesale structure.

**Figure 4.** The relationship between $\gamma$ and $\pi_{R/A} - \pi_R$

With regard to retailer profits, as illustrated in Figure 4, given any $\beta$, $\pi_{R/A} - \pi_R$ is negative for $\gamma \in (\gamma', 1)$, and is positive for $\gamma \in [0, \gamma')$. To find $\gamma'$, we evaluate $\pi_{R/A} = \pi_R$ at $\beta = 0$, and obtain $\gamma' = 3/4$. \(\square\)